

## A Survey of Advanced Logics

**Karthik P. Chandrasekaran and Poovammal Eswaran**

*Computer Science and Engineering SRM University  
Kattankulathur, Kancheepuram District Chennai, Tamilnadu India PIN – 603203  
Tel: +91-044-43066280 [pc\\_karthik@yahoo.com](mailto:pc_karthik@yahoo.com)  
Computer Science and Engineering SRM University  
Kattankulathur, Kancheepuram District Chennai, Tamilnadu India PIN-603203  
[poovammals@gmail.com](mailto:poovammals@gmail.com)*

### Abstract

Knowledge representation uses symbols to represent knowledge. Any manipulation can be done in an automated manner by reasoning systems. The reasoning system uses knowledge base (Knowledge Representation) to carry out manipulation effectively and automatically. In this paper, the suitability of advanced logics in typical applications is highlighted. Advanced logics such as temporal, modal, fuzzy, quantum and causal logics are analyzed. Each and every logic is highly suitable for specific applications and may not be suitable for some other application. It is to logic that scientists, architects, and experts turn to help address the issues of setting up formal properties. The significance of logic comes, to some extent, on account of it's all inclusive character and the rich arrangement of results encompassing it. Logic can likewise assume various parts in programming frameworks. For example, logic can be utilized to formally build up rightness of programs.

**Keywords:** Artificial Intelligence, modal, temporal, fuzzy, quantum, causal, Directed Acyclic Graphs

### 1. Introduction

Information Representation is the area of Artificial Intelligence (AI) concerned with how learning can be exhibited typically and controlled in an automated way by logic programs. It is at the very center of a radical thought regarding how to comprehend intelligence. As opposed to attempting to comprehend or construct brains in the bottom up methodology, we attempt to comprehend or manufacture intelligent behavior from top down methodology. Specifically, we ask what specialists would need to know to carry on insightfully, and what computational components could permit this learning to be made accessible to the operators as needed.

There are simple logics such as the propositional logic and the first-order logic. Propositional logic consists of statements that can take boolean values such as true or false and the conjunctions and disjunctions of their statements. First-order logic consists of two additional quantifiers: “there exists” and “for all”. Propositional and first-order logic can be applied to normal statements. However complicated statements having time-ordering or with qualifiers or cause-effect relationships cannot be represented by these two logics. It is in such cases that the advanced logics are used.

Advanced logics have lots of applications in knowledge representation and reasoning. Temporal logic has applications in time series data such as stock price in stock exchanges etc. Modal logic is extensively used in linguistics. Fuzzy logic is used in control systems such as air conditioners and semi-automatic and automatic washing machines. Quantum logic has applications in quantum encryption and decryption and quantum secret sharing protocol. Causal logic has wide applications in establishing causal relationships between variables in statistics and inference in the medical domain.

## 2. Temporal Logic

In the paper Shakarian and Simari (2009), the authors discuss Annotated Probabilistic Temporal (ATP) logic. Logic of time and probability are expressed through probability distribution over threads. The creators present the linguistic structure of annotated probabilistic logic (ATP) programs which present the idea of a frequency function. An arrangement of complexity results for checking whether the statements are consistent and derivable from basic premises in ATP projects is indicated in their paper. They portray an application utilizing ATP logic programs.

### 2.1 Applications of ATP programs

#### Stock Market Prediction

Many banks find out how stock prices in the future change. They achieve this goal by learning about a lot of indicators and also the historical data on these indicators.

#### Terror Groups

Information on terror groups is available in the website SOMA terror organization.

#### Trains

Railways, airplane and ship operators have a requirement to keep note of the schedule of airplane, the ship and train.

#### Power Grid

Failure analysis of electric lines and transformers is a must in Power grids. The reason is that once the failure of an electric line or transformer is predicted, the company can replace that electrical component with a working component.

Temporal logic can be applied in removing doubts in problems pertaining to philosophy on time. It may also be used for representing semantically, temporal

sentences in natural language. It could also be applied in providing a representation of temporal knowledge in AI. It may also be used in specifying, analyzing formally and verifying the running of computer programs.

### 3. Modal Logic

Modal logic is a formal logic that extends predicate logic to include operators expressing modality. For example, the statement "A is healthy" can be qualified by saying that "A is usually healthy", in which case the term "usually" is a modal. The addition of syntactic property to an axiom in modal logic makes the property true in all circumstances. In current research, Hans van Ditmarsch and Wiebe van der Hoek, (2011), a method is proposed to make the statement true only in local situations, but not in all states. Also all the agents need not know that the statement is true. One can attain the result by adding relational atoms to the language that represent quantification over all formulas. The authors show that this can be achieved for a large modal logic class and a number of syntactic properties.

#### Language and Semantics

The authors work on three languages that are interpreted over Kripke models. Three languages are used, namely an extended modal language  $L$ , first-order language  $L^1$  and second-order language  $L^2$ . A set of modality labels  $A = \{a_1, a_2, \dots, a_A\}$  are assumed. The modal language will provide modalities  $[a]$  and in the other two languages, a binary relation  $R_a$  for each  $a \in A$ . The last two languages assume a set of variables  $X = \{x, y, \dots\}$ . Both  $L_1$  and  $L_2$  don't have constants. The authors in addition use  $S = \{P, P_1, P_2, \dots, Q, Q_1, Q_2\}$  as unary predicates. The authors assume that for each predicate  $P$  in  $S$ , there exists an atomic proposition  $p$  belonging to  $S$ , that forms the building block for the modal language  $L$ . Apart from that, a finite set  $c = \{Z_1, Z_2, \dots, Z_n\}$  of relational atoms is assumed to exist for the modal language  $L$ . The authors denote the three languages as  $L(A, s, c)$ ,  $L_1(A, X)$  and  $L_2(A, S, X)$ .

#### Definition (Modal Language)

Let the sets  $A$ ,  $S$  and  $c$  be defined as above. The modal language  $L(A, S, c)$  is defined as follows:

$$Z ::= p \mid \neg p \mid p \wedge p \mid [a]p \mid c(a_1, a_2, \dots, a_n).$$

Where  $a, a_1, a_2, \dots, a_n \in A$ ,  $p \in S$  and  $S$  is an  $n$ -ary relational atom in  $c$ . Formula  $\langle a \rangle p$  denotes  $\neg[a] \neg p$  and the usual definitions for disjunction, implication and bi-implication follow.

A formula without occurrences of relational atoms is called a purely modal formula. The authors use the notation  $a^1$  for the list  $a_1, \dots, a_n$  and  $p^1$  for  $p_1, \dots, p_k$ . The expression  $a \in a^1$  means that  $a$  is one of the labels for the list  $a^1$  and similarly for  $p$  and  $p^1$ . Any list  $x^1 = x_1, \dots, x_n$  with each  $x_i$  taken from some set  $X$ .

### Definition (First and Second Order Logic)

Let A and X be provided. Define a language  $L^+(a, X)$  as

$$q = R_{ax}y \mid V y q \mid \neg q \mid q \& q$$

with  $a \in A$  and  $x, y \in X$ . The first order language  $L^1(A, X)$  is one free-variable sublanguage of  $L^+$  i.e. the sublanguage of  $L^+$  that consists of all formulas with at most one variable not in the scope of a quantifier. If  $q \in L^1(A, X)$ , has  $x$  as its only free variable, and if  $a_1, a_2, \dots, a_n$  are all the modality labels occurring in  $q$ , we will also write  $q(a')(x)$  for  $q$ .

Finally given A, S and X, the authors define the second-order language  $L^2(A, S, X)$  as the one free variable fragment of

$$q' = P(x) \mid R_{ax}y \mid V y q' \mid V P q' \mid \neg q' \mid q' \& q'$$

with  $P \in S$ ,  $x, y \in X$  and  $a \in A$ . In  $L^1(A, X)$ , and  $L^2(A, S, X)$ , existential quantification and implication are defined in a standard way.

Modal logic may be applied in belief logic. It may also be used in temporal expressions such as present or past tense. Modal logic finds application in moral or deontic sentences like “it is imperative that” and “it may be allowed that” and other such expressions. Understanding the modal logic leads to analyzing formally arguments in philosophy. Formal arguments are generally in common usage and may also be ambiguous. Modal logic may be used for resolving such ambiguity. Modal logic is extensively used in linguistics.

## 4.. Fuzzy Logic

In the paper Schockaert and Vermeir (2012), the authors define fuzzy equilibrium logic as a generalized form of Pearce logic and fuzzy answer set programming. The authors then reduce the problem in discovering fuzzy equilibrium logic models to settling a specific bi-level mixed integer program (biMIP).

### 4.1 Answer Set Programming

An answer set system is a set of sentences such as

$$c \ d_1, \dots, d_n, \text{not } e_1, \dots, \text{not } e_m$$

where  $c, d_1, \dots, d_n, e_1, \dots, e_m$  are atoms from fixed set  $A_t$ . An interpretation is a subset of atoms. A problem is represented by an answer set program. The fundamental thinking behind the answer set programming is to assign specific models of a system as resolutions of the program. The subsequent models form the answer sets.

## 4.2 Equilibrium Logic

Pearce presented the equilibrium logic for expansion to propositional statements of the answer sets. The hypothesis is framed taking into account expansion of logic with solid negatives. The logic that is expanded, is incorporated in traditional logic.

## 4.3 Fuzzy Answer Set Programs (FASP)

FASP adds to ASP the idea of fuzzy logic. Let  $F_n$  for each  $n \in N$  be an arrangement of  $[0, 1]$  to  $[0, 1]$  capacities that are strictly increasing or decreasing and let  $F = \bigcup_n F_n$ .  $F$  contains connectives from fuzzy logic. Every atom  $a \in A_t$  is an equation. Also if  $a_1, \dots, a_n$  are equations, and  $f \in F$ , then  $f(a_1, \dots, a_n)$  is a function. A FASP program is defined by rules as shown below:

$a \alpha,$

where a formula is denoted by  $\alpha$  and an atom is denoted by  $a \in A_t$ . The guidelines given above are called constraints. If a constant from  $[0, 1]$  is represented by  $\alpha$ , the rule is named fact.

Fuzzy logic may be applied in mathematics in the area of logic, algebra, topology and graph theory. Fuzzy logic can be used in algorithms such as clustering, control algorithms and programming in mathematics. It also finds application in well-known models like maintenance, inventory control and the model of transportation. Finally fuzzy logic can be applied to real-world problems of various types. Fuzzy logic is used in control systems such as air conditioners and semi-automatic and automatic washing machines.

## 5. Quantum Logic

The formal portrayal of quantum mechanics implies a speculation of traditional probability in which the Boolean algebra of events is changed by the projection operators on a Hilbert space. The quantum-probabilistic formalism understands that each physical framework is limited to a Hilbert space  $H$ , whose range is the set of conceivable values of  $C$ . The observables are denoted by the operators  $C$  and  $D$ . The observables are comparable if and only if  $C$  and  $D$  commute.

### 5.1 Probability Measures and Gleason's Theorem

#### Definition

A likelihood measure on  $L(H)$  maps  $z$  to  $L[0, 1]$  such that  $z(l) = l$  for a consecutive pair-wise orthogonal projections  $P_i$ ,  $i = 1, 2, \dots$

$$Z(+_i P_i) = \sum_i (P_i).$$

$U$  is a unit vector of  $H$  and  $Z_u(P) = \langle P_u, u \rangle$ . The solution is the likelihood that state  $u$  assigns the value 1 to  $P$ .

### Gleason's Theorem

In the paper Richman F. and Bridges D, 1999,  $H$  represents a measurement more than two. At that point each countable likelihood measure on  $L(H)$  has the structure  $Z(P) = \text{Tr}(WP)$ , for a density operator  $W$  on  $H$ .

### 5.2 Operational Quantum Logic

Quantum mechanics is a speculation about the possible quantifiable distributions of consequences of particular estimations, and its non-settled "rationale" essentially reflects the way that not all recognizable phenomena can be observed in the meantime. In perspective of this, the course of action of probability bearing events (or propositions) is less rich than it would be in customary probability speculation, and the set of possible distributions, in like way, less constrained.

Quantum logic can be applied to provide proofs of correctness for teleportation, quantum key distribution, super-dense coding and quantum secret-sharing protocol. Teleportation means transferring quantum information like the atom state from one place to other place. Quantum key distribution allows two persons to create a secret key which is randomly generated. The secret key is known only to the two parties. The secret key is used for encryption and decryption. One of the important properties of quantum encryption is that the users can find out whether somebody is trying to listen in the communication channel. Super-dense coding is a method for sending two bits of information with a single qubit. Quantum secret sharing is a technique for sending a single secret to a number of participants. All the participants are assigned a portion of the secret.

### 6.Causal Reasoning

Causality, Judea Pearl (2009), means law-like necessity, though probabilities indicate exceptionality, uncertainty and absence of consistency. Still, there are two convincing purposes behind beginning with, and focusing on probabilistic investigation of causality; one is genuinely direct, the other more inconspicuous. We say, for instance, (Suppes 1970) "neglectful driving results in accidents" or "you will fall on account of your apathy". One knows that the precursors just have a tendency to make the outcomes more probable, not completely certain. Any hypothesis of causality that deals with such expressions should in this manner be cast in a dialect that recognizes different shades of probability-to be specific, the dialect of probabilities. We take note of the fact that probability hypothesis can be applied in the utilization of causal demonstrations, including financial aspects, the study of disease transmission, humanism, and brain science. In these areas, examiners are concerned not simply with the vicinity or deficiency of causal associations, but with the relative qualities of those associations and with methods for gathering those associations from perceptions. Likelihood hypothesis gives both the standards and the method for adapting to and drawing conclusions from such perceptions. Probability hypothesis permits us to concentrate on the primary issues of causality without needing to adapt to mysteries of this kind.

## 6.1 Theory of Inferred Causation

A self-governing framework endeavoring to construct a workable model of its surroundings can't depend only on prearranged causal information(Cartwright 1995, Humphreys and Freedman 1996, Cartwright 1999, Korb and Wallace 1997, McKim and Turner 1997, Robins and Wasserman 1999). Rather, it must have the capacity to make an interpretation of direct perceptions to circumstances and cause-effect relationships. We should locate a computational model that imitates this discernment. Priority in time is typically thought to be crucial for characterizing causation, and it is without a doubt a standout amongst the most critical signs that individuals utilize to recognize causal from different sorts of affiliations. Appropriately, most speculations of causation provide a necessity that a cause goes before its effect in time. Yet temporal data alone can't recognize certifiable causation from spurious associations. The barometer indicator falls before it rains yet does not bring about the downpour. Hence, the factual and philosophical writing has stubbornly cautioned experts that, unless one knows ahead of time all causally pertinent elements or unless one can deliberately control a few variables, no veritable causal surmising is conceivable. Neither one of the conditions is feasible in typical learning situations and the inquiry remains how causal information is ever obtained for a fact.

## 6.2 The Causal Modeling Framework

### 6.2.1 Causal Structures

A causal structure (Rebane and Pearl, 1987, Pearl 1988) of an arrangement of variables A corresponds to a non-cyclic graph where each node identifies with a segment of A and each association gives a causal relationship among the variables. A causal structure serves as an outline for confining a "causal model"-a definite determination of how every variable is influenced by its parents in the acyclic graph.

### 6.2.2 Inferred Causation

If a directed path exists between A and B in all minimum structures that correspond with the information, A is said to causally influence B. Here we liken a causal structure with a scientific hypothesis, since both contain an arrangement of free parameters that can be changed in accordance to fit the information.

Causal reasoning has applications in medicine in the determination of the efficacy of new medicines. Medicines can be tested in two groups of patients with one group taking the medicine and the other a placebo. One can determine if the medicine indeed cured the illness. Causal reasoning may also be used in law where a judge has to determine based on the available evidence, whether the person who is convicted is indeed the cause of the crime. It is applicable to statistics where we need to determine if one variable is the cause of the other variable or both the variables are just correlated without a causal relationship.

## 7.DISCUSsION

Temporal logic can be applied in removing doubts in problems pertaining to philosophy on time. It may also be used for representing semantically, temporal

sentences in natural language. It could in addition be used in providing a representation of temporal knowledge in AI. It may also be used in specifying, analyzing formally and verifying the running of computer programs. Temporal logic cannot be used in the area of belief logic since it contains only temporal operators and not operators representing degree of belief. Modal logic can be applied in belief systems. Modal logic may also be used in temporal expressions such as present or past tense. Modal logic finds application in moral or deontic sentences like "it is imperative that" and "it may be allowed that" and other such expressions. Understanding the modal logic leads to analyzing formally arguments in philosophy. Formal arguments are generally in common usage and may also be ambiguous. Modal logic may be used for resolving such ambiguity.

Modal logic cannot be applied to situations where one needs to represent imprecise or fuzzy data. Fuzzy data means data which represents a range of values. Fuzzy logic can be applied in such places. Fuzzy logic may be applied in mathematics in the area of logic, algebra, topology and graph theory. Fuzzy logic can be used in algorithms such as clustering, control algorithms and programming in mathematics. It also finds application in well-known models like maintenance, inventory control and the model of transportation. Finally fuzzy logic can be applied to real-world problems of various types. Fuzzy logic is used in control systems such as air conditioners and semi-automatic and automatic washing machines. Fuzzy logic cannot be used for denoting logic which can contain variables existing in several states at the same time. Quantum logic can be applied here.

Quantum logic can be applied to provide proof of correctness for teleportation, quantum key distribution, super-dense coding and quantum secret-sharing protocol. Teleportation means transferring quantum information like the atom state from one place to other place. Quantum key distribution allows two persons to create a secret key which is randomly generated. The secret key is known only to the two parties. The secret key is used for encryption and decryption. One of the important property of quantum encryption is that the users can find out whether somebody is trying to listen in the communication channel. Super-dense coding is a method for sending two bits of normal information with a single qubit. Quantum secret sharing is a technique for sending a single secret to a number of participants. All the participants are assigned a portion of the secret. Quantum logic cannot be applied in areas of logic where we need to represent cause-effect relationship among the logical variables. Causal reasoning may be applied here.

Causal reasoning has applications in medicine in the determination of the efficacy of new medicines. Medicines can be tested in two groups of patients with one group taking the medicine and the other a placebo. One can determine if the medicine indeed cured the illness. Causal reasoning may also be used in law where a judge has to determine based on the available evidence, whether the person who is convicted is indeed the cause of the crime. It is applicable in statistics where we need to determine if one variable is the cause of the other variable or both the variables are just correlated without a causal relationship.

## 8. CONCLUSION

In this paper, Annotated Probabilistic Temporal (ATP) logic is discussed. The logic of time and probability are expressed through probability distribution over threads. Three languages namely an extended modal language  $L$ , first-order language  $L^1$  and second-order language  $L^2$  are interpreted in Kripke logic. Fuzzy equilibrium logic is produced as a speculation of Pearce equilibrium logic and fuzzy answer set programming. Quantum and causal logics are explained.

## REFERENCES

- [1] Cartwright N, 1995, "False idealization: A philosophical threat to scientific method", *Philosophical Studies*, vol. 77, pp. 339-352.
- [2] Cartwright N. 1999, "Causality: Independence and Determinism", *Causal Models and Intelligent Data Management*, pp. 51-63, Springer-Verlag, Berlin.
- [3] Ditmarsch H. and van der Hoek W., 2012. "Local Properties in Modal Logic", *Artificial Intelligence*, pp. 133-155.
- [4] Humphreys. P and Freedman D, 1996. "The grand leap", *British Journal for the Philosophy of Science*, Vol. 47, pp. 113-123.
- [5] Korb K.B and Wallace C.S, 1997, "In search of the philosopher's stone: Remarks on Humphrey's and Freedman's critic on critique of causal discovery", *British Journal for the Philosophy of Science*, vol. 48, pp. 543-553.
- [6] Mckim V.R. and Turner S.P., 1997, "Causality in crisis?", University of Notre Dame press, Notre Dame.
- [7] Pearl J., 1988, "Graphs, Causality and Structural equation models", *Sociological methods*, pp.226-284.
- [8] Pearl J., 2009, "Causality – Models, Reasoning and Inference", Cambridge University Press.
- [9] Rebane G. and Pearl J, "The recovery of causal poly-trees from statistical data", *Proceedings of the third workshop on Uncertainty in AI*, pp. 222-228, Seattle, WA.
- [10] Richman F. and Bridges D, 1999, "A constructive proof of Gleason's theorem", *Journal of Functional Analysis*, Volume 162, Issue 2, pp. 287-312.
- [11] Robins J.M. and Wasserman L, "On the impossibility of inferring causation from association without background knowledge", *Computation, Causation and Discovery*, pp. 305-321, AAAI / MIT press, Cambridge MA.
- [12] Schockaert, S., Janssen, J., and Vermeir, D. 2012, "Fuzzy equilibrium logic: Declarative problem solving in continuous domains", *ACM Transactions on Computational Logic*, Volume 13, Issue 4, Article 33, 39 pages.
- [13] Shakarian. P and Simari G, 2009. "Annotated Probabilistic Temporal Logic", *ACM Transactions on Computational Logic*, Vol. 5, pp. 1-42.
- [14] Suppes P, 1970, "A Probabilistic Theory of Causality", North-Holland Publishing company, Amsterdam.

