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A Ranking And Selection Approach For Volatility In Financial Market

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Abstract

In this paper, ranking and selection approach of Shanthi S. Gupta (1962) is used to select a subset of stock market populations containing the population with the smallest variance, i.e., volatility. Let $T_1, T_2, ..., T_k$ be a k given normal population of stock market return with unknown volatility $\sigma_1^2, \sigma_2^2, ..., \sigma_k^2$ respectively, where each $\sigma_i^2 > 0$ and with all means known or all means unknown. Using the procedure R, the volatilities are ranked to get a subset from k populations which include the best population, i.e., one with the smallest volatility.

Key words Ranking and selection, procedure R, variance, stock market return, volatility.

1.Introduction

In recent times, the issue of volatility and risk have become increasingly important to financial practitioners, market participants, regulators and researchers. As the world's financial system has become more unstable, it is important for a cautious investor to predict the volatility and best investment.

Suppose we want to compare k stock market return data populations $\pi_1, \pi_2, ..., \pi_k$ with the same underlying distribution where each population π_i is characterized by the same population parameter θ_i. $\theta_{[1]} \le \theta_{[2]} \le ... \le \theta_{[k]}$ denote the true, unknown ordering of the parameters. The goal is to select the best population, where the best is decided in terms of θ, the population parameter of our interest.

Gupta and Sobel[1962], has given a multiple decision approach to the problem of selecting a subset from k given normal populations which includes the best population. The population variances are unknown and the population means may be known or unknown. Based on a common number of observations from each

population, a procedure R is defined, which selects and assigns ranks for the unknown population variances and finally gives a best population with smallest variance.
This paper is organized as follows. Section 2 gives the procedure R. In section 3, the financial data sets from S&P 500, DJIA, Oil Companies, spectrum and mobile companies, whose closing prices ranging from 03.06.2014 to 14.08.2014, are considered for the ranking purpose and they are ranked using the Procedure R. Section 4 gives the conclusion.

2. The procedure R [Gupta &Sobel 1962]

Let $\pi_1, \pi_2, ..., \pi_k$ denote k given normal populations with unknown variances $\sigma_1^2, \sigma_2^2, ..., \sigma_k^2$ respectively, where each $\sigma_i^2 > 0$, and with all means known or all means unknown. The ordered variances are denoted by $\sigma_{[1]} \le \sigma_{[2]}^2 \le ... \le \sigma_{[k]}^2$ (equalities being allowed for mathematical convenience). It is assumed that there is no a priori information available about the correct pairing of the k given populations and the ordered scale parameters $\sigma_{[i]}^2$.
The population with variance equal to $\sigma_{[1]}^2$ is called the best population. The goal is to select a subset of the k populations containing the best population. Any such selection will be called a correct selection. Then the problem is to find a rule R such that for a pre-assigned probability P^*
P(correct selection / R) $\ge P^*$regardless of the true unknown values of the population variances. It is assumed that the same number of n observations will be taken from each population.
From each population π_i, i = 1,2,…,k we take n observations and if the mean μ_i is known, we compute the statistic $s_i^2 = \frac{1}{n}\sum_{j=1}^{n}(x_{ij} - \mu_i)^2$, i = 1,2,…,k which, when multiplied by $\frac{\gamma}{\sigma^2}$ (here $\gamma = n$) has the χ^2distribution with γ degrees of freedom. If the population means are unknown then we use $s_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \bar{x}_i)^2$ and the values of γ is taken as n-1. These statistics s_i^2,
i = 1,2,…,k from a set of sufficient statistics for the problem and the rule R depends only on these statistics.
Let the ordered values of the k observed sample variances s_i^2(i=1,2,…,k), all based on a common number γ of degrees of freedom, be denoted by $s_{[1]}^2 \le s_{[2]}^2 \le ... \le s_{[k]}^2$.
The procedure R is then defined as follows:

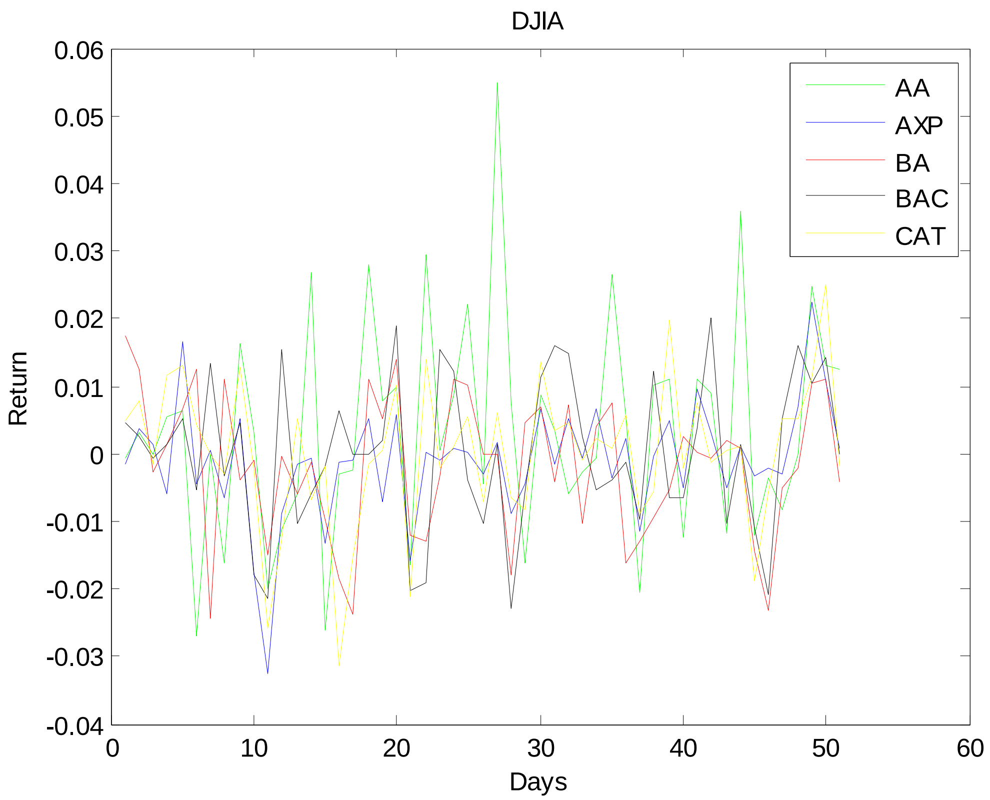
Procedure R:

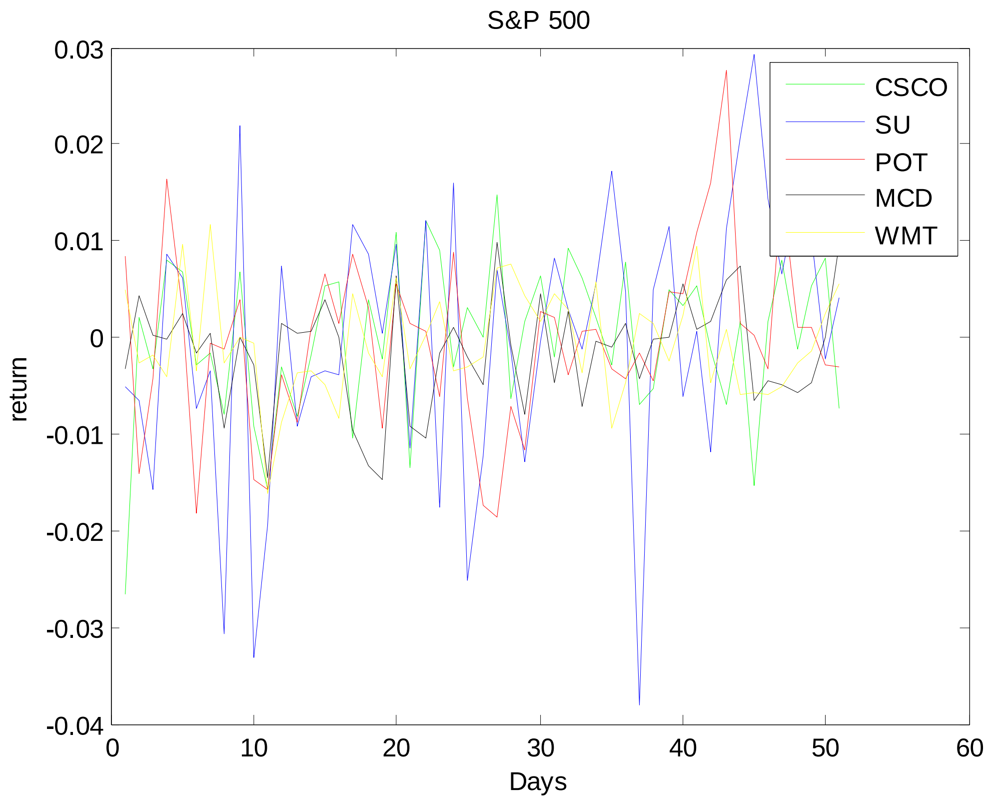
Retain π_i in the selected subset if and only if $s_i^2 \le \frac{s_{[1]}^2}{c}$, where c ($\gamma$,k, P^*) is a constant with $0 < c \leq 1$ which is determined in advance of experimentation. The constant c is chosen to be the largest value which satisfies the basic probability requirement P(correct selection / R) $\geq$ P^* for all true configuration $\sigma^2 = (\sigma_1{}^2, \sigma_2{}^2, \ldots, \sigma_k{}^2)$. Tables of c values for γ =2(2)50, k=2(1)11 and P^* = 0.75, 0.90, 0.95 and 0.99 are given in appendix(Gupta&Sobel,1962). These c values can also be regarded as percentage points of a smallest studentized χ^2 statistic.

3. Data Analysis

The data used in this study consists of daily closing prices ranging from 03.06.2014 to 14.08.2014 of S&P 500, DJIA, Oil Companies, Spectrum and Mobile companies The data are obtained from the web page www.yahoofiance.com. The stock market return is calculated using the formula, $r_t = \ln \frac{p_t}{p_{t-1}}$,where p_t and p_{t-1} refer to the level of index at date t and t-1 respectively.

Daily returns ranging from 03.06.14 to 14.08.14




S&P 500
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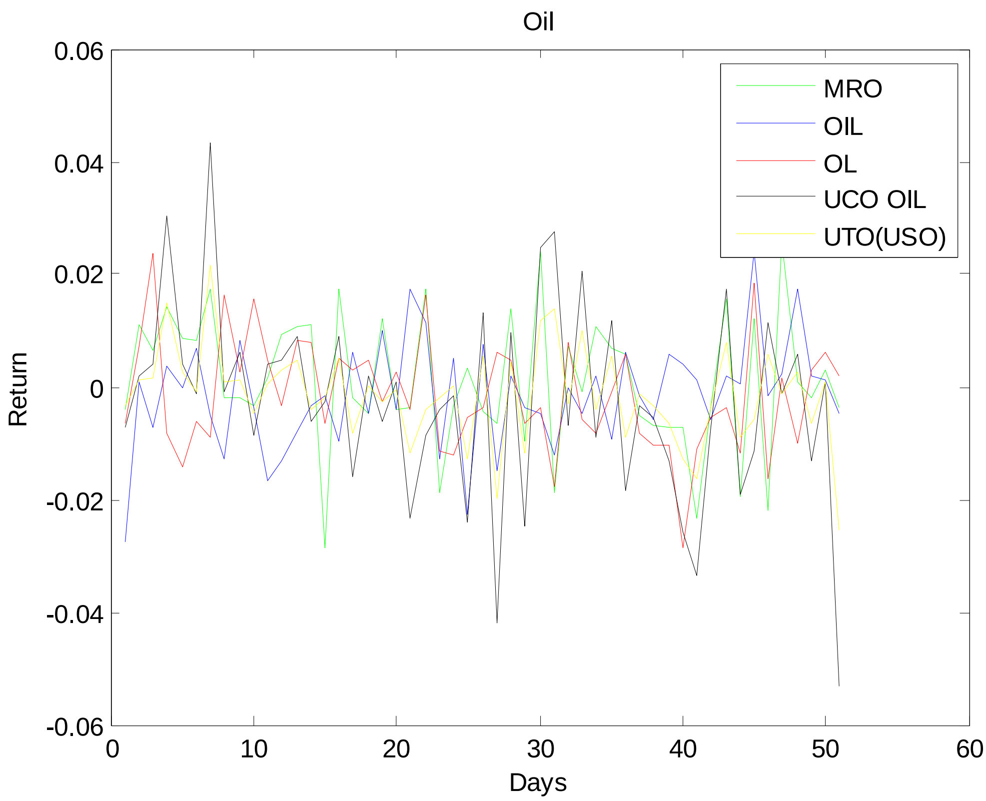
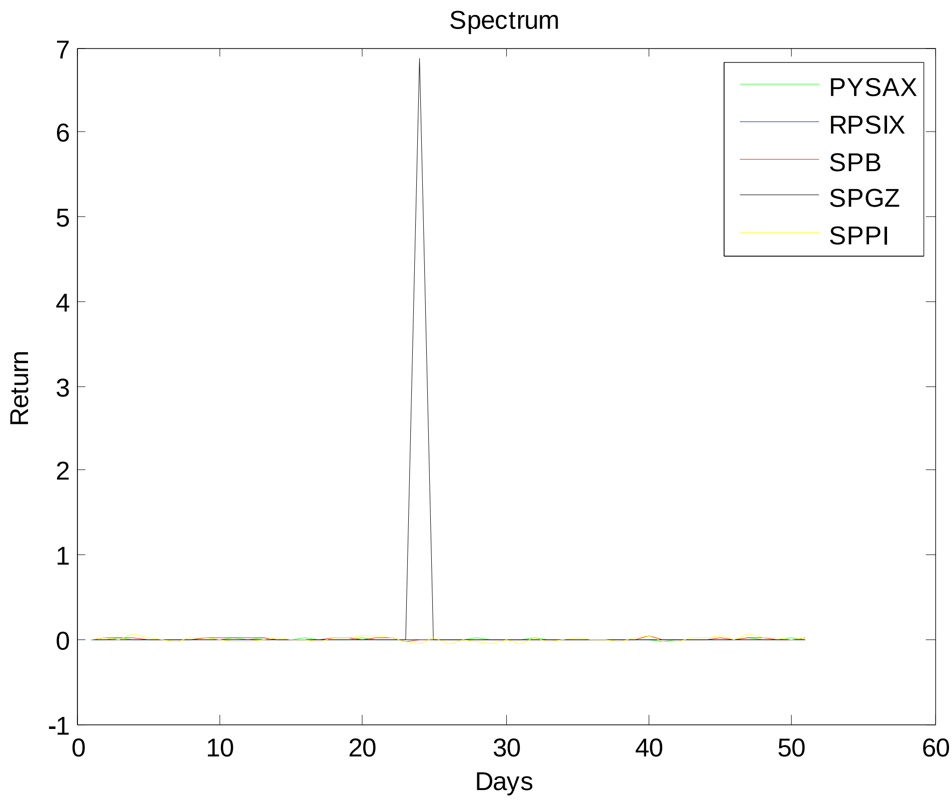
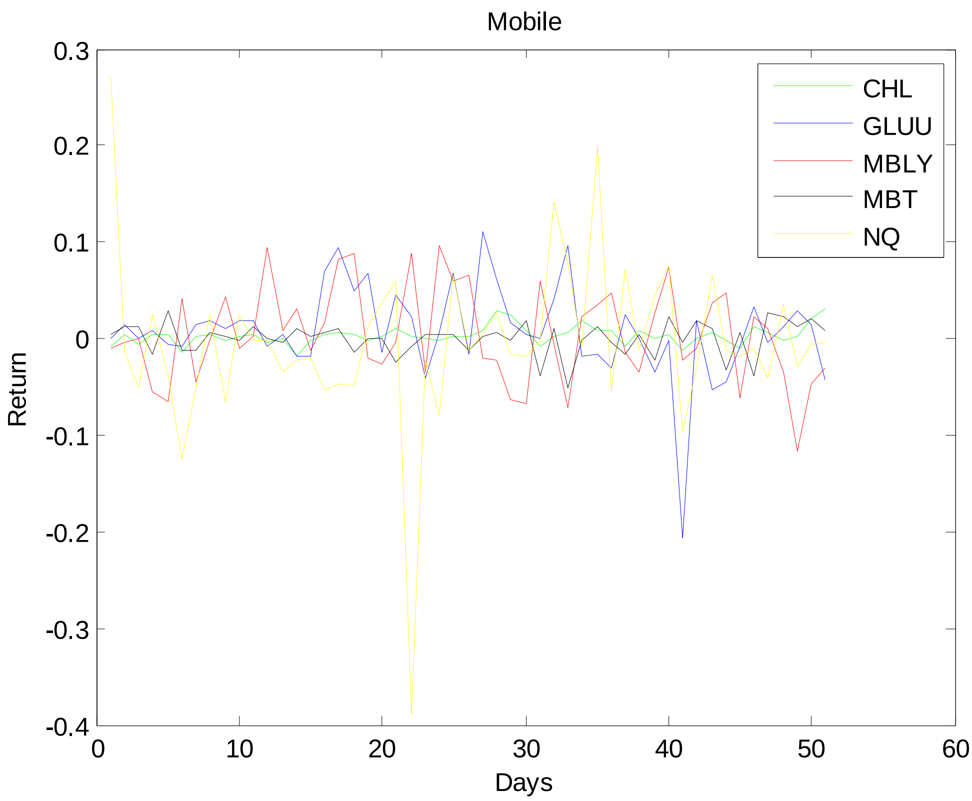
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
return
0
10
20
30
40
50
60
Days
CSCO
SU
POT
MCD
WMT


Oil
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Return
0
10
20
30
40
50
60
Days
MRO
OIL
OL
UCO OIL
UTO(USO)


Spectrum
PYSAX
RPSIX
SPB
SPGZ
SPPI
Return
Days
7
6
5
4
3
2
1
0
-1
0
10
20
30
40
50
60


Mobile
CHL
GLUU
MBLY
MBT
NQ
Return
Days
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
10
20
30
40
50
60

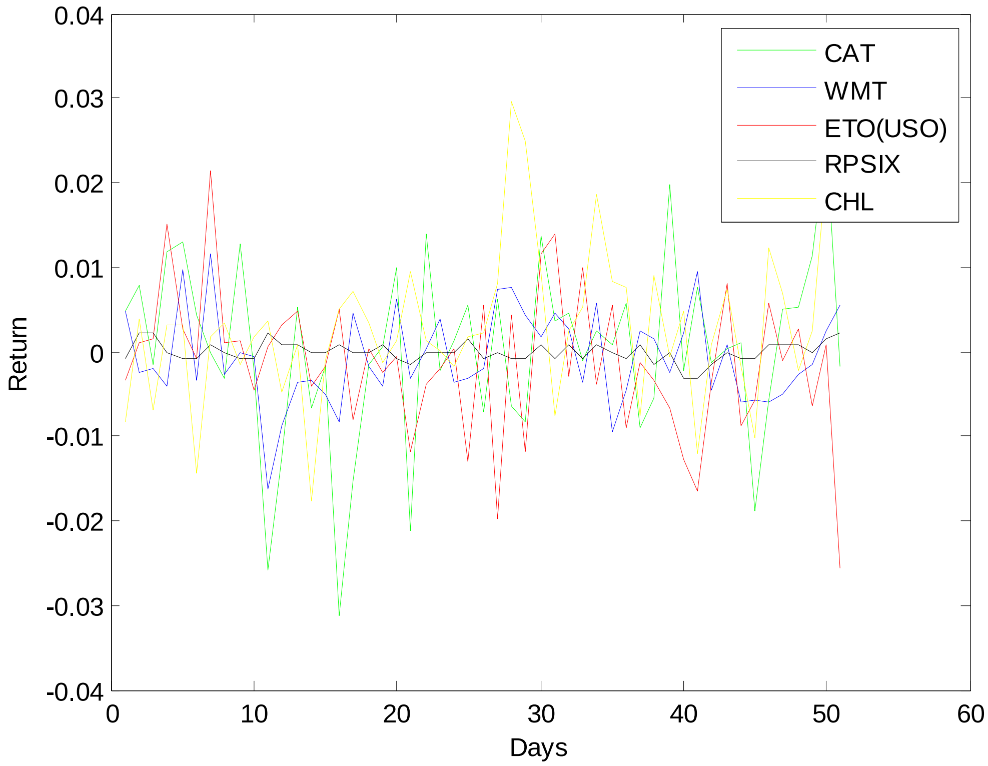


Table 3.1 Dowjones Industrial Average (DJIA)

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
AA	-0.000846	0.000150	0.007513
AXP	-0.000151	0.000207	0.010353
BA	0.000654	0.000230	0.011509
BAC	0.001463	0.000204	0.010239
CAT	0.000070	0.000130	0.006518

c (50,5,0.95) = 0.5345

$$\frac{T_{[1]}}{c} = \frac{0.006518}{0.5345} = 0.012195$$

Applying the procedure R, from the above table values we find that the population in the selected subsets have T_i values 0.006518, 0.007513, 0.010239, 0.010353 and 0.011509. The best population is corresponding to Dow CAT, where the best is decided in terms of volatility.

Table 3.2 Standard & Poor 500

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
CSCO	0.001545	0.000080	0.004012
SU	-0.000373	0.000072	0.003610
POT	-0.005976	0.001003	0.050196
MCD	0.001371	0.000153	0.007658
WMT	0.000183	0.000052	0.002614

$$\frac{T_{[1]}}{c} = \frac{0.002614}{0.5345} = 0.004891$$

Applying the procedure R, from the above table values we find that the population in the selected subsets are those having T_i values 0.002614, 0.003610 and 0.004012. In terms of volatility, the best population is corresponding to S&P WMT.

Table 3.3 Oil Companies

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
MRO	0.001133	0.000146	0.007282
OIL	-0.001449	0.000084	0.004210
OL	-0.001517	0.000102	0.005110
UCO OIL	-0.002666	0.000308	0.015421
ETO(USO)	-0.001364	0.000073	0.003673

$$\frac{T_{[1]}}{c} = \frac{0.003673}{0.5345} = 0.006871$$

From the above table we select the populations with 0.003673, 0.004210 and 0.005110 as T_i values. The population corresponding to ETO (USO) is the best.

Table 3.4 Spectrum

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
PYSAX	-0.000187	0.000045	0.002283
RPSIX	0.000000	0.000001	0.000066
SPB	0.002269	0.000112	0.005577
SPGZ	0.134679	0.925071	46.25352
SPPI	0.000417	0.000627	0.031345

$$\frac{T_{[1]}}{c} = \frac{0.000066}{0.5345} = 0.000123$$

From the above table values we find that the only one population selected is RPSIX corresponding to the value 0.000066.

Table 3.5 Mobile

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
CHL	0.003229	0.000094	0.004712
GLUU	0.007804	0.002247	0.112337
MBLY	0.003676	0.002408	0.120403
MBT	0.000114	0.000286	0.014285
NQ	-0.003416	0.007596	0.379806

$$\frac{T_{[1]}}{c} = \frac{0.004712}{0.5345} = 0.008816$$

From the above table values it is obvious that the population selected is the singleton subset CHL with T_i value 0.004712.

Once again the procedure R is used to obtain the best population among the selected populations in the previous steps.

Table 3.6

Company	$\bar{r}_i$ (mean return)	$s_i^2 = \frac{1}{50}\sum_{j=1}^{51}(r_{ij} - \bar{r}_i)^2$	$T_i = \gamma s_i^2$
CAT	0.000070	0.000130	0.006518
WMT	0.000183	0.000052	0.002614
ETO(USO)	-0.001364	0.000073	0.003673
RPSIX	0.000000	0.000001	0.000066
CHL	0.003229	0.000094	0.004712

$$\frac{T_{[1]}}{c} = \frac{0.000066}{0.5345} = 0.000123$$ $T_{[1]}$ / c = 0.000066 / 0.5345 = 0.000123

It is found that the population RPSIX giving the value 0.000066 for T_i is the best with minimum volatility.

4. Conclusion

In this paper, the procedure R is used to to rank the volatility for the DJIA, S&P 500, Oil, Spectrum and Mobile indices. Each population has sub populations. The sub populations are ranked according to their volatility and the best sub population from each population is selected. Considering the resultant sub populations as main populations, using R procedure, the best population in terms of minimum volatility is obtained. On the basis of selection, a suitable proportion of investment can be decided by an investor according to his / her expectation

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APPENDIX

Table 3. *Lower 100α percentage points of* $Y = \chi^2_{\min}/\chi^2_0 = \min(F_1, F_2, \ldots, F_p)$, *the degrees of freedom*, ν, *for all* $p+1$ *independent chi-squares being the same*

ν \ p	1	2	3	4	5	6	7	8	9	10
				(A) $1-\alpha = P^* = 0{\cdot}75$						
2	0·3333	0·1667	0·1111	0·0833	0·0667	0·0556	0·0476	0·0417	0·0310	0·0333
4	·4844	·3168	·2494	·2112	·1860	·1678	·1540	·1430	·1340	·1264
6	·5611	·4040	·3369	·2973	·2704	·2505	·2350	·2225	·2121	·2033
8	·6099	·4628	·3978	·3587	·3317	·3116	·2957	·2828	·2720	·2627
10	·6446	·5060	·4434	·4054	·3788	·3588	·3430	·3301	·3192	·3098
12	0·6711	0·5395	0·4794	0·4424	0·4165	0·3968	0·3813	0·3684	0·3576	0·3483
14	·6921	·5667	·5087	·4728	·4475	·4283	·4130	·4004	·3898	·3806
16	·7094	·5892	·5332	·4984	·4737	·4550	·4400	·4276	·4171	·4081
18	·7239	·6084	·5542	·5203	·4963	·4779	·4633	·4511	·4408	·4319
20	·7364	·6250	·5724	·5394	·5160	·4980	·4837	·4718	·4616	·4529
22	0·7472	0·6395	0·5883	0·5562	0·5333	0·5158	0·5017	0·4900	0·4801	0·4715
24	·7568	·6523	·6026	·5712	·5488	·5317	·5179	·5064	·4967	·4882
26	·7653	·6635	·6153	·5847	·5628	·5460	·5325	·5212	·5117	·5034
28	·7729	·6742	·6268	·5969	·5754	·5590	·5457	·5347	·5253	·5171
30	·7798	·6836	·6373	·6080	·5870	·5708	·5578	·5470	·5377	·5297
32	0·7861	0·6922	0·6469	0·6182	0·5976	0·5817	0·5689	0·5583	0·5492	0·5413
34	·7919	·7001	·6558	·6276	·6074	·5918	·5792	·5687	·5598	·5520
36	·7972	·7074	·6640	·6363	·6164	·6011	·5887	·5784	·5696	·5619
38	·8021	·7142	·6715	·6444	·6248	·6098	·5976	·5874	·5788	·5712
40	·8067	·7205	·6786	·6519	·6327	·6178	·6058	·5958	·5873	·5799
42	0·8109	0·7264	0·6852	0·6590	0·6400	0·6254	0·6136	0·6038	0·5953	0·5880
44	·8149	·7319	·6914	·6656	·6470	·6326	·6209	·6112	·6029	·5957
46	·8186	·7371	·6973	·6718	·6534	·6393	·6278	·6182	·6100	·6029
48	·8221	·7420	·7028	·6777	·6596	·6456	·6343	·6248	·6167	·6097
50	·8254	·7466	·7080	·6832	·6654	·6516	·6404	·6311	·6231	·6162
50†	0·8247	0·7485	0·7119	0·6887	0·6720	0·6592	0·6489	0·6403	0·6330	0.6266
				(B) $1-\alpha = P^* = 0{\cdot}90$						
2	0·1111	0·0556	0·0370	0·0278	0·0222	0·0185	0·0159	0·0139	0·0123	0·0111
4	·2435	·1630	·1297	·1106	·0979	·0886	·0816	·0759	·0713	·0674
6	·3274	·2417	·2039	·1813	·1657	·1541	·1450	·1377	·1315	·1263
8	·3862	·3002	·2610	·2370	·2202	·2076	·1976	·1894	·1826	·1766
10	·4306	·3457	·3062	·2818	·2645	·2515	·2410	·2325	·2252	·2190
12	0·4657	0·3825	0·3433	0·3188	0·3014	0·2881	0·2775	0·2688	0·2613	0·2549
14	·4944	·4132	·3744	·3501	·3327	·3194	·3087	·2999	·2924	·2859
16	·5186	·4392	·4011	·3770	·3597	·3464	·3358	·3270	·3194	·3129
18	·5394	·4618	·4243	·4004	·3833	·3702	·3596	·3508	·3433	·3368
20	·5575	·4816	·4447	·4212	·4043	·3913	·3808	·3720	·3646	·3581
22	0·5734	0·4992	0·4629	0·4397	0·4230	0·4101	0·3997	0·3911	0·3837	0·3772
24	·5876	·5149	·4792	·4564	·4399	·4272	·4169	·4083	·4010	·3946
26	·6004	·5291	·4940	·4715	·4553	·4427	·4325	·4240	·4168	·4104
28	·6119	·5420	·5076	·4854	·4693	·4569	·4468	·4384	·4312	·4250
30	·6225	·5539	·5199	·4981	·4822	·4700	·4600	·4517	·4446	·4384
32	0·6322	0·5648	0·5314	0·5098	0·4942	0·4820	0·4722	0·4640	0·4570	0·4508
34	·6411	·5749	·5419	·5207	·5052	·4933	·4836	·4754	·4684	·4624
36	·6493	·5842	·5518	·5308	·5156	·5037	·4941	·4861	·4792	·4732
38	·6570	·5929	·5609	·5402	·5252	·5135	·5040	·4960	·4892	·4833
40	·6642	·6011	·5695	·5491	·5342	·5227	·5133	·5054	·4987	·4928
42	0·6709	0·6087	0·5776	0·5574	0·5427	0·5313	0·5220	0·5142	0·5076	0·5017
44	·6772	·6159	·5852	·5653	·5508	·5394	·5303	·5226	·5160	·5102
46	·6831	·6227	·5924	·5727	·5583	·5472	·5381	·5304	·5239	·5182
48	·6887	·6291	·5992	·5797	·5655	·5544	·5454	·5379	·5314	·5258
50	·6940	·6352	·6056	·5863	·5723	·5614	·5525	·5450	·5386	·5330
50†	0·6934	0·6373	0·6092	0·5914	0·5785	0·5683	0·5601	0·5532	0·5474	0·5422

† First term of normal approximation based on (5·2) and (5·3).

Table 3 (*cont.*)

ν \ p	1	2	3	4	5	6	7	8	9	10
					(C) $1-\alpha = P^* = 0{\cdot}95$					
2	0·0526	0·0263	0·0175	0·0132	0·0105	0·0088	0·0075	0·0066	0·0058	0·0053
4	·1565	·1062	·0851	·0728	·0646	·0586	·0540	·0504	·0473	·0448
6	·2334	·1749	·1486	·1327	·1217	·1134	·1069	·1017	·0972	·0935
8	·2909	·2293	·2007	·1830	·1706	·1612	·1537	·1476	·1424	·1379
10	·3358	·2732	·2436	·2250	·2119	·2018	·1938	·1872	·1815	·1767
12	0·3722	0·3096	0·2796	0·2606	0·2470	0·2366	0·2283	0·2214	0·2155	0·2104
14	·4026	·3405	·3103	·2911	·2774	·2668	·2583	·2512	·2452	·2399
16	·4285	·3671	·3370	·3178	·3039	·2933	·2847	·2775	·2714	·2661
18	·4510	·3903	·3604	·3413	·3274	·3168	·3081	·3009	·2947	·2894
20	·4708	·4109	·3813	·3622	·3484	·3378	·3291	·3219	·3157	·3104
22	0·4883	0·4294	0·4000	0·3811	0·3674	0·3568	0·3481	0·3409	0·3348	0·3294
24	·5041	·4460	·4170	·3982	·3846	·3740	·3654	·3582	·3521	·3467
26	·5184	·4611	·4324	·4138	·4003	·3898	·3812	·3741	·3680	·3626
28	·5313	·4749	·4465	·4281	·4147	·4043	·3958	·3887	·3826	·3773
30	·5432	·4876	·4595	·4413	·4280	·4177	·4093	·4022	·3962	·3909
32	0·5542	0·4993	0·4716	0·4536	0·4404	0·4302	0·4218	0·4148	0·4088	0·4036
34	·5643	·5102	·4828	·4649	·4519	·4418	·4335	·4265	·4206	·4154
36	·5737	·5203	·4932	·4756	·4627	·4526	·4444	·4375	·4316	·4264
38	·5825	·5298	·5030	·4855	·4728	·4628	·4546	·4478	·4419	·4368
40	·5907	·5387	·5122	·4949	·4822	·4724	·4643	·4575	·4517	·4466
42	0·5984	0·5470	0·5208	0·5037	0·4912	0·4814	0·4734	0·4667	0·4609	0·4558
44	·6057	·5549	·5290	·5120	·4996	·4899	·4820	·4753	·4696	·4646
46	·6126	·5624	·5367	·5199	·5076	·4980	·4901	·4835	·4778	·4729
48	·6190	·5694	·5440	·5274	·5152	·5057	·4979	·4913	·4857	·4808
50	·6252	·5761	·5510	·5345	·5224	·5130	·5053	·4988	·4932	·4883
50†	0·6250	0·5784	0·5545	0·5394	0·5284	0·5199	0·5124	0·5064	0·5013	0·4968
					(D) $1-\alpha = P^* = 0{\cdot}99$					
2	0·0101	0·0051	0·0034	0·0025	0·0020	0·0017	0·0014	0·0013	0·0011	0·0010
4	·0626	·0434	·0351	·0302	·0269	·0245	·0226	·0211	·0199	·0189
6	·1181	·0907	·0779	·0701	·0646	·0605	·0572	·0545	·0522	·0503
8	·1659	·1339	·1186	·1089	·1024	·0968	·0926	·0891	·0862	·0837
10	·2062	·1717	·1548	·1440	·1362	·1303	·1255	·1215	·1181	·1152
12	0·2407	0·2046	0·1867	0·1752	0·1668	0·1604	0·1552	0·1508	0·1472	0·1439
14	·2704	·2334	·2149	·2029	·1942	·1874	·1820	·1774	·1734	·1700
16	·2966	·2590	·2401	·2278	·2188	·2118	·2061	·2014	·1973	·1937
18	·3197	·2819	·2627	·2501	·2410	·2338	·2280	·2232	·2190	·2153
20	·3404	·3025	·2831	·2704	·2612	·2539	·2480	·2431	·2388	·2351
22	0·3591	0·3212	0·3017	0·2890	0·2796	0·2723	0·2663	0·2613	0·2570	0·2532
24	·3761	·3382	·3188	·3060	·2966	·2892	·2832	·2782	·2738	·2700
26	·3916	·3539	·3344	·3216	·3122	·3048	·2988	·2937	·2894	·2855
28	·4059	·3684	·3490	·3362	·3268	·3194	·3133	·3082	·3038	·3000
30	·4191	·3818	·3625	·3497	·3403	·3329	·3268	·3217	·3173	·3135
32	0·4314	0·3943	0·3750	0·3623	0·3529	0·3455	0·3395	0·3344	0·3300	0·3261
34	·4428	·4060	·3868	·3741	·3648	·3574	·3513	·3462	·3418	·3380
36	·4535	·4169	·3979	·3852	·3759	·3685	·3625	·3574	·3530	·3492
38	·4636	·4272	·4083	·3957	·3864	·3791	·3730	·3680	·3636	·3598
40	·4730	·4369	·4181	·4056	·3963	·3890	·3830	·3780	·3736	·3698
42	0·4819	0·4461	0·4274	0·4149	0·4057	0·3984	0·3925	0·3874	0·3831	0·3793
44	·4903	·4548	·4362	·4238	·4146	·4074	·4014	·3964	·3921	·3883
46	·4983	·4630	·4445	·4322	·4231	·4159	·4100	·4050	·4007	·3969
48	·5059	·4709	·4525	·4402	·4312	·4240	·4181	·4132	·4089	·4051
50	·5131	·4784	·4601	·4479	·4389	·4318	·4259	·4210	·4167	·4129
50†	0·5144	0·4816	0·4644	0·4530	0·4446	0·4380	0·4326	0·4281	0·4241	0·4207

These four tables A, B, C and D give the values of c for which $\int_0^\infty [1-G_\nu(cx)]^p \, g_\nu(x)\, dx = 1-\alpha$, where $G_\nu(x)$ and $g_\nu(x)$ refer to the C.D.F. and P.D.F., respectively, of a central χ^2 with ν degrees of freedom.

† First term of normal approximation based on (5·2) and (5·3).