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On Bi-bi ideal and Quasi-bi ideal of Bi-Near Subtraction Ordered Semigroup

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Abstract

In this paper we introduce the notion of Bi-bi ideal, Quasi-bi ideals of bi-near subtraction Ordered semigroup. Also we give characterizations of Bi-bi ideal, Quasi-bi ideals of bi-near subtraction Ordered semigroup.

Mathematical subject classification: 06F35

Key words: subtraction Ordered semigroup, near subtraction Ordered semigroup, Bi-near subtraction Ordered semigroup Bi-bi ideal Quasi-bi ideal

1.Introduction
In 2007, Dheena [1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz [5]. S.Lekkoksung [4] introduced the notation of Fuzzy ideals in Near Subtraction Ordered semigroups. Zekiye Ciloglu, Yilmaz Ceven [6] gave the notation of Fuzzy Near Subtraction semigroups.. Recently Firthous et.al [2, 3] introduced the notation of Bi bi-ideals and Quasi-bi ideal in Bi near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of Bi bi-ideals and Quasi-bi ideal of Bi-near subtraction ordered semigroups.
Given two subsets A and B of X the product AB={ ab /a$\in$A and b$\in$B}. Also we define another operator "$*$" on the class of subsets of X given by A$*$B={ab-a(a′-b)/a, a′$\in$A, b$\in$B}.

2. Preliminaries :

Definition2.1

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction semigroup If

(i) (X, –) is a subtraction algebra

(ii) (X,.) is a semi group

(iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X.

Definition2.2

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction Ordered semigroup If

(i) (X, –) is a subtraction algebra

(ii) (X,., ≤) is a Ordered semi group

(iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X.

Definition2.3

A non-empty subset X together with two binary operations "–" and "." is said to be near subtraction semigroup if

(i) (X, –) is a subtraction algebra

(ii) (X,.) is a semi group and

(iii) (x–y)z= xz–yz for every x, y, z∈X.

Definition2.4

A non-empty subset X together with two binary operations "–" and "." is said to be near subtraction Ordered semigroup if

(i) (X, –) is a subtraction algebra

(ii) (X,., ≤) is a Ordered semi group and

(iii) (x–y)z= xz–yz for every x, y, z∈X.

Definition2.5

A non-empty subset S of X Is said to be Sub algebra if x–y∈ S whenever x, y∈ S.

Definition2.6

A non-empty subset S= $S_1 \cup S_2$ of X Is said to be Bi- Sub algebra if S_1 is a Sub algebra inX_1 and S_2 is a Sub algebra in X_2

Definition 2.7

A non-empty subset I of X is said to be Right ideal if

(i) x–y∈I for every x∈ I and y∈X and

(ii) IX ⊆ I.

Definition2.8

A non-empty subset I of X is said to be Left ideal if

(i) x–y∈I for every x∈ I and y∈X and

(ii) $XI \subseteq I$.

Definition 2.9
A non-empty subset I of X is said to be ideal if
(i) $IX \subseteq I$ and
(ii) $XI \subseteq I$.

Definition 2.10
An element a $\in$X is said to be Regular if for each a $\in$X; a = aba, for some b $\in$X

Definition 2.11
A set $X= X_1 \cup X_2$ Is said to be Bi- Regular, if both X_1 and X_2 is Regular.

Definition 2.12
A subset P of X is said to be Zero symmetric if $XP \subseteq X*P$.

Definition 2.13
A subset B of X is said to be Bi-ideal if $BXB \cap BX*B \subseteq B$.

Definition 2.14
A subset Q of X is said to be Quasi-ideal if $QX \cap XQ \cap X*Q \subseteq Q$.

Definition 2.15
A non-empty subset $X=X_1 \cup X_2$ together with two binary operations "–" and "." is said to be bi-near subtraction semigroup (right) if
(i) $(X_1, -, .)$ is a near-subtraction semigroup
(ii) $(X_2, -, .)$ is a subtraction semigroup.

3. Bi-near Subtraction Ordered Semigroup

Definition 3.1
A non-empty subset $X=X_1 \cup X_2$ together with two binary operations "–" and "." is said to be bi-near subtraction Ordered semigroup (right) if (i) $(X_1, -, .)$ is a near-subtraction Ordered semigroup (ii) $(X_2, -, .)$ is a subtraction Ordered semigroup.

Example 3.2

Let $X_1=\{0, 1, 2, 3\}$ in which "-" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	0	0	3

Then (X_1, -,.) is a near-subtraction Ordered semi group
Let X_2={0, 1, 2, 3} in which "-" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then (X_2, -,.) is a subtraction Ordered semi group.

Note 3.3
Obviously, every Bi -near subtraction Ordered semi group is Bi- near subtraction semi group. But the converse is not true

Example 3.4

Let X_1={0, a, b, c} in which "-" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	0	0	c

Then (X_1, -,.) is a near-subtraction semi group not a near-subtraction Ordered semigroup

Let X_2={0, a, b, 1} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then (X_2, -,.) is a subtraction semi group not a subtraction Ordered semigroup
Hence, every Bi_ near subtraction semi group need not be a Bi_ near subtraction ordered semi group.

4. Bi- bi ideals of Bi-near Subtraction Ordered Semigroup

Definition 4.1

A non-empty subset B= $B_1 \cup B_2$ of X is said to be Bi-bi ideal, if B_1 is Bi-ideal in X_1 and B_2 is ideal in X_2.

Example 4.2

Let X_1={0, a, b, c} in which "-" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	0	0	3

Then B_1={0, 1} is bi-ideal in X_1

Let X_2={0, a, b, 1} in which "-" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then B_2 = {0, 1, 2} is ideal in X_2.

Proposition: 4.3

The set of all Bi-bi ideals of a Bi-near subtraction ordered semi group form a Moore System on X.

Proof:

Let B_i be a set of all Bi-bi ideals in X.

Let B= $\cap B_i$, $i \in I$.Then B= $\cap(B_i' \cup B_i'')$

Since Intersection of all B_i' are Bi-ideal and Intersection of all B_i'' are ideal, B is a Bi-bi ideal of X.

Proposition:4.4

If B is a Bi -bi ideal of a Bi-near subtraction ordered semi group X and S is a subalgebra of X, then $B \cap S$ is a Bi-bi ideal of S.

Proof:
Let B be a Bi-bi ideal of X. Then $B\cap S= (B_1\cup B_2)\cap S = (B_1\cap S)\cup(B_2\cap S)$
Since $B_1\cap S$ is a bi -ideal of S and $B_2\cap S$ is an ideal of S, $B\cap S$ is a Bi -bi ideal of S.

Theorem;4.5
Let $X=X_1\cup X_2$ be a Bi-near subtraction ordered semi group and let B be a Bi -ideal of X. Then B is a Bi -bi ideal of X if and only if there exist two proper subsets X_1 and X_2 of X such that

(i) $X=X_1\cup X_2$ where X_1 and X_2 are proper subsets of X
(ii) $(B \cap X_1)$ is a Interior -ideal of $(B_1, -,.)$
(iii) $(B\cap X_2)$ is a ideal of $(B_2, -,.)$

Proof:
Assume that B is a Bi–bi ideal of X. Thus there exist two subsets B_1 and B_2 of B such that $B= B_1\cup B_2$ Where B_1 is a Bi -ideal of X_1 and B_2 is a ideal of X_2.
Taking $B_1=B \cap X_1$ and $B_2= B \cap X_2$.
Conversely, let B be a nonempty subset of X a satisfying conditions (i), (ii) and (iii).
Hence
$(B \cap X_1)\cup (B \cap X_2)$
$= ((B \cap X_1)\cup B)\cap ((B \cap X_2)\cup X_2)$
$= ((B \cup B)\cap(X_1\cup B)) \cap ((B \cup X_2) \cap (X_1\cup X_2))$
$= (B \cap(B \cup X_1))\cap((B \cup X_2)\cap X)$
$= B \cap(B \cup X_2)$ (since $B \subseteq B \cup X_1$ and $B \cup X_2\subseteq B$)
$= B$. (since $B \subseteq B \cup X_2$)

Thus, $(B \cap X_1)\cup (B \cap X_2) = B$.
Hence, B is a Bi -bi-ideal of X.

Proposition:4.6
Let X be a Zero- Symmetric Near subtraction ordered semi group. A Subset B of X is a Bi ideal then $BXB \subseteq B$.

Proof:
Let $B = B_1\cup B_2$ Where B_1 is a Bi ideal of X_1 and B_2 is a ideal of X_2. Since X is a Zero- Symmetric, $XB\subseteq X*B$.
Now, $BXB= (B_1\cup B_2)x(B_1\cup B_2) =(B_1X\cup B_2X) (B_1\cup B_2) \subseteq(B_1X\cup B_2) (B_1\cup B_2) =B_1X (B_1\cup B_2) \cup B_2 (B_1\cup B_2) \subseteq B_1XB_1\cup B_2X B_2\subseteq B_1\cup B_2= B$
Thus $BXB \subseteq B$.

Proposition:4.7
Let X be a Zero- Symmetric Near subtraction ordered semi group. If Subset B is a Bi-bi ideal of X, then Bx and x^1B are Bi_bi ideal of X, where x, $x^1\in X$ and x^1 is the distributive element in X.

Proof:
Clearly Bx is a bi- subalgebra of X and BxXBx $\subseteq$BXBx $\subseteq$Bx.
Bx is a Bi-bi ideal of X. Again x^1B is a bi-Subalgebra. Since x^1 is the distributive element in X and $x^1BX\,x^1B \subseteq x^1BX\,B \subseteq x^1B$. Thus x^1B is a Bi-bi ideal of x.

5. Quasi-bi ideals of Bi-near Subtraction Ordered Semigroup

Definition 5.1
A non-empty subset Q= $Q_1 \cup Q_2$ of X is said to be Quasi-bi ideal, if Q_1 is Quasi-ideal in X_1 and Q_2 is ideal in X_2.

Example 5.2

Let X_1={0, a, b, c} in which "–" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	0	0	3

Then Q_1={0, b} is Quasi-ideal in X_1
Let X_2={0, a, b, 1} in which "–" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then Q_2 = {0, a, b} is ideal in X_2.

Note 5.3
Obviously, every quasi-bi ideal is Bi-bi ideal in a bi-near subtraction ordered semigroup. But the converse is not true

Example 5.4

Let X_1={0, a, b, c } in which "–" and "." be defined by

-	0	1	2	3
0	0	0	0	0
1	1	0	3	2
2	2	2	0	2
3	3	0	3	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	0	0	3

Here S_1={0, 1} is bi-ideal but not Quasi-ideal in X_1

Let X_2={0, a, b, 1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then S_2={0, a, b} is an ideal in $X_{2.}$ Hence, every Bi_ bi ideal need not be a Quasi_bi ideal.

Proposition: 5.5

The set of all Quasi-bi ideals of a Bi-near subtraction ordered semi group form a Moore System on X.

Proof:

Let Q_i be a set of all Quasi-bi ideals in X.

Let Q= $\cap Q_i$, $i \in I$.Then Q= $\cap(Q_i' \cup Q_i'')$

Since Intersection of all Q_i' are Quasi-ideal and Intersection of all Q_i'' are ideal, B is a Quasi -bi ideal of X.

Proposition:5.6

If Q is a Quasi -bi ideal of a Bi-near subtraction ordered semi group X and S is a subset of X, then $Q \cap S$ is a Quasi -bi ideal of S.

Proof:

Let Q be a Quasi -bi ideal of X. Then $Q \cap S = (Q_1 \cup Q_2) \cap S = (Q_1 \cap S) \cup (Q_2 \cap S)$

Since $Q_1 \cap S$ is a Quai-ideal of S and $Q_2 \cap S$ is an ideal of S, $B \cap S$ is a Quasi -bi ideal of S.

Theorem;5.7
Let $X=X_1 \cup X_2$ be a Bi-near subtraction ordered semi group and let Q be a Quasi -ideal of X. Then Q is a Quasi -bi ideal of X if and only if there exist two proper subsets X_1 and X_2 of X such that
(i) $X=X_1 \cup X_2$ where X_1 and X_2 are proper subsets of X
(ii) $(Q \cap X_1)$ is a Quasi i-ideal of $(X_1, -,.)$
(iii) $(Q \cap X_2)$ is a ideal of $(X_2, -,.)$

Proof:
Assume that Q is a Quasi –bi ideal of X. Thus there exist two subsets Q_1 and Q_2 of Q such that $Q = Q_1 \cup Q_2$ Where Q_1 is a Quasi -ideal of X_1 and Q_2 is a ideal of X_2.
Taking $Q_1 = Q \cap X_1$ and $Q_2 = Q \cap X_2$.
Conversely, let Q be a nonempty subset of X a satisfying conditions (i), (ii) and (iii). Hence
$(Q \cap X_1) \cup (Q \cap X_2)$
$= ((Q \cap X_1) \cup Q) \cap ((Q \cap X_2) \cup X_2)$
$= ((Q \cup Q) \cap (X_1 \cup Q)) \cap ((Q \cup X_2) \cap (X_1 \cup X_2))$
$= (Q \cap (Q \cup X_1)) \cap ((Q \cup X_2) \cap X)$
$= Q \cap (Q \cup X_2)$ (since $Q \subseteq Q \cup X_1$ and $Q \cup X_2 \subseteq X$)
$= Q$. (since $Q \subseteq Q \cup X_2$)

Thus, $(Q \cap X_1) \cup (Q \cap X_2) = B$.
Hence, Q is a Quasi -bi-ideal of X.

Theorem:5.8
Let X be a Zero- Symmetric Bi-Near subtraction ordered semi group. If Q is a Quasi-bi ideal of X. If the element of Q are bi- regular then Q is a Quasi-ideal.

Proof:
Let $x \in QX \cap XQ$ Then $x=bx=x'b'$ for some b, b' in Q and x, x' in X.
Since Q is bi-regular, (i.e.,) $bb_1b=b_1$ for some b_1 in Q.
Now, $x = bx = (bb_1b)\, x = (bb_1)\,(b\, x) = (bb_1)\,(x'b) \in QX \cap XQ \subseteq Q$.
Hence B is a Quasi-ideal.

Theorem:5.9
Let X be a Zero- Symmetric Bi- Near subtraction ordered semigroup. If Q is a bi-ideal of X. If the element of Q are bi- regular then Q is a Quasi-bi ideal.

Proof:
Let $x \in Q$. Since Q is Quasi-bi ideal, therefore $Q=Q_1 \cup Q_2$ Where Q_1 is a Quasii-ideal of X_1 and Q_2 is a ideal of X_2. Every element of Q is a Bi-regular, then every element of Q_1 and Q_2 is a Bi-regular By above theorem, Q_1 is a Quasi-ideal of X_1 and Q_2 is a ideal of X_2. Hence Q is a Quasi-bi ideal.

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