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$C_m \cup K_{1,r} \cup P_n$ is even graceful for $r \geq 0, n \geq 2$ and $m = 3, 4$.

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ABSTRACT

Moussa [1] has proved that $C_m \cup P_n$ is odd graceful if *m* is even. Mahalakshmi et al. [2] has proved that the graph $C_m \cup P_n$ is even graceful if *m* is odd with some certain conditions. Mary and Saranya [3] have proved that $C_m \cup P_n$ is even graceful if *m* is an even positive integer and *m*> *4* for every *n* = *m+2*. In this paper, we prove that the following graphs are even graceful.

(i) $C_3 \cup P_n$ *for* $n \geq 2$
(ii) $C_3 \cup K_{1,r} \cup P_n$ *for* $r \geq 1$, $n \geq 2$
(iii) $C_4 \cup P_n$ for $n \geq 2$
(iv) $C_4 \cup K_{1,r} \cup P_n$ *for* $r \geq 1, n \geq 2$.

KEY WORDS: Graceful labelling , Even graceful labelling , $C_3 \cup K_{1,r}$,$C_4 \cup K_{1,r}$

1. INTRODUCTION

A function *f* is called graceful labelling of a graph *G(p, q)* if *f : V* → *{0, 1,..., q}* is injective and the induced function *f∗ :E →{1,..., q}* defined as *f∗(uv)* = |*f(u)−f(v)*| is bijective. A graph which admits graceful labelling is called a *graceful graph*. Rosa introduced such labelling in 1967 and named it as a β − *valuation* of graph while Golomb independently introduced such labelling and called it as *graceful labelling*. *H*ere we consider the type of graceful labelling known as *Even-Graceful labelling*. A function *f* is called an *Even-Graceful Labelling* of a graph *G* with *p* vertices and *q* edges , if *f* is an injection from the vertices of *G* to the set *{ 0, 1, 2,----------, 2k-2 }* where *k = max {p, q }* such that when each edge ' *uv'* is assigned the label |*f(u)−f(v)*|, the resulting edge labels are distinct even numbers from *2* to *2k-2*. A graph which admits even graceful labelling is called *even graceful graph*.

2. NOTATIONS

C_n: A cycle of length *n*.
$K_{1,r}$: A bipartite graph whose central vertex is joined with r vertices.
$C_n \cup K_{1,r}$: The union of C_n and $K_{1,r}$ such that one of the vertices of C_n coincides with the central vertex of $K_{1,r}$.
$C_n \cup K_{1,r} \cup P_n$: The disjoint union of $C_n \cup K_{1,r}$ and P_n .

3. Theorem 3. 1

$C_3 \cup P_n$ is even graceful for $n \geq 2$.

Proof : Let $V(C_3) = \{u_1, u_2, u_3\}$ is the vertex set of cycle C_3.
Let $V(P_n) = \{v_1, v_2, v_3, ------, v_n\}$ is the vertex set of the path P_n. Let $q = n+2$ denote the total number of edges by the disjoint union of C_3 and P_n. Let f be a vertex label of $C_3 \cup P_n$ defined by

$$f(u_i) = \begin{cases} 0 & , \quad i = 1 \\ 2q & , \quad i = 2 \\ 2q-2 & , \quad i = 3 \end{cases} \text{ and } f(v_j) = \begin{cases} j & , \ j \ is \ odd \\ 2q-j-1 & , \ j \ is \ even \end{cases}$$

The edge labelling of C_3 are given as $f^*(u_1u_2) = 2q$, $f^*(u_2u_3) = 2$, $f^*(u_3u_1) = 2q-2$ and the edge labelling of P_n is given by $f^*(v_{j-1}, v_j) = 2q-2j$. We observe that the label of each edge of the graph is as even number. The cycle C_3 has edge labelling in the set $\{2q, 2q-2, 2\}$ and the path P_n has the edge labelling consisting in the set $\{2q-4, 2q-6, 2q-8, -----, 6, 4\}$. Therefore $C_3 \cup P_n$ has the edge labelling consisting of the set $\{2q, 2q-2, 2q-4, 2q-6, 2q-8, -----, 6, 4, 2\}$. Hence $C_3 \cup P_n$ is even graceful for $n \geq 2$.

ILLUSTRATION-3. 2 [Theorem-3. 1]

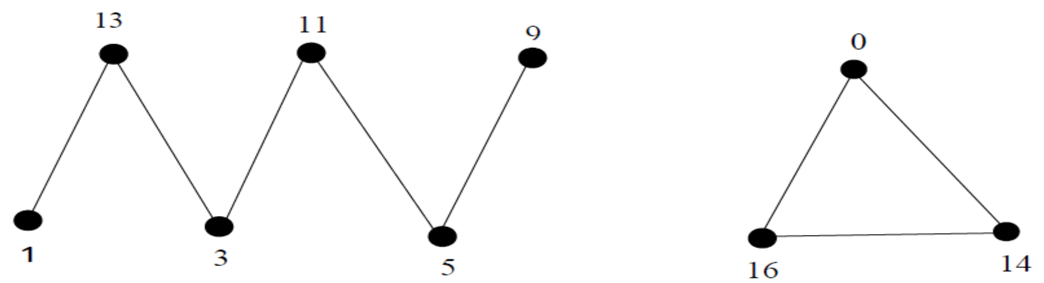

Figure-1 : $C_3 \cup P_6$ is even graceful

Theorem 3. 3

$C_3 \cup K_{1,r} \cup P_n$ is even graceful for $r \geq 1, n \geq 2$.

Proof: Let $V(C_3) = \{u_1, u_2, u_3\}$ is the vertex set of cycle C_3. Let $V(P_n) = \{v_1, v_2, v_3, - - - - -, v_n\}$ is the vertex set of the path P_n. Let $V(K_{1,r}) = \{w_1, w_2, - - -, w_r\}$ is the vertex of $K_{1,r}$. Let $q = n + r + 2$ denote the total number of edges of $C_3 \cup K_{1,r} \cup P_n$. Let f be the vertex labelling of $C_3 \cup K_{1,r} \cup P_n$ defined as follows

$$f(u_i) = \begin{cases} 0 & , \quad i = 1 \\ 2q & , \quad i = 2 \\ 2 & , \quad i = 3 \end{cases}, \quad f(v_j) = \begin{cases} j & , \ j \text{ is odd} \\ 2q - 2r - j - 1 & , \ j \text{ is even} \end{cases}$$

and $f(w_k) = 2q - 2k - 2, \quad k = 1, 2, \ldots\ldots\ldots, r$.

The edge labelling of C_3 are given by $f^*(u_1 u_2) = 2q$, $f^*(u_2 u_3) = 2q - 2$ $f^*(u_3 u_1) = 2$. The edge labelling of $K_{1,r}$ is given by $2q - 2r - 2$. Here all the r vertices are joined to the one central vertex which coincides with one vertex of C_3 having the common vertex labelling as 0. The set of edge labels of P_n are given by $f^*(v_{j-1} v_j) = 2q - 2r - 2j - 2$. So the edge labels of P_n consists of the set $\{2q - 2r - 4, 2q - 2r - 6, 2q - 2r - 8, - - - - - -, 6, 4\}$. The set of all the edge labels of $C_3 \cup K_{1,r} \cup P_n$ is the set $\{2q, 2q - 2, 2q - 2r - 4, 2q - 2r - 6, 2q - 2r - 8, - - - - - -, 6, 4, 2\}$. Hence $C_3 \cup K_{1,r} \cup P_n$ is even graceful for $r \geq 1, n \geq 2$.

ILLUSTRATION-3. 4 [Theorem-3. 3]

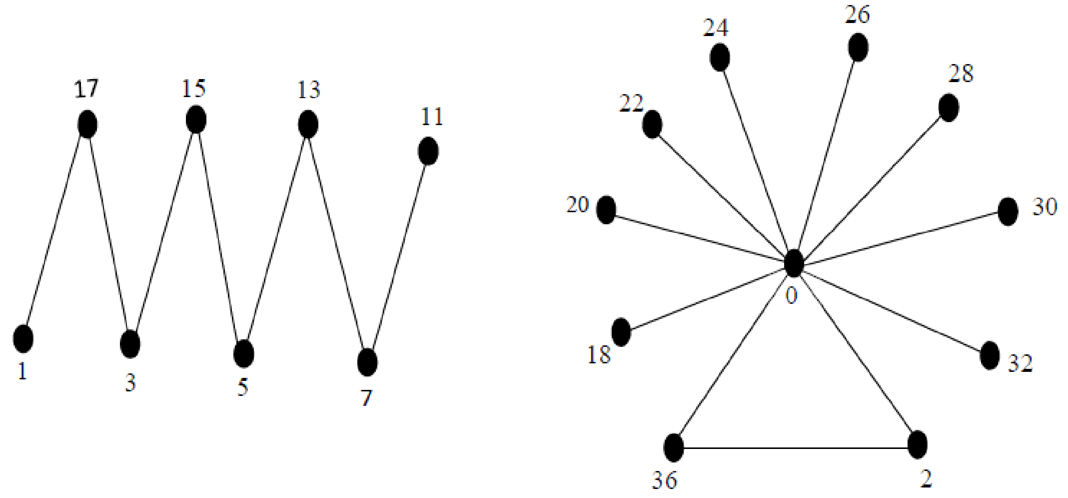

Figure-2 : $C_3 \cup K_{1,8} \cup P_8$ is even graceful.

Theorem-3. 5

$C_4 \cup P_n$ is even graceful for $n \geq 2$.

Proof: Let $V(C_4) = \{u_{1,} u_{2}, u_{3,} u_4\}$ is the vertex set of cycle C_4.
Let $V(P_n) = \{v_{1,} v_{2}, v_{3}, ------, v_n\}$ is the vertex set of the path P_n. Let $q = n + 3$ denote the total number of edges of $C_4 \cup P_n$. For every $u_i, i = 1, 2, 3, 4$ and $v_j, j = 1, 2, ------, n$ their vertex labelling are denoted by the functions $f(u_i)$, $f(v_j)$ respectively defined as follows

$$f(u_i) = \begin{cases} 0 & , \quad i = 1 \\ 2q & , \quad i = 2 \\ 4 & , \quad i = 3 \\ 2q - 2 & , \quad i = 4 \end{cases} \quad \text{and} \quad f(v_j) = \begin{cases} j & , \; j \text{ is odd} \\ 2q - j - 5 & , \; j \text{ is even} \end{cases}$$

The edge labelling of C_4 are given by $f^*(u_1u_2) = 2q$, $f^*(u_2u_3) = 2q - 4$, $f^*(u_3u_4) = 2q - 6$, $f^*(u_4u_1 = 2q - 2)$. The edge labelling of P_n are given by $f^*(v_{j-1,} v_j) = 2q - 2j - 4$. So the edge labelling of $C_4 \cup P_n$ consists of the labelled set $\{2q, 2q - 2, 2q - 4, 2q - 6, 2q - 8, -----, 6, 4, 2\}$. Hence $C_4 \cup P_n$ is even graceful.

ILLUSTRATION-3. 6 [Theorem-3. 5]

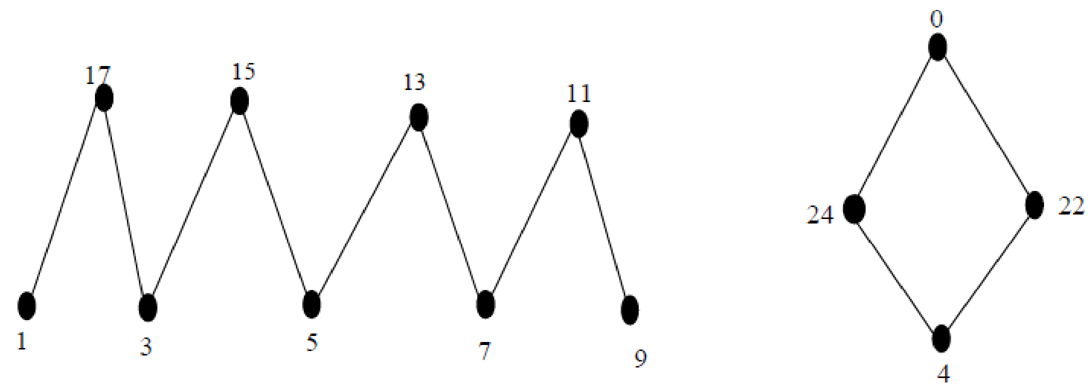

Figure-3: $C_4 \cup P_9$ **is even graceful**

Theorem-3. 7

$C_4 \cup K_{1,r} \cup P_n$ is even graceful for $r \geq 1, n \geq 2$.

Proof: Let $V(C_4) = \{u_{1,} u_{2}, u_3, u_4\}$ is the vertex set of cycle C_3. Let $V(P_n) = \{v_{1,} v_{2}, v_{3}, ----, v_n\}$ is the vertex set of the path P_n. Let $V(K_{1,r}) = \{w_1, w_2, ---, w_r\}$ is the vertex set of $K_{1,r}$. Let $q = n + r + 3$ denote the total number of edges of $C_4 \cup K_{1,r} \cup P_n$. For every $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, ------, n$ their vertex labelling are the functions $f(u_i), f(v_j), f(w_r)$ respectively defined as follows

$$f(u_i) = \begin{cases} 0 & , i = 1 \\ 2q & , i = 2 \\ 4 & , i = 3 \\ 2q-2 & , i = 4 \end{cases} \text{ and } f(v_j) = \begin{cases} j & , j \text{ is odd} \\ 2q-2r-j-5 & , j \text{ is even} \end{cases}$$

$f(w_k) = 2q - 2k - 6$, $k = 1, 2, ..., r$.

The edge labels of C_4 are given by $f^*(u_1u_2) = 2q$, $f^*(u_2u_3) = 2q - 4$, $f^*(u_3u_4) = 2q - 6$, $f^*(u_4u_1) = 2q - 2$. The edge labelling of $K_{1,r}$ are $f^*(u_1w_k) = 2q - 2k - 6$, *k=1, 2,------, r*. Here all the r vertices are joined to the central vertex which coincides with one vertex of C_3 having the common vertex labelling as 0. The edge labelling of P_n are given by $f^*(v_{j-1,} v_j) = 2q - 2r - 2(j+3)$. The set of edge labels of $C_4 \cup K_{1,r} \cup P_n$ is the set $\{2q, 2q-2, 2q-4, 2q-6, \ldots., 2q-2r-6, \ldots\ldots, 6, 4, 2\}$. Hence $C_4 \cup K_{1,r} \cup P_n$ is even graceful for $r \geq 1, n \geq 2$.

ILLUSTRATION-3. 8: [Theorem-3. 7]

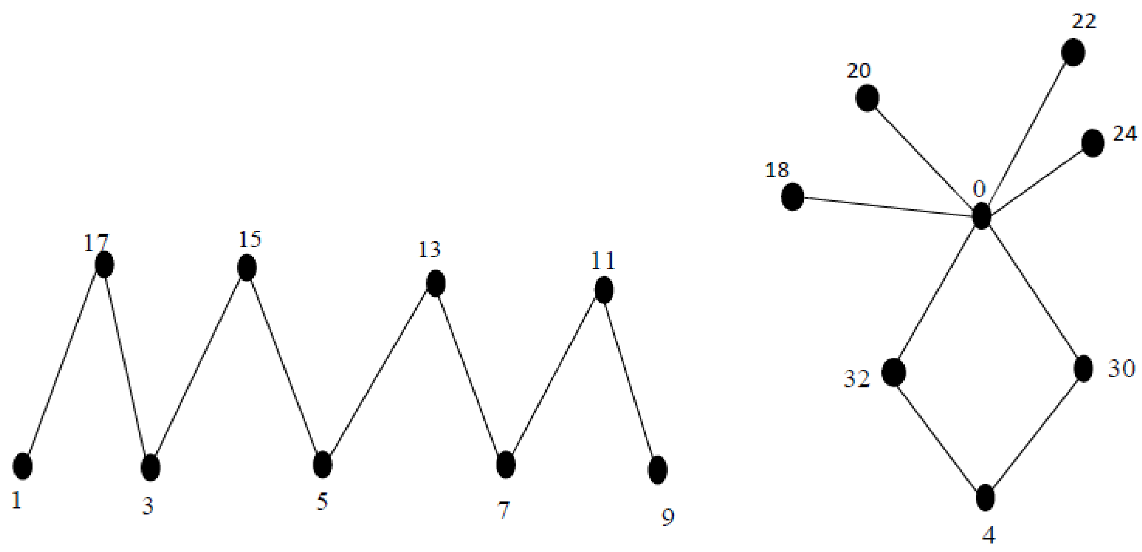

Figure 4 : $C_4 \cup K_{1,4} \cup P_9$ is even graceful

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