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# Level Method For Fuzzy Assignment Problems

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**ABSTRACT**

A new term namely, realistic assignment of a fuzzy assignment problem is defined. For finding a realistic assignment of the fuzzy assignment problem with costs of triangular fuzzy numbers, a new method namely, level method is proposed. Numerical examples are presented to understand the solution procedure and to show the importance of the proposed method. Further, the level method is extended to intuitionistic fuzzy assignment problems.

**Keywords:** Fuzzy numbers, Intuitionistic fuzzy numbers, Fuzzy assignment problem, Intuitionistic fuzzy assignment problem, Realistic assignment, Level method.

## 1.INTRODUCTION

The assignment problem is a special case of linear programming (LP) problem [8] in which the main objective is to assign *n* number of jobs to *n* number of machines (persons) at a minimum total cost in such a way that one job is assigned to only one machine(person) and one machine(person) is assigned to only one job. Such problems play an important role in assigning persons to jobs, or classes to rooms, operators to machines, drivers to trucks, trucks to routes, or problems to research teams, etc. In the literature, various methods are available for solving classical assignment problems of real life situations. Hungarian algorithm developed by Kuhn[9] is one of the popular algorithms for solving assignment problems.

In real life, some of the costs of an assignment problem are uncertain instead of precise numbers because of cost for doing a job by a machine/person that might vary due to various reasons. An assignment problem with fuzzy costs is known as a fuzzy assignment (FA) problem. In the literature, different types of FA problems [1, 2, 5, 6, 11, 12, 20] have been considered and solved by using the concepts of decision making in fuzzy environment developed by Zadeh [21] and Belmann and Zadeh [4]. In general, the FA problem is solved by using ranking function, or has been transformed into one or a series of classical assignment problems and then, it obtained an optimal solution. Amit Kumar and Anila Gupta [1] presented a method for assignment and travelling salesman problems with cost coefficients as LR fuzzy parameters. A method for solving fully FA problems with triangular fuzzy numbers was developed by Amit Kumar et al. [2]. In [15], Sathi Mukherjee et al. discussed the application of fuzzy ranking method for solving assignment problems with fuzzy costs. An efficient algorithm based on labeling method for solving FA problems was proposed by Lin Chi-Jen and Wen Ue-Pyng [10]. Thorani and Ravi Sankar [19] developed a method for solving FA problems with generalized fuzzy numbers. In [13, 18], a fuzzy version of Hungarian algorithm for solving an FA problem in which the problem is not converted to classical assignment problems.

Atanassov [3] proposed the concept of intuitionistic fuzzy (IF) sets which is found to be highly useful to deal with vagueness. An intuitionistic fuzzy assignment (IFA) problem is an assignment problem in which costs are IF numbers. Sathi Mukherjee and Kajla Babu [16] proposed a heuristic method for solving a class of IFA problems by using similarity measures. Senthil Kumar and Jahir Hussain [17] developed a method for solving IFA problems, using ranking function and the Hungarian method. Prabakaran and Ganesan [14] have developed a fuzzy version of Hungarian algorithm for finding an optimal solution of IFA problems with costs of triangular IF numbers.

This paper is organized as follows: Section 2 deals with some basic terminology of fuzzy sets and IF fuzzy sets. In Section 3, the mathematical formulation of a fuzzy assignment problem is presented, realistic assignment of the FA problem is defined, level method for finding realistic assignment of FA problems is proposed and the mathematical proof for optimality of the assignment of the FA problem obtained by the proposed method is given and also, numerical examples for finding a realistic assignment of the FA problem are illustrated. Section 4 discusses the works which are done in Section 3 in intutionistic fuzzy environment and finally, the conclusion is given in Section 5.

## 2. FUZZY SETS AND INTUITIONISTIC FUZZY SETS

The following definitions and concepts related to fuzzy set theory and IF set theory are needed which can be found in [21, 7, 3].

**Definition 2.1** Let X denote a universe of discourse and $A \subseteq X$ . Then, a fuzzy set of $A$ in X, $\tilde{A}$ with the membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$

where $\mu_A(x) : X \to [0,1]$.

For each $x$ in $X$ , $\mu_{\tilde{A}}(x)$ represents the membership degree of value of $x$ in the set $A \subseteq X$ .

**Definition 2.2** A triangular fuzzy number $\tilde{a}$ is a fuzzy number denoted by $(a_1, a_2, a_3)$ where $a_1, a_2$ and $a_3$ are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & : a_1 \le x \le a_2 \\ (a_3 - x)/(a_3 - a_2) & : a_2 \le x \le a_3 \\ 0 & : \text{Otherwise} \end{cases}$$

**Definition 2.3** Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers and $k$ be a non-negative real number. Then

(i) $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ and

(ii) $k\tilde{a} = (ka_1, ka_2, ka_3)$.

**Definition 2.4** Let $\tilde{a} = (a_1, a_2, a_3)$ be a triangular fuzzy number. Then,

(i) $\tilde{a}$ is said to be non-negative if $a_1 \ge 0$ and

(ii) $\tilde{a}$ is said to be integer if $a_i$, i =1, 2, 3, are integers.

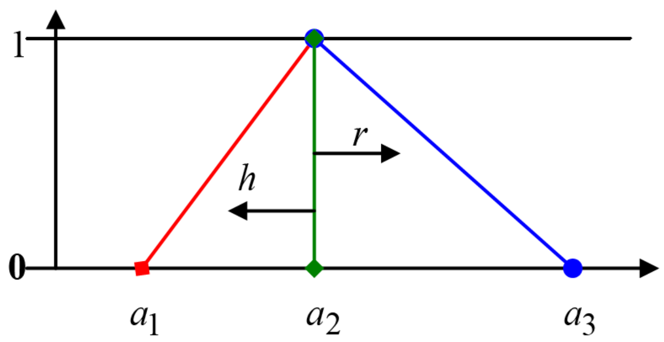
**Definition 2.5** Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers.

(i) $\tilde{a} \approx \tilde{b}$ iff $a_i = b_i, i = 1,2,3$ and

(ii) $\tilde{a} \preceq \tilde{b}$ iff $a_i \le b_i, i = 1,2,3$.

The triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can be represented as an interval number form as follows:

$[\tilde{a}] = [a_2 - (a_2 - a_1)h, a_2 + (a_3 - a_2)r]; 0 \le r, h \le 1.$




Note that $r$ and $h$ are the level of pessimistic and the level of optimistic of the fuzzy number $\tilde{u}$ respectively.

**Definition 2.6** Let X denote a universe of discourse and $A \subseteq X$. Then, an IF set of $A$ in X, $\tilde{A}^I$ is defined as follows:

$\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)); x \in X\}$

where $\mu_A(x), \vartheta_A(x)) : X \to [0,1]$ are functions such that $0 \le \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$ for all $x \in X$.

For each $x$ in $X$, $\mu_{\tilde{A}^I}(x)$ and $\vartheta_{\tilde{A}^I}(x)$ represent the membership and non-membership grades of $x$ in the set $A \subseteq X$.

**Definition 2.7** A triangular IF number $\tilde{a}^I$ is an IF number denoted by $(a_2, a_3, a_4)(a_1, a_3, a_5)$ where $a_1, a_2, a_3, a_4$ and $a_5$ are real numbers such that $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ and its membership function $\mu_{\tilde{a}^I}(x)$ and its non-membership function $\vartheta_{\tilde{a}^I}(x)$ are given below.

$$\mu_{\tilde{a}^I}(x) = \begin{cases} \dfrac{x - a_2}{a_3 - a_2} : a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} : a_3 \le x \le a_4 \\ 0 \quad : otherwise \end{cases} \text{ and } \vartheta_{\tilde{a}^I}(x) = \begin{cases} \dfrac{a_3 - x}{a_3 - a_1} : a_1 \le x \le a_3 \\ \dfrac{x - a_5}{a_5 - a_3} : a_3 \le x \le a_5 \\ 0 \quad : otherwise \end{cases}.$$

**Definition 2.8** Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ and $\tilde{b}^I = (b_2, b_3, b_4)(b_1, b_3, b_5)$ be two triangular IF numbers and let $k$ be a non-negative real number. Then,

(i) $\tilde{a}^I \oplus \tilde{b}^I = (a_2 + b_2, a_3 + b_3, a_4 + b_4)(a_1 + b_1, a_3 + b_3, a_5 + b_5)$ and

(ii) $k\tilde{a}^I = (ka_2, ka_3, ka_4)(ka_1, ka_3, ka_5)$.

**Definition 2.9** Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ be a triangular IF number. Then,

(i) $\tilde{a}^I$ is said to be non-negative ( $\tilde{a}^I \succeq \tilde{0}^I$ ) if $a_1 \ge 0$ and

(ii) $\tilde{a}^I$ is said to be integer if $a_i$, i =1, 2, 3, 4, 5 are integers.

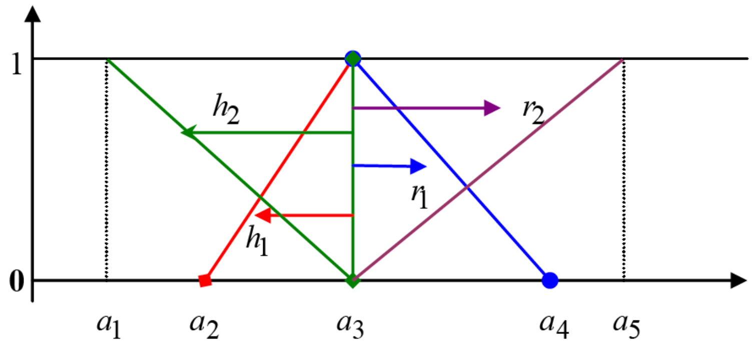
**Definition 2.10** Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ and $\tilde{b}^I = (b_2, b_3, b_4)(b_1, b_3, b_5)$ be two triangular IF numbers. Then,

(i) $\tilde{a}^I \approx \tilde{b}^I$ iff $a_i = b_i, i = 1,2,3,4,5$ and

(ii) $\tilde{a}^I \preceq \tilde{b}^I$ iff $a_i \le b_i, i = 1,2,3,4,5$.

A triangular IF number $\tilde{u}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ can be represented as an interval number form as follows:

$[\tilde{u}^I] = [a_3 - (a_3 - a_2)h_1, a_3 + (a_4 - a_3)r_1][a_3 - (a_3 - a_1)h_2, a_3 + (a_5 - a_3)r_2]$, where $0 \le r_1, r_2, h_1, h_2 \le 1$.




Note that $r_1$ and $h_1$ are the level of pessimistic and the levels of optimistic of the membership part of the IF number $\tilde{u}^I$ respectively and $r_2$ and $h_2$ are the level of pessimistic and the levels of optimistic of the non-membership part of the IF number $\tilde{u}^I$ respectively.

## 3. FUZZY ASSIGNMENT PROBLEM

Consider the situation of assigning $n$ machines to $n$ jobs in a company. Each machine in the company is capable of doing any job at different costs. Let $\tilde{c}_{ij} = (c_{ij}{}^1, c_{ij}{}^2, c_{ij}{}^3)$ be a fuzzy cost of assigning the jth job to the ith machine. Let $x_{ij}$ denote the decision variable of the assignment of the ith machine to the jth job. As the policy of the company, one job is assigned to only one machine and vice versa. The objective of the company is to determine an assignment schedule of all jobs to the available machines at the least total fuzzy cost.

The above situation can be represented as a fuzzy LP model as follows:

(P) Minimize $\tilde{z} = (z_1, z_2, z_3) = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \tag{2}$$

$$x_{ij} \in \{0,1\},\ i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,n. \tag{3}$$

A set $X = \{x_{ij}, i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,n\}$ is said to be an assignment or a feasible solution to the problem (P) if $X$ satisfies (1), (2) and (3). An assignment $X$ of the

problem (P) is said to be an optimal assignment or optimal solution of the problem (P) if $\tilde{z}(X) \leq \tilde{z}(U)$, for all feasible U of the problem (P).
Now, the FA problem (P) can be represented by a tabular form called an assignment table. For $n = 3$, the structure of the fuzzy assignment table is

| | J1 | J2 | J3 |
|---|---|---|---|
| M1 | $\tilde{c}_{11}$ | $\tilde{c}_{12}$ | $\tilde{c}_{13}$ |
| M2 | $\tilde{c}_{21}$ | $\tilde{c}_{22}$ | $\tilde{c}_{23}$ |
| M3 | $\tilde{c}_{31}$ | $\tilde{c}_{32}$ | $\tilde{c}_{33}$ |

Now, we can construct three levels of crisp assignment problems from the FA problem (P) namely, lower level assignment problem, (L); truth level assignment problem, (T) and upper level assignment problem, (U) as given below:

(L) Minimize $z_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^1 x_{ij}$ subject to (1), (2) and (3) ;

(T) Minimize $z_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^2 x_{ij}$ subject to (1), (2) and (3) and

(U) Minimize $z_3 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^3 x_{ij}$ subject to (1), (2) and (3).

Note that the FA problem (P) has an optimal solution, only if the problem (T) has an optimal solution.
Now, we need the following theorem that connects the optimal assignment of problem (T) and the optimal assignment of FA problem which is used in the proposed method.

**Theorem 3.1:** Let $X_L$, $X_T$ and $X_U$ be optimal assignments of the problem (L), the problem (T) and the problem (U) respectively. If $X_T = X_L = X_U$, then $X_T$ is an optimal assignment of the FA problem (P) with FA cost $\tilde{z}(X_T)$.

**Proof:** Now, since $X_T$ is optimal assignment of the problem (P), then $X_T$ is an assignment of the FA problem (P).
Let $Y$ be an assignment to the FA problem (P).
Now, since $Y$ is an assignment to the FA problem (P), $Y$ is an assignment to the problems (L), (T) and (U).
Now, since $X_L$, $X_T$ and $X_U$ are optimal assignments of problems (L), (T) and (U) respectively and $X_T = X_L = X_U$, we conclude that
$\tilde{z}(Y) \succeq (z_1(X_L), z_2(X_T), z_3(X_U))$
$\approx (z_1(X_T), z_2(X_T), z_3(X_T)) \approx \tilde{z}(X_T)$.

Therefore, $X_T$ is an optimal assignment of the FA problem (P) with FA cost $\tilde{z}(X_T)$. Hence, the theorem is proved.

**Remark 3.1** If $X_T = X_L = X_U$, we say that the problem (P) is a **realistic model** and its optimal solution is a **realistic assignment**. Otherwise, the problem (P) is called non-realistic and it has no realistic assignment.

**3.1 Construction of a Realistic Fuzzy Assignment Model**

Now, we propose a method for constructing a realistic model from a non-realistic FA model. In this method, we modify the lower and upper values of the fuzzy costs of the FA problem in the optimal allotted cells of the problem (T) in such a way that the optimal solutions of the problems (L), (T) and (U) are the same.

Now, using the interval representation of fuzzy number and for making the same assignment at all level problems, we obtain the following two relations which connect the optimal values of the problems (L), (T) and (U).

$$z_2(X_T) - ((z_2(X_T) - z_1(X_T))u = z_1(X_L) \quad (4)$$

and

$$z_2(X_T) + ((z_3(X_T) - z_2(X_T))v = z_3(X_U) \quad (5)$$

where *u* and *v* are real numbers.

Now, from (4) and (5), we obtain the values of *u* and *v* which are given below:

$$u = \begin{cases} \dfrac{z_2(X_T) - z_1(X_L)}{z_2(X_T) - z_1(X_T)} & : z_2(X_T) \neq z_1(X_T) \\ 0 & : \text{Otherwise} \end{cases}$$

and

$$v = \begin{cases} \dfrac{z_3(X_U) - z_2(X_T)}{z_3(X_T) - z_2(X_T)} & : z_3(X_T) \neq z_2(X_T) \\ 0 & : \text{Otherwise} \end{cases}$$

Now, we construct a new FA model from the non-realistic FA model, called a modified FA model with cost matrix $[\tilde{\beta}_{ij}]$ where

$$\tilde{\beta}_{ij} = \begin{cases} (c_{ij}^2 - (c_{ij}^2 - c_{ij}^1)u, c_{ij}^2, c_{ij}^2 + (c_{ij}^3 - c_{ij}^2)v) & : (i, j) \text{ is an optimal} \\ & \quad \text{assignment cell of the problem (T)} \\ \tilde{c}_{ij} & : \text{Otherwise} \end{cases}$$

Now, from (4) and (5) and the construction of the values of *u* and *v*, we conclude that $X_T$ is an optimal solution of the lower level and upper level problems (L) and (U) of the modified FA problem. Therefore, the modified FA model is realistic. Thus, the modified FA model is the realistic FA model that corresponds to the given non-realistic FA model.

Now, we propose a new method namely, level method for obtaining a realistic assignment of FA problem (P).

The proposed method proceeds as
**Step 1:** Construct the three crisp assignment problems (L), (T) and (U) from the given FA problem (P).
**Step 2:** Solve the problems (L), (T) and (U) obtained in Step 1. by the Hungarian method and obtain the optimal assignment of each one of the problems. Let $X_L$, $X_T$ and $X_U$ be optimal assignments of the problem (L), the problem (T) and the problem (U) respectively.
**Step 3:** If $X_T = X_L = X_U$, then $X_T$ is a realistic assignment of the given FA problem (P) with FA cost $\tilde{z}(X_T)$ by the Theorem 3.1.. If not, go to the Step 4..
**Step 4:** The given problem is not realistic.
**Step 5:** Construct a realistic FA model from the FA model obtained in the Step 4. as per the method given in the Section 3.1..
**Step 6:** Compute a realistic assignment of the realistic model obtained in the Step 5. using the Step 1. and the Step 2..

Now, the solution procedure of the level method is demonstrated, using the following numerical examples.

**Example 3.1** Consider the following FA problem with costs of triangular fuzzy numbers:

| Person → Job↓ | A | B | C |
|---|---|---|---|
| 1 | (1, 5, 9) | (8, 9, 10) | (2, 3, 4) |
| 2 | (7, 8, 9) | (6, 7, 8) | (6, 8, 10) |
| 3 | (5, 6, 7) | (6, 10, 14) | (10, 12, 14) |

Now, by the Step 1. and Step 2. of the level method, optimal assignments of the three crisp assignment problems are given below ;
(i) (L) problem: $X_L : 1 \to C ; 2 \to B$ and $3 \to A$ with assignment cost 13
(ii) (T) problem: $X_T : 1 \to C ; 2 \to B$ and $3 \to A$ with assignment cost 16 and
(iii) (U) problem: $X_U : 1 \to C ; 2 \to B$ and $3 \to A$ with assignment cost 19.

Now, since $X_T = X_L = X_U$ and by the Step 3. of the level method, the realistic assignment of the FA problem is $1 \to C ; 2 \to B$ and $3 \to A$ with FA cost (13, 16, 19).

**Remark 3.2** The optimal solution of the Example 3.2. obtained by the level method is the same as in Amit Kumar at al.[2].

**Example 3.2** Consider the following FA problem with costs of the triangular fuzzy numbers:

| Person → Job↓ | A | B | C |
|---|---|---|---|
| 1 | (4.5, 5, 5.5) | (8.1, 9, 9.9) | (2.7, 3, 3.3) |
| 2 | (7.2, 8, 8.8) | (6.3, 7, 7.7) | (7.2, 8, 8.8) |
| 3 | (5.4, 6, 6.6) | (9, 10, 11) | (10.8, 12, 13.2) |

Now, by the level method, the realistic assignment of the FA problem is $1 \to C; 2 \to B$ and $3 \to A$ with FA cost (14.4, 16, 17.6).

**Remark 3.3** The optimal solution of the Example 3.3. obtained by the proposed method and by the method proposed in De and Bharti Yadav [6] are the same.
Using the level method, we can solve FA problem with costs of trapezoidal fuzzy numbers. In the case of trapezoidal fuzzy numbers, we consider two truth level assignments problems instead of one in the case of triangular fuzzy numbers. The solution procedure is demonstrated with the help of the following numerical example:

**Example 3.3** Consider the following FA problem with costs of trapezoidal fuzzy numbers:

| Person → Job↓ | A | B | C | D |
|---|---|---|---|---|
| 1 | (3, 5, 6, 7) | (5, 8, 11, 12) | (9, 10, 11, 15) | (5, 8, 10, 11) |
| 2 | (7, 8, 10, 11) | (3, 5, 6, 7) | (6, 8, 10, 12) | (5, 8, 9, 10) |
| 3 | (2, 4, 5, 6) | (5, 7, 10, 11) | (8, 11, 13, 15) | (4, 6, 7, 10) |
| 4 | (6, 8, 10, 12) | (2, 5, 6, 7) | (5, 7, 10, 11) | (2, 4, 5, 7) |

Now, using the Step 1. and the Step 2. of the level method, optimal solutions of the level assignment problems are given below ;

(i) (L) problem: $X_L : 1 \to C;\ 2 \to B\ ;\ 3 \to A$ and $4 \to D\, 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 16

(ii) $(T_1)$ problem: $X_{T_1} : 1 \to C;\ 2 \to B\ ;\ 3 \to A$ and $4 \to D\, 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 23

(iii) $(T_2)$ problem: $X_{T_2} : 1 \to C;\ 2 \to B\ ;\ 3 \to A$ and $4 \to D\, 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 27 and

(iv) (U) problem: $X_U : 1 \to C;\ 2 \to B\ ;\ 3 \to A$ and $4 \to D\, 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 35.

Now, by the Step 3. of the level method, the realistic assignment of the FA problem is $1 \to C; 2 \to B\ ;\ 3 \to A$ and $4 \to D$ with minimum FA cost (16, 23, 27, 35)

**Remark 3.4** The optimal solution of the Example 3.4. got by the level method is the same as in Sathi Mukherjee and Kajla Basu [15], Manimaran and Ananthanarayanan [11] and Nagoor Gani and Mohamed [12].
Now, we present an example of FA model which is non realistic, and illustrate the method of constructing a realistic FA model from it.

**Example 3.4:** Consider the following FA problem with costs of triangular fuzzy numbers:

| | J1 | J2 | J3 |
|---|---|---|---|
| M1 | (7, 21, 29) | (7, 20, 57) | (12, 25, 56) |
| M2 | (8, 9, 16) | (4, 12, 35) | (6, 14, 28) |
| M3 | (5, 9, 22) | (10, 15, 20) | (4, 16, 19) |

Now, by the Steps 1. to 2. of the level method, optimal solutions of the level assignment problems related to the given FA problem are given below ;

(i) (L) problem: $X_L : M1 \to J1$; $M2 \to J2$ and $M3 \to J3$ with assignment cost 15

(ii) (T) problem: $X_T : M1 \to J2$; $M2 \to J3$ and $M3 \to J1$ with assignment cost 43 and

(iii) (U) problem: $X_U : M1 \to J1$; $M2 \to J3$ and $M3 \to J2$ with assignment cost 77.

Now, since $X_T \neq X_L \neq X_U$, the given problem is not realistic and has no optimal solution.
Now, we construct a realistic model from the non-realistic FA model and obtain a realistic assignment of the realistic model.
Now, $\tilde{z}(X_L) = (z_1(X_L), z_2(X_L), z_2(X_L)) = (15,49,83)$;
$\tilde{z}(X_T) = (z_1(X_T), z_2(X_T), z_2(X_T)) = (18,43,107)$ and
$\tilde{z}(X_U) = (z_1(X_U), z_2(X_U), z_2(X_U)) = (23,50,77)$;
Now, $u = \dfrac{z_2(X_T) - z_1(X_L)}{z_2(X_T) - z_1(X_T)} = 1.12$ and $v = \dfrac{z_3(X_U) - z_2(X_T)}{z_3(X_T) - z_2(X_T)} = 0.53123$.
Now, by the Section 3.1., the cost matrix of the modified FA model is $[\tilde{\beta}_{ij}]$ where

$$\tilde{\beta}_{ij} = \begin{cases} (c_{ij}^2 - (c_{ij}^2 - c_{ij}^1)u, c_{ij}^2, c_{ij}^2 + (c_{ij}^3 - c_{ij}^2)v) : (i,j) \text{ is an optimal} \\ \qquad\qquad \text{assignment cell of the problem (T)} \\ \tilde{c}_{ij} \qquad : \text{Otherwise} \end{cases}$$

Now, the modified FA problem from the given FA problem is given below

| | J1 | J2 | J3 |
|---|---|---|---|
| M1 | (7, 21, 29) | **(5.44, 20, 39.7)** | (12, 25, 56) |
| M2 | (8, 9, 16) | (4, 12, 35) | **(5.04, 14, 21.4)** |
| M3 | **(4.52, 9, 15.9)** | (10, 15, 20) | (4, 16, 19) |

Now, by the level method, the realistic assignment of the modified FA problem is $M1 \to J2$; $M2 \to J3$ and $M3 \to J1$ with minimum FA cost (15, 43, 77).
Therefore, the modified FA problem is realistic and it has a realistic assignment.

**4. INTUTIONISITIC FUZZY ASSIGNMENT PROBLEM**

Now, we consider an assignment problem in a company. In the company, there are n workers and n jobs. Each worker is capable of doing any job at different costs. The company has a policy that one job is assigned to only one worker and vice versa.. Let $\tilde{c}_{ij}^{I} = (c_{ij}^{2}\ , c_{ij}^{3}\ ,\ c_{ij}^{4})(c_{ij}^{1}, c_{ij}^{3}, c_{ij}^{5})$ be an IF cost of assigning the jth job to the ith worker. Let $x_{ij}$ be the decision variable that denotes the assignment of the worker i to the job j. The main objective of the company is to find an assignment schedule between jobs and workers such that the total IF fuzzy assignment cost is minimum.

The IF mathematical programming model for the above said IFA problem is given below:

(Q) Minimize $\tilde{z}^{I} = (z_2, z_3, z_4)(z_1, z_3, z_5) = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij}^{I}\ x_{ij}$

subject to (1), (2) and (3).

A set $X = \{x_{ij}, i = 1,2,...,n$ and $j = 1,2,...,n\}$ is said to be an assignment or a feasible solution to the problem (Q) if $X$ satisfies (1), (2) and (3). An assignment $X$ of the problem (Q) said to be an optimal solution of the problem (P) if $\tilde{z}^{I}(X) \leq \tilde{z}^{I}(U)$, for all feasible U of the problem (Q).

The IFA problem can be put in a table form which is called assignment table. For $n = 3$, the structure of the assignment table is

| | J1 | J2 | J3 |
|---|---|---|---|
| M1 | $\tilde{c}_{11}^{I}$ | $\tilde{c}_{12}^{I}$ | $\tilde{c}_{13}^{I}$ |
| M2 | $\tilde{c}_{21}^{I}$ | $\tilde{c}_{22}^{I}$ | $\tilde{c}_{23}^{I}$ |
| M3 | $\tilde{c}_{31}^{I}$ | $\tilde{c}_{32}^{I}$ | $\tilde{c}_{33}^{I}$ |

Now, we can decompose the given IFA problem (Q) into five level crisp assignment problems namely, non-membership lower problem, (NL) ; membership lower level problem, (ML) ; truth level problem, (T) ; membership upper level problem, (MU) and non-membership upper level problem, (NU) which are given below:

(NL) Minimize $z_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^1 x_{ij}$ subject to (1), (2) and (3) ;

(ML) Minimize $z_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^2 x_{ij}$ subject to (1), (2) and (3) ;

(T) Minimize $z_3 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^3 x_{ij}$ subject to (1), (2) and (3) ;

(MU) Minimize $z_4 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^4 x_{ij}$ subject to (1), (2) and (3)

and

(NU) Minimize $z_5 = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^5 x_{ij}$ subject to (1), (2) and (3).

Note that the IFA problem (Q) has an optimal solution, only if the problem (T) has an optimal solution.
Now, the following theorem connects the optimal assignment of problem (T) and the optimal assignment of IFA problem (Q) which is used in the proposed method.

**Theorem 4.1:** Let $X_{NL}$, $X_{ML}$, $X_T$, $X_{MU}$ and $X_{NU}$ be optimal assignments of the problem (NL), the problem (ML), the problem (T), the problem (MU) and the problem (NU) respectively. If $X_T = X_{NL} = X_{ML} = X_{MU} = X_{NU}$, then $X_T$ is an optimal assignment of the IFA problem (Q) with IFA cost $\tilde{z}^I(X_T)$.

**Proof:** It is similar to the proof of the Theorem 3.1. in the Section 3..

**Remark 4.1.** If $X_T = X_{NL} = X_{ML} = X_{MU} = X_{NU}$, we say that the problem (Q) is a realistic model and its optimal solution is a realistic assignment. Otherwise, it is called non-realistic, and has no optimal.

### 4.1 Construction of a Realistic Intutionistic Fuzzy Assignment Model

Now, we propose a method to construct a realistic IFA model from a non-realistic IFA model. In this approach, we modify the lower and upper values of the IF costs in the optimal allotted cells of the problem (T) such that the optimal solutions of all the level crisp assignment problems are the same.
Now, using the interval representation of a triangular IF number and for getting same assignment, we obtain the following four relations connecting the optimal values of the problems (NL), (ML), (T), (MU) and (NU)

$z_3(X_T) - ((z_3(X_T) - z_1(X_T))u_1 = z_1(X_{NL})$ (6)

$z_3(X_T) - ((z_3(X_T) - z_2(X_T))u_2 = z_2(X_{ML})$ (7)

$$z_3(X_T)+((z_4(X_T)-z_3(X_T))v_1=z_4(X_{MU}) \quad (8)$$

and

$$z_3(X_T)+((z_5(X_T)-z_3(X_T))v_1=z_5(X_{NU}) \quad (9)$$

where $u_1$, $u_2$, $v_1$ and $v_2$ are real numbers.

Now, from (6), (7), (8) and (9), we obtain the values of $u_1$, $u_2$, $v_1$ and $v_2$ which are given below:

$$u_1=\begin{cases}\dfrac{z_3(X_T)-z_1(X_{NL})}{z_3(X_T)-z_1(X_T)} & : z_3(X_T)\neq z_1(X_T)\\ 0 & :\text{Otherwise}\end{cases} \quad (10)$$

$$u_2=\begin{cases}\dfrac{z_3(X_T)-z_2(X_{ML})}{z_3(X_T)-z_2(X_T)} & : z_3(X_T)\neq z_2(X_T)\\ 0 & :\text{Otherwise}\end{cases} \quad (11)$$

$$v_1=\begin{cases}\dfrac{z_4(X_{MU})-z_3(X_T)}{z_4(X_T)-z_3(X_T)} & : z_4(X_T)\neq z_3(X_T)\\ 0 & :\text{Otherwise}\end{cases} \quad (12)$$

and

$$v_2=\begin{cases}\dfrac{z_5(X_{NU})-z_3(X_T)}{z_5(X_T)-z_3(X_T)} & : z_5(X_T)\neq z_3(X_T)\\ 0 & :\text{Otherwise}\end{cases} \quad (13)$$

Now, we construct a new IFA model called a modified IFA model from the given IFA model with cost matrix $[\tilde{\mu}_{ij}^I]$ where

$$\tilde{\mu}_{ij}^I=\begin{cases}\tilde{\eta}_{ij}^I & :(i,j)\text{ is an optimal assignment cell of the problem (T)}\\ \tilde{c}_{ij}^I & :\text{Otherwise}\end{cases} \quad (14)$$

where

$$\tilde{\eta}_{ij}^I=(c_{ij}^3-(c_{ij}^3-c_{ij}^2)u_2,c_{ij}^3,c_{ij}^3+(c_{ij}^4-c_{ij}^3)v_1)(c_{ij}^1-(c_{ij}^3-c_{ij}^1)u_1,c_{ij}^3,c_{ij}^3+(c_{ij}^5-c_{ij}^3)v_2).$$

Now, from (6) to (9) and the construction of the values of $u_1$, $u_2$, $v_1$ and $v_2$, we conclude that $X_T$ is an optimal solution of the lower level problems and upper level problems of the modified IFA problem. Therefore, the modified IFA model is realistic. Thus, the modified IFA model is a realistic IFA model corresponding to the given non-realistic IFA model.

Now, we propose a new method namely, level method for obtaining a realistic assignment of IFA problem (Q) from its level assignment problems.

The proposed method proceeds as

**Step 1:** Decompose the given IFA problem (Q) into five crisp assignment problems (NL), (ML), (T), (MU) and (NU).

**Step 2:** Solve the problems (NL), (ML), (T), (MU) and (NU) obtained in the Step1. by the Hungarian method and obtain the optimal solution for each of the problems.

Let $X_{NL}$, $X_{ML}$, $X_T$, $X_{MU}$ and $X_{NU}$ be optimal solutions of the problems (NL), (ML), (T), (MU) and (NU) respectively.
**Step 3:** If $X_T = X_{NL} = X_{ML} = X_{MU} = X_{NU}$, then $X_T$ is a realistic assignment of the given problem (Q) with IFA cost $\tilde{z}^I(X_T)$ by the Theorem 4.1.. If not, go to the Step 4..
**Step 4**: The given problem (Q) is not realistic.
**Step 5**: Construct a realistic IFA model from the IFA model obtained in the Step 4. as per the method given in the Section 4.1.
**Step 6**: Find a realistic assignment of the realistic IFA model obtained in the Step 5. using the Step 1. and the Step 2..

Now, the proposed method is illustrated for solving IFA problems by means of numerical examples.

**Example 4.1** Consider the following IFA problem with cost of triangular IF numbers:

| Person → Job↓ | A | B | C |
|---|---|---|---|
| 1 | (2, 5, 8)(1, 5, 9) | (8.5, 9, 9, 5)(8, 9, 10) | (2.5, 3, 3.5)(2, 3, 4) |
| 2 | (7.5, 8, 8.5)(7, 8, 9) | (6.5, 7, 7.5)(6, 7, 8) | (7, 8, 9)(6, 8, 10) |
| 3 | (5.5, 6, 6.5)(5, 6, 7) | (7, 10, 13)(6, 10, 14) | (11, 12, 13)(10, 12, 14) |

Now, by the Steps 1. to 2. of the level method, optimal solutions of the crisp assignment problems are given below ;

(i) (NL) problem: $X_{NL}: 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 13 ;

(ii) (ML) problem: $X_{MT}: 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 14.5 ;

(iii) (T) problem: $X_T: 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 16;

(iv) (MU) problem: $X_{MU}: 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 17.5 and

(v) (NU) problem: $X_{NU}: 1 \to C; 2 \to B$ and $3 \to A$ with assignment cost 19.

Now, since $X_T = X_{NL} = X_{ML} = X_{MU} = X_{NU}$ and by the level method, $1 \to C; 2 \to B$ and $3 \to A$ is a realistic assignment of the IFA problem with IFA cost (14.5, 16, 17.5) (13, 16, 19).
Now, we present an example of non-realistic IFA model and illustrate the method of obtaining a realistic IFA model from it.

**Example 4.2.** Consider the following IFA problem with costs of triangular IF numbers:

| Worker $\to$ Job$\downarrow$ | W1 | W2 | W3 |
|---|---|---|---|
| J1 | (7, 21, 29) (2, 21, 34) | (7, 20, 57) (3, 20, 61) | (12, 25, 56) (8, 25, 60) |
| J2 | (8, 9, 16) (2, 9, 22) | (4, 12, 35) (1, 12, 38) | (6, 14, 28) (3, 14, 31) |
| J3 | (5, 9, 22) (2, 9, 25) | (10, 15, 20) (5, 15, 25) | (4, 16, 19) (1, 16, 22) |

Now, using the Step 1. and the Step 2. of the level method, optimal solutions of the crisp assignment problems are given below ;

(i) (NL) problem: Optimal assignment, $X_{NL}: J1 \to W1; J2 \to W2$ and $J3 \to W3$ with assignment cost 4 ;

(ii) (ML) problem: Optimal assignment, $X_{MT}: J1 \to W1; J2 \to W2$ and $J3 \to W3$ with assignment cost 15 ;

(iii) (T) problem: Optimal assignment, $X_T$: $J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 43;

(iv) (MU) problem: Optimal assignment, $X_{MU}: J1 \to W1; J2 \to W3$ and $J3 \to W2$ with assignment cost 77 and

(v) (NU) problem: Optimal assignment, $X_{NU}: J1 \to W1; J2 \to W3$ and $J3 \to W2$ with assignment cost 90.

Now, since $X_{NL} = X_{ML} \neq X_T \neq X_{MU} = X_{NU}$, the given problem is not realistic.
Now, $z_3(X_T) = 43$; $z_1(X_T) = 8$; $z_1(X_{NL}) = 4$; $z_2(X_T) = 18$; $z_2(X_{ML}) = 15$; $z_4(X_T) = 107$; $z_4(X_{MU}) = 77$; $z_5(X_T) = 117$ and $z_5(X_{NU}) = 90$
Now, using (10) to (13), we obtain the u's and v's values which are given below:
$u_1 = 1.1143$; $u_2 = 1.12$; $v_1 = 0.5313$ $v_2 = 0.635$
Now, using (14), we obtain the following realistic IFA model corresponding to the given IFA model:

| $\to$ Worker Job$\downarrow$ | W1 | W2 | W3 |
|---|---|---|---|
| J1 | (7, 21, 29)(2, 21, 34) | (5.44, 20, 39.65)(1.06, 20, 46.04) | (12, 25, 56)(8, 25, 60) |
| J2 | (8, 9, 16)(2, 9, 22) | (4, 12, 35)(1, 12, 38) | (5.04, 14, 21.44)(1.74, 14, 24.8) |
| J3 | (4.52, 9, 15.91)(1.2, 9, 19.16) | (10, 15, 20)(5, 15, 25) | (4, 16, 19) (1, 16, 22) |

Now, by the Step 1. and the Step 2. of the level method, optimal assignments of the crisp assignment problems corresponding to the modified IFA model are given below ;

(a) (NL) problem: Optimal assignment, $X_{NL}: J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 4 ;

(b) (ML) problem: Optimal assignment, $X_{MT}: J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 15 ;

(c) (T) problem: Optimal assignment, $X_T$: $J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 43;

(d) (MU) problem: Optimal assignment, $X_{MU}: J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 77 and

(e) (NU) problem: Optimal assignment, $X_{NU}: J1 \to W2; J2 \to W3$ and $J3 \to W1$ with assignment cost 90.

Now, since $X_T = X_{NL} = X_{ML} = X_{MU} = X_{NU}$ and by the level method, $J1 \to W2; J2 \to W3$ and $J3 \to W1$ is a realistic assignment of the modified IFA problem with minimum IFA cost (15, 43, 77) (4, 43, 90).

**Remark 4.2** The optimal solution of the Example 4.2 in Senthil Kumar and Jahir Hussain [17] is $J1 \to W1$; $J2 \to W2$ and $J3 \to W3$ with minimum total IFA cost (15, 49, 83)(4, 49, 94). It can be noted that, when the membership value is one, the optimal solution of the problem is 49, but the actual optimal solution is 43. The optimal solution of the Example 4.2. in Prabharan and Ganesan [14] is $J1 \to W2$; $J2 \to W3$ and $J3 \to W1$ with minimum total IFA cost $(43,13-13r,37-37r;43,17-17r,41-41r)$, a function of $r, r \in [0,1]$. It can be noted that, when $r = 0$, the optimal assignment cost is (30, 43, 80)(26, 43, 84), but the actual value is (18, 43, 107)(8, 43, 117). Now, according to the proposed method, the given IFA model is not realistic. So it is converted into realistic, and the optimal solution of the realistic model related to given IFA model, that is, the realistic assignment, is $J1 \to W2; J2 \to W3$ and $J3 \to W1$ with IFA cost (15, 43, 77)(4, 43, 90).

## 5. CONCLUSION

In this paper, we consider an assignment problem in which cost coefficients are impressive (triangular fuzzy numbers / triangular IF numbers). Realistic assignment of an impressive assignment problem (FA problem / IFA problem) is defined. Level method is presented for finding a realistic assignment of a impressive assignment problem. We prove mathematically, the optimality of the solution obtained by the level method. The method of construction of a realistic impressive assignment model from the non-realistic assignment model is presented. The proposed method is easy to understand and apply in which ranking functions are not applied. Numerical examples are presented to show the importance of the level method by comparing with latest proposed methods. The level method helps decision makers to understand and to apply for finding appropriate realistic assignment of impressive assignment problems occurring in real life situations.

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