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Soret and Dufour Effects on MHD Boundary Layer Flow over Stretching Sheet with Heat Source/Sink

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Abstract

The effect of radiation on Magneto-hydrodynamics (*MHD*) boundary layer flow of a viscous fluid over an exponentially stretching sheet was studied. The governing system of partial differential equations was transformed into ordinary differential equations before being solved numerically by a fourth order Runga-Kutta method along with shooting technique. Numerical results are obtained for the skin-friction coefficient, the local Nusselt number and local Sherwood number as well as the velocity, temperature and concentration profiles for different values of the governing parameters, namely, the magnetic parameter, heat source parameter, radiation parameter, Soret number, Dufour number, Schmidt number and Prandtl number.
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Keywords: Heat and Mass Transfer, radiation, MHD, Stretching sheet, heat source /sink

I. Introduction

The study of heat transfer over a stretching sheet has gained considerable attention due to its applications in industrial manufacturing processes, such as paper production, the aerodynamics extrusion of plastic sheet, glass blowing and metal spinning. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. The quality of the final product depends on the rate of heat transfer at the stretching surface. Numerous investigations have been conducted on the Magneto-hydrodynamics (MHD) flows and heat transfer. MHD was initially known in the field of astrophysics and geophysics and later becomes very important in engineering and industrial processes. For example, MHD can be found in MHD accelerators and generators, electric transformers, power generators, refrigeration coils, pumps, meters, bearing, petroleum production and metallurgical processes which involve cooling of continuous strips or filaments. In metallurgical processes, the rates of cooling and stretching of the strips can be controlled by drawing the strips in an electrically conducting fluid subject to a magnetic field, so that a final product of desired characteristics can be achieved [1-3]. Hassanpour *et al.* [4] investigated numerically the MHD mixed convective flow in a lid-laden cavity filled with porous medium using Lattice Boltzmann method. They found that the fluid circulation within the cavity is reduced by increasing magnetic field strength and the heat transfer depends on the magnetic field strength and the Darcy number. Chamkha [5] and Abo-Eldahab [6] considered MHD problem in three-dimensional flow, while Ishak *et al.* [7] studied the effect of a uniform transverse magnetic field on the stagnation-point flow over a stretching vertical sheet.

Coupled heat and mass transfer finds applications in a variety of engineering applications, such as the migration of moisture through the air contained in fibrous insulation and grain storage installations, filtration, chemical catalytic reactors and processes, spreading of chemical pollutants in plants and diffusion of medicine in blood veins. Free convection flow of an incompressible viscous fluid past an infinite or semi-infinite vertical plate has been studied since long because of its technological importance. Callahan and Marner [8] solved the problem of transient free convection with mass transfer on an isothermal vertical plate using an explicit finite difference scheme. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [9]. Soundalgekar [10] studied the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. In these studies the Magneto-hydrodynamics phenomena is ignored. However in *metallurgical* transport systems, by drawing plates in an electrically conducting fluid subjected to a transverse magnetic field, the rate of cooling can be controlled and the final desired characteristics can be further refined. Gangadhar and Bhaskar Reddy [11] has analyzed by chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction.

Many processes in engineering occur at high temperatures and the full understanding of the effect of radiation on the rate of heat transfer is necessary in the design of equipment. The effect of radiation on the boundary layer flow was studied by Elbashbeshy and Dimian [12], Hossain *et al.* [13], Bataller [14] and Cortell [15]. The radiation effect is considered by Bataller [14] in the study of boundary layer flow over a static flat plate (Blasius flow) and Cortell [15] in the study of boundary layer flow over a moving flat plate (Sakiadis flow) in a quiescent fluid. The problems of Bataller [14] and Cortell [15] have been extended by Ishak [16] and he found the existence of dual solutions when the plate and the fluid move in the opposite directions. Soid et al. [17] studied the Magneto-hydrodynamics Boundary Layer Flows over a Stretching Surface with Radiation Effect and Embedded in Porous Medium.

In all these studies Soret / Dufour effects are assumed to be negligible. Such effects are significant when density differences exist in the flow regime. For example when species are introduced at a surafce in fluid domain, with different (lower) density than the surrounding fluid, both Soret and Dufour effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air), the diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake [18]). In view of the importance of these above mentioned effects, Dursunkaya and Worek [19] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [20] studied the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity.

The study of heat generation or absorption in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions (see Vajravelu and Hadjinicolaou, [21] and Vajravelu and Nayfeh, [22]). In addition, natural convection with heat generation can be applied to combustion modeling (Westphal et al., [23]). Although, exact modeling of the internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behavior for most physical situations. Heat generation or absorption has been assumed to be constant, space-dependent or temperature dependent. Sparrow and Cess [24] have considered temperature dependent heat absorption in their work on steady stagnation point flow and heat transfer. Moalem [25] has studied the effect of temperature dependent heat sources taking place in electrical heating on the heat transfer within a porous medium. Vajravelu and Nayfeh [22] have reported on the hydromagnetic convection at a cone and a wedge in the presence of temperature-dependent heat generation or absorption effects. Chamkha [26] has considered linear variation with temperature dependent heat sources or sinks in his work on mixed convection in a channel filled with a porous medium. Crepeau and Clarksean [27] have used a space-dependent exponentially decaying heat generation or absorption model in their work on flow and heat transfer from a vertical plate.

The aim of the present paper is to analyze the steady Magneto-hydrodynamics (MHD) boundary layer flow due to an exponentially stretching sheet with radiation in the presence of mass transfer, Soret and Dufour effects, and heat source or sink. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runga-Kutta method along with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, concentration, and Nusselt number are shown in figures and analyzed in detail

II. Mathematical Formulation

Consider a steady two-dimensional MHD flow of viscous, incompressible, electrically-conducting and radiating fluid over a vertical stretching surface in the presence of heat source/sink and mass transfer. The *x*-axis is coincident with the vertical surface and the *y*-axis is perpendicular to the surface. *u* and *v* are defined as the velocity components along the *x*-and *y*-axes, respectively. The stretching sheet velocity is assumed to be in the form of $u = ax^m$ where *a* is a positive constant. The velocity at a short distance from the surface allows a thin boundary layer to develop near the surface. The surface temperature, T_w is assumed to follow the power law $T_w = T_\infty + bx^n$ where b is a constant and T_∞ is the ambient temperature. The surface concentration, C_w is assumed to follow the power law $C_w = C_\infty + cx^n$ where *c* is a constant and C_∞ is the ambient concentration. It is also assumed that the magnetic Reynolds number is small in such a way that the induced magnetic field is negligible. Both viscous dissipation and Ohmic heating terms are neglected because their values are generally small. Under these assumptions along with Boussinesq and boundary layer approximations, the governing equations of partial differential equations for the conservation of mass, momentum, energy and Species are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B(x)}{\rho}u\pm g\beta_T(T-T_\infty)\pm g\beta_C(C-C_\infty) \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{D_M k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+\frac{q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

Species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u=ax^m, v=0, T=T_w(x), C=C_w(x) \text{ at } y=0$$

$u \to 0, T \to T_\infty, C \to C_\infty$ as $y \to \infty$ (5)

where T and C is the fluid temperature and temperature in the boundary layer, B(x) is the variable magnetic field strength, ν is the kinematic viscosity, ρ is the fluid density, $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity, σ is the electric conductivity, k is the thermal conductivity, β is the coefficient of thermal expansion, q_0 is the heat generation or absorption coefficient such that $q_0 > 0$ corresponds to heat generation while $q_0 < 0$ corresponds to heat absorption, g is the acceleration due to gravity, "+" and "-" signs correspond to the buoyancy assisting and the buoyancy opposing flow regions, respectively, c_p is the specific heat at constant pressure, q_r is the radioactive heat flux, k_T - the thermal diffusion ratio, c_s - the concentration susceptibility, T_m - the mean fluid temperature and D_m - the mass diffusivity.

By using the Rosseland approximation (Brewster [28]), the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ* and K are the Stefan-Bolzman constant and Rosseland mean absorption coefficient, respectively. The assumption that the temperature differences within the flow are sufficiently small allows T_4 to be expressed as a linear function of temperature by Taylor series expansion,

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4 \tag{7}$$

In view of equations (6) and (7), equation (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\ 1 + N\ \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + \frac{q_0}{\rho c_p}(T - T_\infty) \tag{8}$$

Where

$R = \frac{16\sigma^* T_\infty^3}{3kK}$ is the radiation parameter.

The continuity equation (1) is satisfied by the Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{9}$$

where $\psi(x, y)$ is the stream function.When the variable magnetic field $B(x) = B_0 x^{(m-1)/2}$, the system (1)-(4) admits similarity solutions. The momentum, energy and spices equations along with the boundary conditions can be transformed into a system of coupled ordinary differential equations by the following transformation:

$$\psi = a\upsilon x^{m-1} \tfrac{1}{2} f(\eta), \quad \eta = \left(\frac{ax^{m-1}}{\upsilon}\right)^{1/2} y, \quad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w - C_\infty},$$

$$M = \frac{\sigma B_0^2}{\rho a x^{(m-1)/2}} \ \lambda = \pm\frac{G_{rx}}{\mathrm{Re}_x^2}, Gr_x = \frac{g\beta\, a(T_w - T_\infty)}{\upsilon^2 x^3}, \delta = \pm\frac{G_{cx}}{\mathrm{Re}_x^2}, Gc_x = \frac{g\beta^* a(C_w - C_\infty)}{\upsilon^2 x^3},$$

$$\Pr = \frac{\upsilon}{\alpha}, Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p (T_w - T_\infty)}, Sr = \frac{D_m k_T (T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)}, Q = \frac{q_0 \upsilon}{\rho c_p a x^{m-1}}, Sc = \frac{\upsilon}{D_m} \quad (10)$$

where $f(\eta)$ is the dimensionless stream function, θ - the dimensionless temperature, ϕ - the dimensionless concentration, η - the similarity variable, M - the magnetic parameter, λ - the thermal buoyancy parameter, δ - solutal buoyancy parameter, Gr_x - the thermal grashof number, Gc_x - the solutal Grashof number, *Pr* - the Prandtl number, Sc - the Schmidt number and *R*- the radiation parameter.

In view of equations (9) and (10), the equations (2), (4) and (8) transform into

$$f''' + \frac{m+1}{2} ff'' - mf'^2 - M^2 f' + \lambda\theta + \delta\phi = 0 \quad (11)$$

$$\frac{1}{\Pr}\left(1+R\right)\theta'' + \frac{m+1}{2} f\theta' - nf'\theta + Du\phi'' + Q\theta = 0 \quad (12)$$

$$\frac{1}{Sc}\phi'' + \frac{m+1}{2} f\phi' - nf'\phi + Sr\theta'' = 0 \quad (13)$$

The transformed boundary conditions can be written as

$$f = 0,\ f' = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0$$

$$f' = \theta = \phi = 0 \text{ as } \eta \to \infty \quad (14)$$

The physical quantities of interest are the skin-friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x are defined by

$$C_f = \frac{\tau_w}{\rho U_\infty^2/2}, Nu_x = \frac{xq_w}{k\,(T_w - T_\infty)}, Sh_x = \frac{xm_w}{D(C_w - C_\infty)} \quad (15)$$

Respectively, where the surface shear stress τ_w, and the surface heat flux and surface mass flux is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\left(\frac{a^{3/2} x^{\frac{3m-1}{2}} f''(0)}{\upsilon^{1/2}}\right) \quad (16)$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\left(bx^n \theta'(0)\left(\frac{ax^{m-1}}{\upsilon}\right)^{1/2}\right) \quad (17)$$

$$m_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} = -D\left(cx^n\phi'(0)\left(\frac{ax^{m-1}}{\upsilon}\right)^{1/2}\right) \tag{18}$$

with μ is the dynamic viscosity. Using the non-dimensional variables, we obtain

$$\frac{1}{2}C_f \operatorname{Re}_x^{1/2} = f''(0),$$
$$Nu_x \operatorname{Re}_x^{-1/2} = -\theta'(0),$$
$$Sh_x \operatorname{Re}_x^{-1/2} = -\phi'(0) \tag{19}$$

III. Method of solution

The set of coupled non-linear governing boundary layer equations (11) - (13) together with the boundary conditions (14) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (11) - (13) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al.*[29]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta$=0.05 is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

IV. Result and Discussions

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters viz., the thermal Grashof number λ, solutal Grashof number δ, velocity exponent parameter (m), magnetic field parameter M, Prandtl number Pr, radiation parameter (R), heat source/sink parameter (Q), temperature exponent (n), Dufour number Du, Schmidt number Sc, Soret number Sr.

For various values of the magnetic parameter M, the velocity, temperature and concentration profiles are plotted in Fig.1 (a)-1(c). It can be seen that as M increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow. It is observed that an increase in the magnetic parameter results in a decrease in the temperature and concentration profiles. Figures 2(a)-2(c) shows that as the radiation parameter R increases, velocity and temperature increases whereas the concentration decrease.

Figs.3(a) and 3(b) illustrate the velocity and temperature profiles for different values of the Prandtl number Pr. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Fig.3 (a)). From Fig.3 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

As the heat source/sink parameter Q increases, the velocity and temperature boundary layer thickness increases whereas concentration increase. This is seen from Figure 4(a)-4(c). For different values of the Dufour number Du, the velocity, temperature and concentration profiles are plotted in Figs. 5(a), 5(b) and 5(c) respectively. The Dufour number Du signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in the Dufour number causes a rise in the velocity and temperature throughout the boundary layer where as reduce the concentration boundary layer. Figs. 6(a), 6(b) and 6(c) depict the velocity, temperature and concentration profiles for different values of the Soret number Sr. The Soret number Sr defines the effect of the temperature gradients inducing significant mass diffusion effects. It is noticed that an increase in the Soret number Sr results in an increase in the velocity and concentration within the boundary layer where as reduce the temperature boundary layer.

The influence of the Schmidt number Sc on the velocity and concentration profiles is plotted in Figs. 7(a) and 7(b) respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figs. 7(a) and 7(b).

Table 1, 2 and 3 show the excellent agreement between the numerical results of $f''(0)$ and $-\theta'(0)$ by fourth order Runga-Kutta method along with shooting technique and the results via Keller-Box method of Ishak et al. [7] and also Soid et al. [17]. The effects of various governing parameters on the skin-friction coefficient C_f, Nusselt number Nu and the Sherwood number Sh are shown in Tables 4. From Table 4, it is observed that as M increases, there is a fall in the skin-friction coefficient, the Nusselt number and the Sherwood number. Also, it is noticed that as the radiation parameter or Dufour number or heat source/sink parameter increases, the Nusselt number decreases, while the skin-friction coefficient and Sherwood number increase. It is noticed that an increase in Soret number leads to a fall in the Sherwood number while a rise in the skin-friction coefficient and Nusselt number.

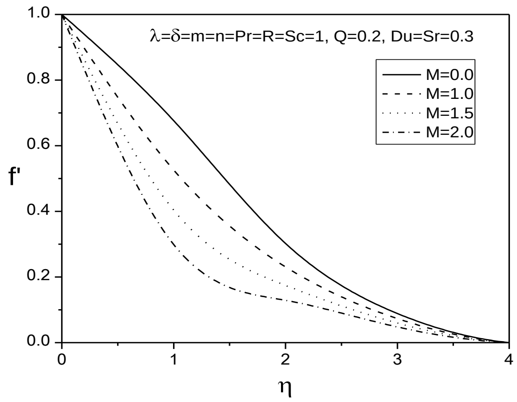

Fig.1(a) Velocity profiles for different values of *M*

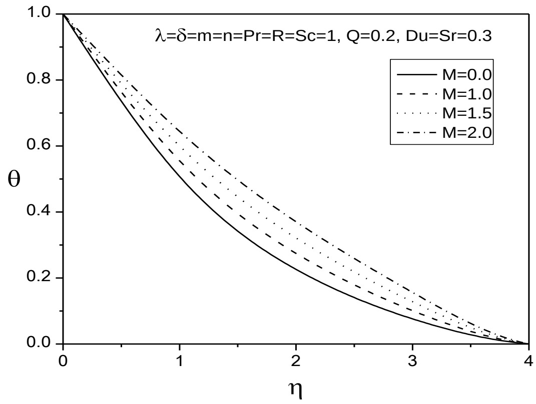

Fig.1(b) Temperature profiles for different values of *M*

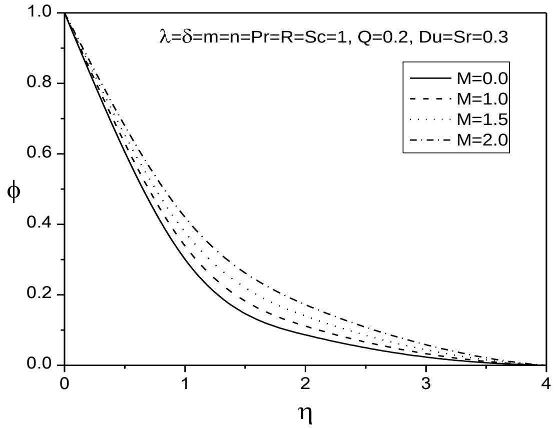

Fig.1(c) Concentration profiles for different values of *M*

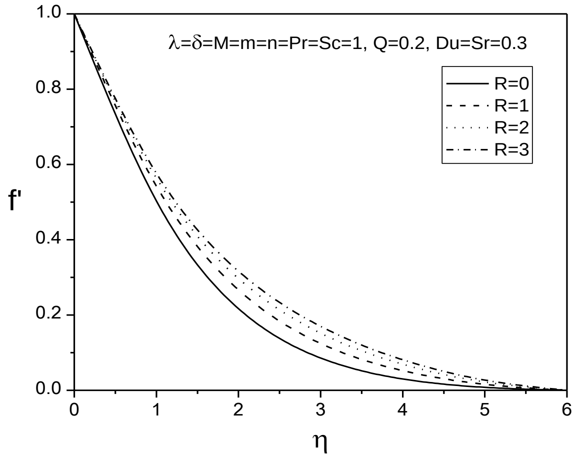

Fig.2(a) Velocity profiles for different values of *R*

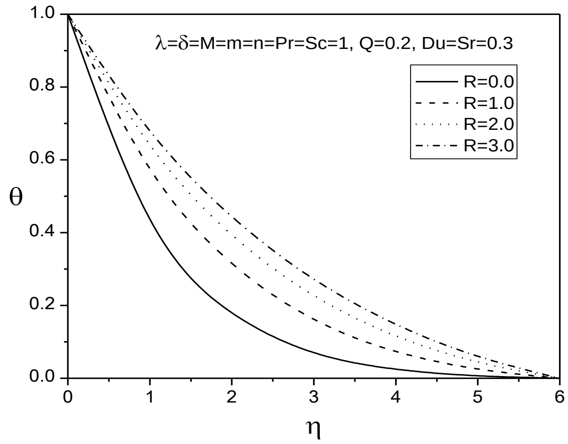

Fig.2(b) Temperature profiles for different values of *R*

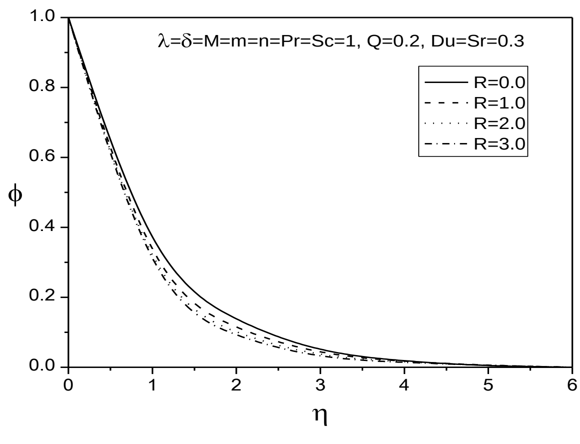

Fig.2(c) Concentration profiles for different values of *R*

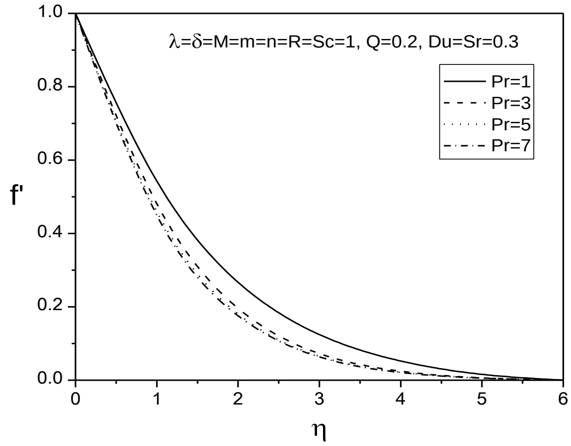

Fig.3(a) Velocity profiles for different values of *Pr*

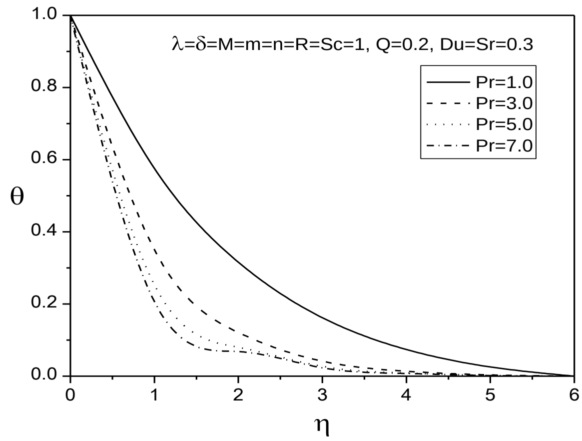

Fig.3(b) Temperature profiles for different values of *Pr*

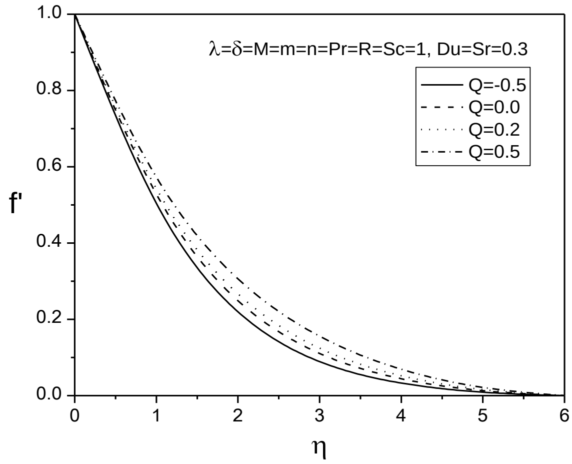

Fig.4(a) Velocity profiles for different values of *Q*

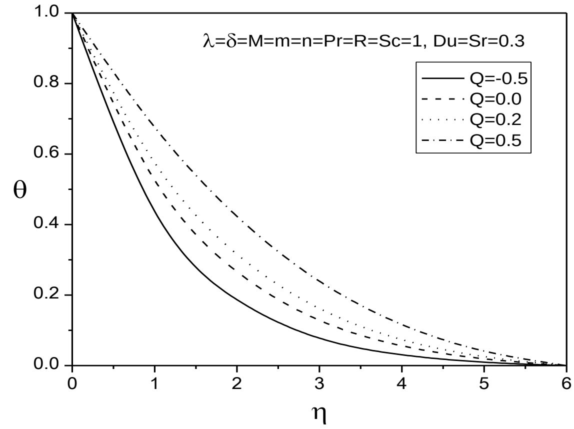

Fig.4(b) Temperature profiles for different values of *Q*

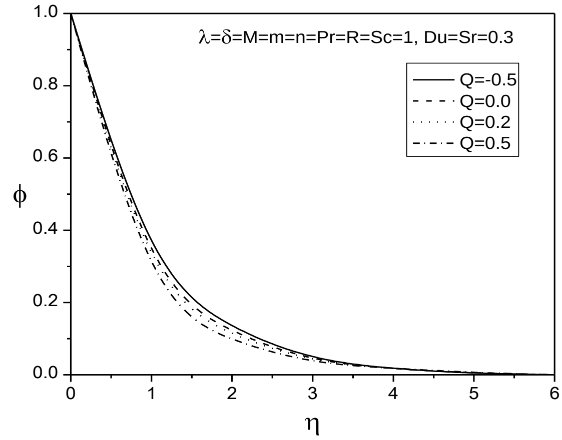

Fig.4(c) Concentration profiles for different values of *Q*

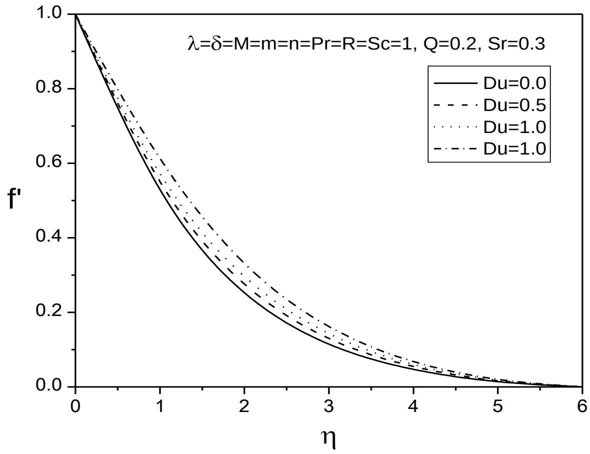

Fig.5(a) Velocity profiles for different values of *Du*

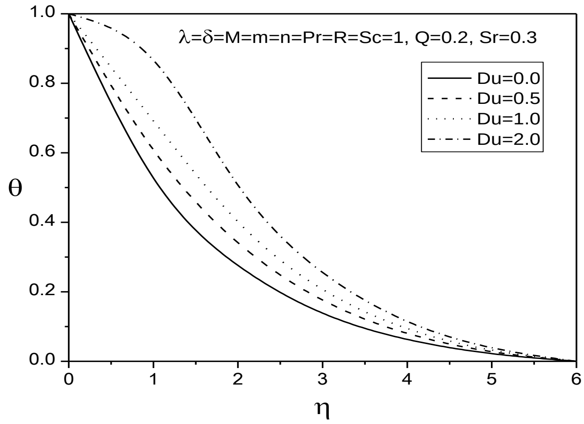

Fig.5(b) Temperature profiles for different values of *Du*

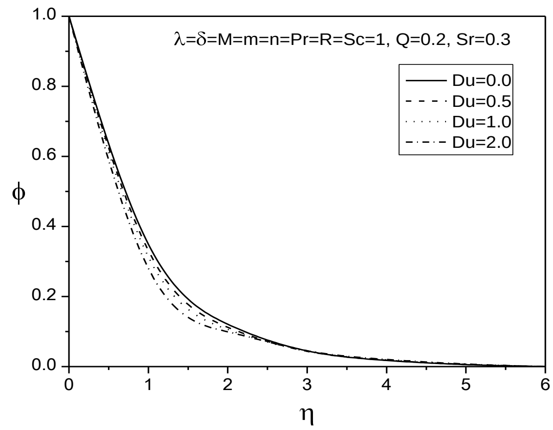

Fig.5(c) Concentration profiles for different values of *Du*

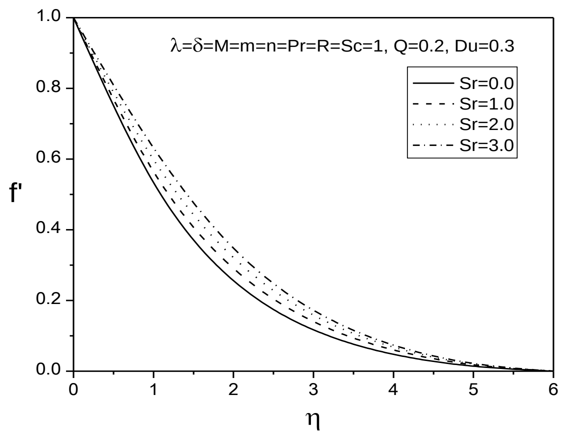

Fig.6(a) Velocity profiles for different values of *Sr*

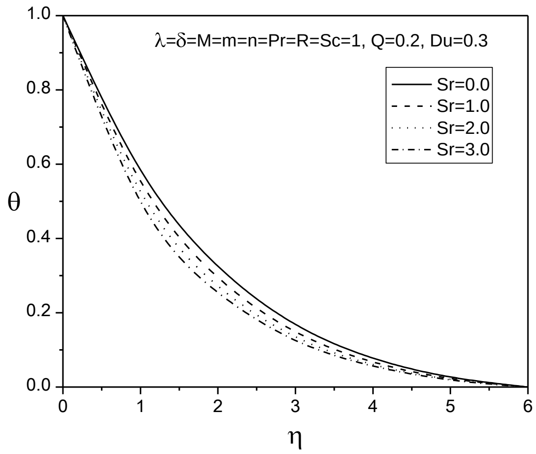

Fig.6(b) Temperature profiles for different values of *Sr*

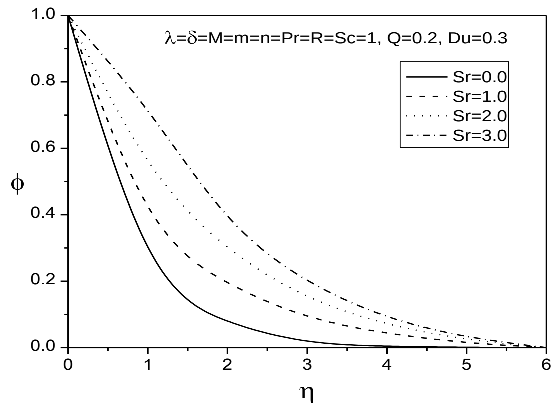

Fig.6(c) Concentration profiles for different values of *Sr*

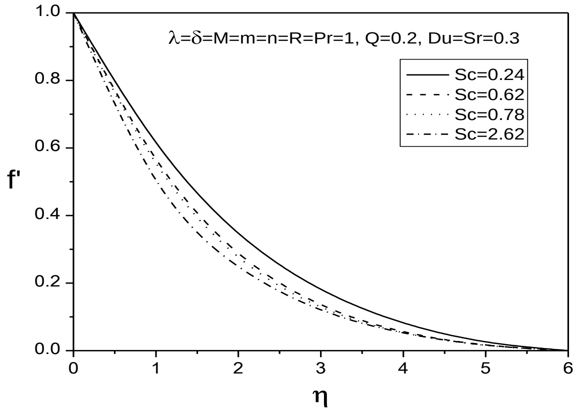

Fig.7(a) Velocity profiles for different values of *Sc*

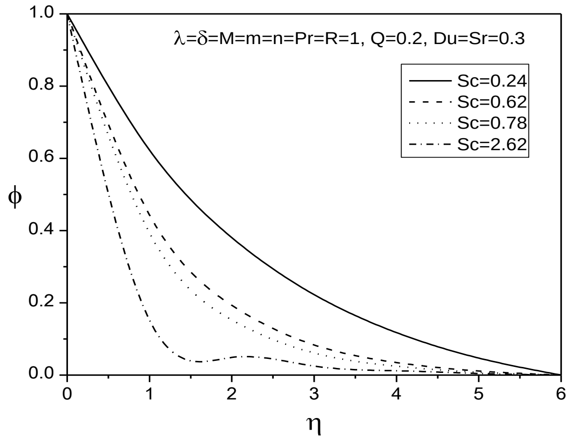

Fig.7(b)Concentration profiles for different values of *Sc*

Table 1 Numerical values of $f''(0)$ and $-\theta'(0)$ at the sheet for different values of M when $\Pr=\lambda=m=n=1$ and $\delta=R=Du=Sr=Sc=0$, Comparison of the present results with that of Ishak et al. [7] and Soid et al. [17]

M	Ishak et al.[7]		Soid et al.[17]		Present results	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	-0.5607	1.0873	-0.56075	1.08727	-0.560811	1.087250
0.1	-0.5658	1.0863	-0.56585	1.08626	-0.565906	1.086250
0.2	-0.5810	1.0833	-0.58103	1.08326	-0.581081	1.083240
0.5	-0.6830	1.0630	-0.68303	1.06301	-0.683073	1.063010
1	-1.0000	1.0000	-1.00000	1.00000	-1.000060	1.000060
2	-1.8968	0.8311	-1.89683	0.83113	-1.901350	0.850574
5	-4.9155	0.4702	-4.91553	0.47027	-4.926900	0.758217

Table 2 Numerical values of $f''(0)$ and $-\theta'(0)$ at the sheet for different values of M when $\Pr = \lambda = m = n = R = 1$ and $\delta = Du = Sr = Sc = 0$, Comparison of the present results with that of Soid et al. [17]

M	Soid et al.[17]		Present results	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	-0.4578	0.7477	-0.458370	0.747544
0.1	-0.4631	0.7468	-0.463753	0.746682
0.2	-0.4792	0.7442	-0.479795	0.744110
0.5	-0.5873	0.7267	-0.587860	0.726717
1	-0.9236	0.6721	-0.924170	0.672646
2	-1.8594	0.5342	-1.869960	0.578604
5	-4.9108	0.2975	-4.923950	0.630072

Table 3 Numerical values of $-\theta'(0)$ at the sheet for different values of *Pr* and *n* when $m=1$ and $M = \lambda = \delta = R = Du = Sr = Q = 0$, Comparison of the present results with that of Soid et al. [17]

n	Soid et al.[17]			Present results		
	Pr=1	*Pr=3*	*Pr=10*	*Pr=1*	*Pr=3*	*Pr=10*
-2	-1.0000	-3.0000	-10.0000	-1.00000	-3.00000	-10.00000
1-	0.0000	0.0000	0.0000	0.00000	0.00000	0.00000
0	0.5820	1.1652	2.3080	0.58197	1.16525	2.30800
1	1.0000	1.9237	3.7207	1.00000	1.92368	3.72067
2	1.3333	2.5097	4.7969	1.33333	2.50972	4.79672

Table 4 Numerical values of $f''(0), -\theta'(0)$ and $-\phi'(0)$ at the sheet for different values of *M, R, Du, Sr* and *Q* when $m = \lambda = \delta = n = \Pr = Sc = 1$.

M	R	Du	Sr	Q	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	1	0.3	0.3	0.2	-0.0251764	0.611923	1.108830
0.5	1	0.3	0.3	0.2	-0.1588940	0.590275	1.089420
1	1	0.3	0.3	0.2	-0.5119040	0.531115	1.037660
1	0	0.3	0.3	0.2	-0.5629930	0.791896	0.961884
1	1	0.3	0.3	0.2	-0.5119040	0.531115	1.037660
1	2	0.3	0.3	0.2	-0.4856490	0.424224	1.069030
1	1	0	0.3	0.2	-0.5291610	0.635004	1.008930
1	1	1	0.3	0.2	-0.4708340	0.262515	1.109980
1	1	2	0.3	0.2	-0.4089830	-0.206330	1.232910
1	1	0.3	0	0.2	-0.5240650	0.515816	1.104510
1	1	0.3	1	0.2	-0.4816400	0.567754	0.863060
1	1	0.3	2	0.2	-0.3795740	0.694278	0.146432
1	1	0.3	0.3	-0.5	-0.5634830	0.813724	0.956313
1	1	0.3	0.3	0	-0.5308730	0.625463	1.010550
1	1	0.3	0.3	0.5	-0.4709280	0.351237	1.088320

V. Conclusions

In the present paper, the steady Magneto-hydrodynamics (MHD) boundary layer flow due to stretching sheet with thermal radiation by taking mass transfer and, heat source/sink, soret and dufour effects into account, are analyzed. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. The self-similar equations are solved by using fourth order Runga-Kutta method along with shooting technique. Our results show a good agreement with the existing work in the literature. The results are summarized as follows:

- The steady reveals that due to increase of the magnetic field parameter reduces the momentum boundary layer thickness.
- The radiation and Prandtl number reduce the temperature.
- The Schmidt number reduces the concentration.
- Dufour number or heat source/sink parameter increases the skin-friction and Sherwood number where as reduce the Nusselt number.

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