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MHD Free Surface Flow Of A Jeffrey Fluid Over A Deformable Porous Layer

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ABSTRACT

Free surface flow of a conducting Jeffrey fluid in a channel is investigated. The channel is bounded below by a finite deformable porous layer. The governing equations are solved in the free flow and deformable porous flow regions. The expressions for the velocity field and solid displacement are obtained. The effects of the Jeffrey parameter, magnetic field parameter, viscosity parameter, the volume fraction component of the fluid on the flow velocity, displacement, mass flux and shear stress are discussed. It is found that the velocity increases with the increase in the non-Newtonian Jeffrey parameter whereas the velocity decreases with the increase in the magnetic field parameter.

Keywords: MHD; free surface flow; Jeffrey fluid; Porous layer; permeable bed.

1. INTRODUCTION

Viscous flow through and past porous media has important applications in engineering and medicine. Most of the research works in flow through porous media available deal with undeformable porous media. But the studies on deformable porous media is very limited. The coupled phenomenon of fluid flow and deformation of porous materials is a problem of prime importance in geomechanics and biomechanics. One application of interaction of free flow and deformable porous media, for example, is the study of haemodynamic effect of the endothelial glycocalyx.

Nomenclature

μ_a	Apparent viscosity of the fluid in the porous material.
K	Drag coefficient.
μ	Lame constant.
μ_f	Coefficient of viscosity.
q	Fluid velocity in the free flow region in
x -	direction.
u	Displacement in x-direction.
G_0	Typical pressure gradient.
λ_1	Jeffrey parameter.
M	Magnetic field parameter.
ε	Porous layer thickness.
σ	Electrical conductivity
τ_1	Shear stress.
ϕ^{β}	Volume fraction of component β and
$\beta = s, f$	for the binary mixture of solid and fluid phases with $\varphi^s + \varphi^f = 1$.
M_2	Mass flow rate in the deformable porous layer.
M_0	Mass flow rate in the non deformable porous layer.
F	Fractional increase in mass flow rate.
v	Velocity of the fluid in the deformable porous layer.
δ	Viscous drag.
η	Viscosity parameter in porous layer.
M_T	Total mass flux in the channel.
B_0	Magnetic field strength

The study of flow through deformable porous materials was initiated by Terzaghi [1] and later continued by Biot [2, 3, 4] into a successful theory of soil consolidation and acoustic propagation. Atkin and Craine [5], Bowen [6] and Bedford and Drumheller [7] made important contributions to the theory of mixtures. Mow et al. [8] developed a similar theory for the study of biological tissue mechanics. Applying the theory proposed by Biot [2] water transport in the artery wall is studied by Jayaraman [9]. Sreenadh et al. [10] analyzed the Couette flow of a viscous fluid in a parallel plate channel in which a finite deformable porous layer is attached to the lower plate. It is found that the increase in the volume fraction component of fluid phase reduces the magnitude of velocity in the free flow region of the horizontal channel. All these works are concerned with Newtonian fluid flow through deformable porous media.

But non-Newtonian behavior of the biofluids such as blood plays an important role in biofluid transport through and past tissue regions of biological systems. In view of this, it is necessary to consider non-Newtonian fluid flow through a deformable porous layer.

Most of the industrial and biofluids are classified as non-Newtonian fluids. Further Jeffrey model is one of the best non-Newtonian fluid models used by researchers to explain the biological fluid flow in living organisms. Kothandapani and Srinivas [11] analyzed the peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Hayat and Ali [12] examined the peristaltic transport of a Jeffrey fluid in a tube under the effect of a magnetic field. Nadeem et al. [13] examined the effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. Makinde and Aziz [14] carried out the MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. Vajravelu et al [15] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Hayat et al. [16] analyzed the boundary layer flow of a Jeffrey fluid with convective boundary conditions. The effect of magnetic field on the peristaltic pumping of a Jeffrey fluid in an inclined channel is analyzed by Krishna Kumari et al. [17]. Bhaskara Reddy et al. [18] studied the flow of a Jeffrey fluid between torsionally oscillating disks. Rudraiah et al. [19] analyzed the Hartmann flow over an undeformable permeable bed.

Motivated by these studies, MHD free surface flow of a Jeffrey fluid over a deformable porous layer is investigated. The fluid velocity, the displacement of the solid matrix, the mass flux and its fractional increase are obtained. The effects of various physical parameters on the flow quantities are discussed through graphs.

2. MATHEMATICAL FORMULATION

Consider the free surface flow of a Jeffrey fluid over a finite deformable porous layer of thickness L (Fig. 1). The deformable porous layer is bounded below by a rigid plate. The flow region is divided into two regions. The flow region between the lower plate $y=-L$ and the interface $y=0$ of deformable porous layer is named as Region-I. The flow region between the interface $y=0$ and the free surface $y=h$ is designated as Region-II. The fluid velocity in the free flow region and the porous flow region are assumed to be $(q,0,0)$ and $(v,0,0)$ respectively. The displacement due to the deformation of the solid matrix is taken as $(u,0,0)$. A pressure gradient $\frac{\partial p}{\partial x}=G_0$ is applied, producing an axially directed flow in the channel. A uniform transverse magnetic field of strength B_0 is applied perpendicular to the rigid plate $y=-L$.

In view of the assumptions mentioned above, the equations of motion in the flow Regions I and II are [19]

$$\mu\frac{\partial^2 u}{\partial y^2}-\phi^s G_0+K\,v=0 \tag{1}$$

$$\frac{2\mu_a}{1+\lambda_1}\frac{\partial^2 v}{\partial y^2}-\phi^f G_0-K\,v-\sigma B_0^{\,2}v=0 \tag{2}$$

$$\frac{\mu_f}{1+\lambda_1}\frac{\partial^2 q}{\partial y^2}-\sigma B_0^2 q=G_0 \tag{3}$$

The boundary conditions are

at $y=-L$: $v=0,\ u=0$

at $y=0$: $q=\phi^f v$

$$\phi^f\mu_f\frac{dq}{dy}=2\mu_a\frac{dv}{dy}$$

$$\mu_f\frac{dq}{dy}=\frac{\mu}{\phi^s}\frac{du}{dy}$$

at $y=h$: $\dfrac{dq}{dy}=0$ (4)

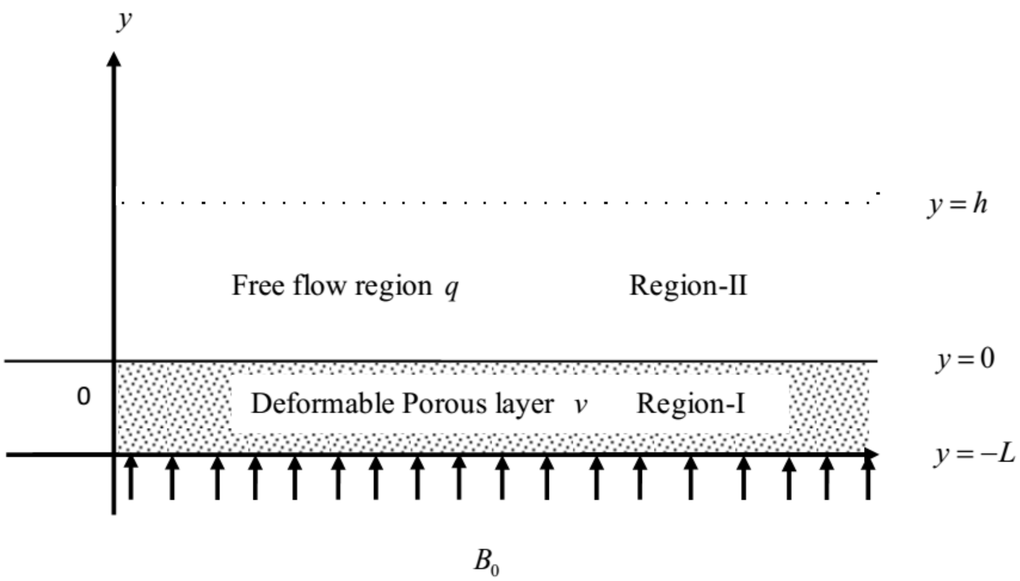

Figure 1: Physical model

3. NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES

It is convenient to introduce the following non-dimensional quantities:

$$y=h\hat{y},\quad u=-\frac{h^2G_0}{\mu}\hat{u},\ v=-\frac{h^2G_0}{\mu_f}\hat{v},\ \ q=-\frac{h^2G_0}{\mu_f}\hat{q},\ \varepsilon=\frac{L}{h},\ \hat{\tau}=-\frac{\tau}{hG_0}$$

In view of the above dimensionless quantities, the equations (1) – (4) takes the following form after the hats $(\wedge)$ are neglected.

$$\frac{d^2u}{dy^2} = -\phi^s - \delta v \tag{5}$$

$$\frac{d^2v}{dy^2} = (1+\lambda_1)\eta\left[(\delta + M^2)v - \phi^f\right] \tag{6}$$

$$\frac{d^2q}{dy^2} - M^2(1+\lambda_1)q = -(1+\lambda_1) \tag{7}$$

where $M^2 = \frac{\sigma B_0^2 h^2}{\mu_f}$, $\delta = \frac{Kh^2}{\mu_f}$, $\hat{G} = \frac{G}{G_0}$, $\eta = \frac{\mu_f}{2\mu_a}$, $G_0 = \frac{dp}{dx}$.

The parameter δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium. The parameter η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are

at $y = -\varepsilon$: $v = 0, u = 0$;

at $y = 0$: $q = \phi^f v$

$$\frac{dq}{dy} = \frac{1}{\eta\phi^f}\frac{dv}{dy}$$

$$\frac{dq}{dy} = \frac{1}{\phi^s}\frac{du}{dy}.$$

at $y = 1$: $\frac{dq}{dy} = 0$ (8)

4. SOLUTION OF THE PROBLEM

Equations (5)-(7) are coupled differential equations that can be solved by using the boundary conditions (8). The solid displacement and fluid velocities in the free flow region and deformable porous layer are obtained as

$$u(y) = -(1-\phi^f)\frac{y^2}{2} - \frac{\delta c_3 e^{by}}{b^2} - \frac{\delta c_4 e^{-by}}{b^2} - \frac{\phi^f \delta}{\delta + M^2}\frac{y^2}{2} + c_5 y + c_6 \tag{9}$$

$$q(y) = c_1 e^{ay} + c_2 e^{-ay} + \frac{1}{M^2} \tag{10}$$

$$v(y) = c_3 e^{by} + c_4 e^{-by} + \frac{\varphi^f}{\delta + M^2} \tag{11}$$

where $a = M\sqrt{(1+\lambda_1)}$ and $b = \sqrt{((1+\lambda_1)\delta + a)\eta}$ the constants c_1, c_2, c_3, c_4, c_5 and c_6 found from the boundary conditions, are

$$c_1 = \frac{(c_3 + c_4)\phi^f - \left(\frac{1}{M^2} - \frac{(\phi^f)^2}{\delta + M^2}\right)}{1 + e^{2a}},\ c_2 = c_1 e^{2a},\ A_1 = \frac{b\left(1 + e^{2a}\right)}{\eta\phi^f} - a\phi^f\left(1 - e^{2a}\right),$$

$$A_2 = \frac{b\left(1+e^{2a}\right)}{\eta\phi^f} + a\phi^f\left(1-e^{2a}\right),\ A_3 = \left(\frac{1}{M^2} - \frac{\left(\phi^f\right)^2}{\delta+M^2}\right)a\left(1-e^{2a}\right),$$

$$c_3 = -\frac{\left(A_3 e^{b\varepsilon} + \frac{A_2\phi^f}{\delta+M^2}\right)}{A_1 e^{b\varepsilon} + A_2 e^{-b\varepsilon}},\ c_4 = -\left(c_3 e^{-b\varepsilon} + \frac{\phi^f}{\delta+M^2}\right)e^{-b\varepsilon},$$

$$c_5 = \left(a(c_1-c_2) + \frac{\delta(c_3-c_4)}{(1-\phi^f)b}\right)\left(1-\phi^f\right),\ c_6 = \frac{\delta}{b^2}\left(c_3 e^{-b\varepsilon} + c_4 e^{b\varepsilon}\right) + \frac{\varepsilon^2}{2}\left(\left(1-\phi^f\right) + \frac{\phi^f\delta}{\delta+M^2}\right) + c_5\varepsilon$$

4.1 Mass Flux

(i) Let M_T denote the dimensionless mass flow rate per unit width of the channel is

$$M_T = M_1 + M_2 \tag{12}$$

where $M_1 = \int_{-\varepsilon}^{0} v\,dy = \frac{c_3-c_4}{b} - \left(\frac{c_3 e^{-b\varepsilon} - c_4 e^{b\varepsilon}}{b} - \frac{\phi^f\varepsilon}{\delta+M^2}\right)$ and

$$M_2 = \int_0^1 q\,dy = \frac{c_1 e^a - c_2 e^{-a} - \left(c_1 - c_2\right)}{a} + \frac{1}{M^2}$$

Here we note that M_1 and M_2 describe the mass flow rates in the porous and free flow regions.

(ii) The fluid velocity q_0 for the MHD free surface flow of a Jeffrey fluid over a rigid plate is obtained on solving the equation (7) subject to the following boundary conditions:

at $y=0$: $q_0 = 0$

at $y=1$: $\frac{dq_0}{dy} = 0$

It can be seen that $q_0 = A\mathrm{e}^{ay} + B\mathrm{e}^{-ay} + \frac{1}{M^2}$

where $A = -\frac{e^{-a}}{M^2\left(e^{-a}+e^{a}\right)}$, $B = -\frac{1}{M^2} - A.$

Then the dimensionless mass flow rate M_0 per unit width of the channel in the free flow region $\left(0 \le y \le 1\right)$ is given by

$$M_0 = \int_0^1 q_0\,dy = \frac{\left(c_1 e^a - c_2 e^{-a} - \left(c_1 - c_2\right)\right)}{a} + \frac{1}{M^2} \tag{13}$$

4.2 Fractional increase

Let F denote the fractional increase in mass flow rate due to deformable porous layer and it is defined by

$$F = \frac{M_2 - M_0}{M_0} \tag{14}$$

4. 3 Shear stress

The shear stress at the deformable interface y = 0 is given by

$$\tau_1 = \frac{1}{1+\lambda_1}\left(\frac{dq}{dy}\right)_{y=0}$$

$$= \frac{a}{1+\lambda_1}\left(c_1 - c_2\right) \tag{15}$$

5. RESULTS AND DISCUSSIONS

The solutions for the fluid velocities q, v in the free flow region and deformable porous layer and solid displacement of solid matrix u are evaluated numerically for different values of physical parameters such as the volume fraction of component ϕ^f, the viscous drag parameter δ, the viscosity parameter η and the thickness of lower wall ε, magnetic field parameter M and Jeffrey parameter λ_1.

The variation of fluid velocities in the channel q, v is calculated for different values of viscosity parameter η and is shown in figure 2 for fixed δ =2. 0, ϕ^f =0. 5, $M = 1$, $\lambda_1 = 0.1$ and ε =0. 2. We observe that the velocities q and v increases with the increasing viscosity parameter η.

The variation of fluid velocities in the channel q, v and solid displacement u in the channel is calculated for different values of volume fraction of component ϕ^f and is shown in figures 3 for fixed δ =2. 0, η =0. 5, $M = 1$, $\lambda_1 = 0.1$ and ε =0. 2. We observe that the velocities q, v increase with the increasing ϕ^f whereas the solid displacement u decreases with the increase in ϕ^f.

The variation of fluid velocities q, v and solid displacement u in the channel is calculated for different values of Jeffrey parameter λ_1 and is shown in figure 4 for fixed δ =2. 0, η =0. 5, $M = 1$, ϕ^f =0. 5and ε =0. 2. We observe that the velocities q, v and solid displacement increase with the increase Jeffrey parameter λ_1.

The variation of fluid velocities q, v and solid displacement u in the channel is calculated for different values of magnetic field parameter M and is shown in figure 5 for fixed δ =2. 0, η =0. 5, ϕ^f =0. 5, $\lambda_1 = 0.1$ and ε =0. 2. We observe that the velocities q, v and solid displacement decreases with the increase magnetic field parameter M .

The variation of dimensionless mass flow rate per unit width of the channel M_T with ϕ^f is calculated from equation (12) and is shown in figure 6 for fixed δ =2. 0, η =0. 5, ε =0. 2 and $\lambda_1 = 0.1$. We observe that the mass flow rate decreases with increasing magnetic field parameter M . Further the effect of magnetic field is to reduce the mass flow rate and the rate of reduction depends on the strength of the magnetic field, which is similar to the observation made by Rudraiah et al. [18] for the Hartmann flow over a non-deformable permeable bed.

The variation of dimensionless mass flow rate per unit width of the channel $_c$ with ϕ^f is calculated from equation (12) and is shown in figure 7 for fixed δ =2. 0, η =0. 5, ε =0. 2 and $M = 1$. We observe that the mass flow rate increases with increasing Jeffrey parameter λ_1.

The variation of fractional increase F with M is calculated from equation (14) and is shown in figure 8 for fixed δ =2. 0, η =0. 5, ε =0. 2 and $\lambda_1 = 0.1$. We observe that the fractional increase decreases with increasing magnetic field parameter M .

The variation of shear stress τ_1 with λ_1 is calculated from equation (14) and is shown in Table 1 for fixed $\delta = 2$, $\varepsilon = 0.2$, $M = 1$ and $\eta = 0.5$. We observe that the shear stress at the wall interface decreases with increasing Jeffrey parameter λ_1.

The variation of shear stress τ_1 with M is calculated from equation (14) and is shown in Table 1 for fixed $\delta = 2$, $\varepsilon = 0.2$, $\lambda_1 = 0.1$ and $\eta = 0.5$. We observe that the shear stress at the wall interface decreases with increasing magnetic field parameter M.

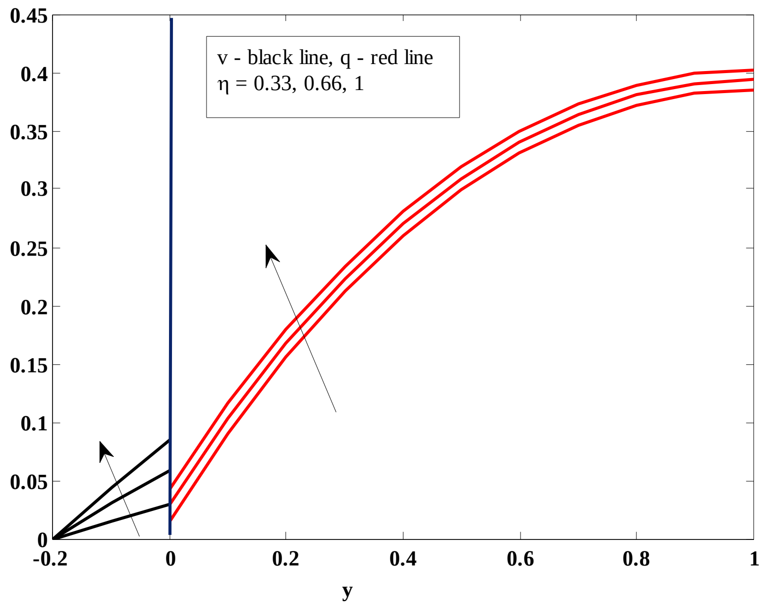

Figure 2: Velocity profiles for deformable porous region $v(y = -0.2 - 0)$ and free flow region $q(y = 0 - 1)$ for different values of η.

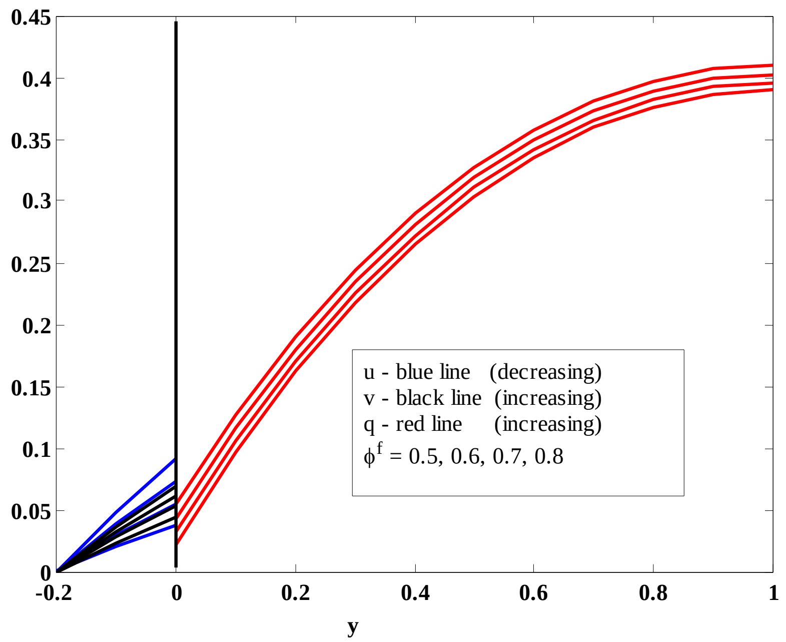

Figure 3: Velocity and displacement profiles for deformable porous region $v(y = -0.2 - 0)$ and free flow region $q(y = 0 - 1)$ for different values of ϕ^f.

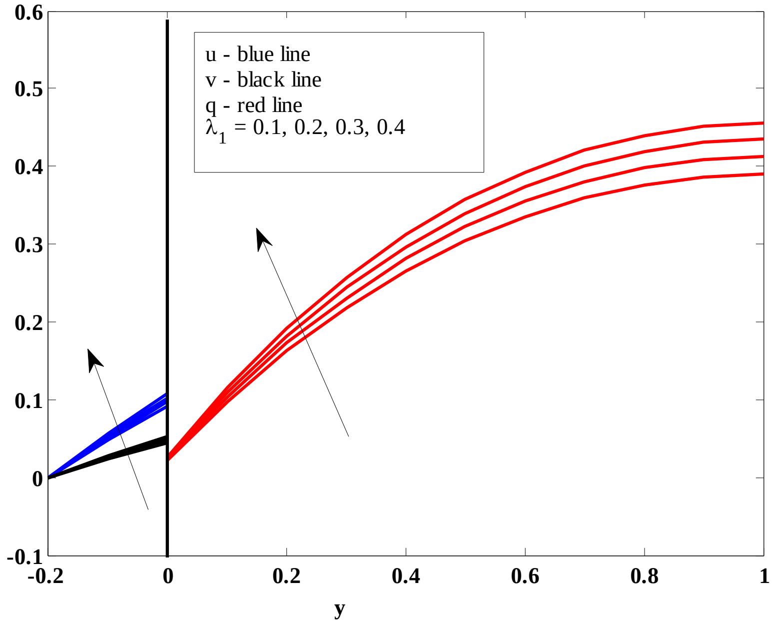

Figure 4: Velocity and displacement profiles for deformable porous region $v(y=-0.2-0)$ and free flow region $q(y=0-1)$ for different values of λ_1.

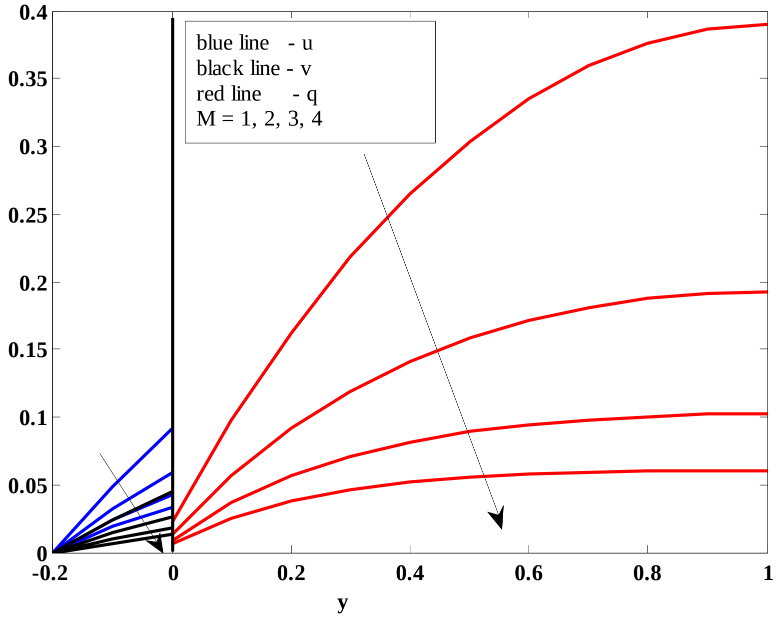

Figure 5: Velocity and displacement profiles for deformable porous region $v(y=-0.2-0)$ and free flow region $q(y=0-1)$ for different values of M.

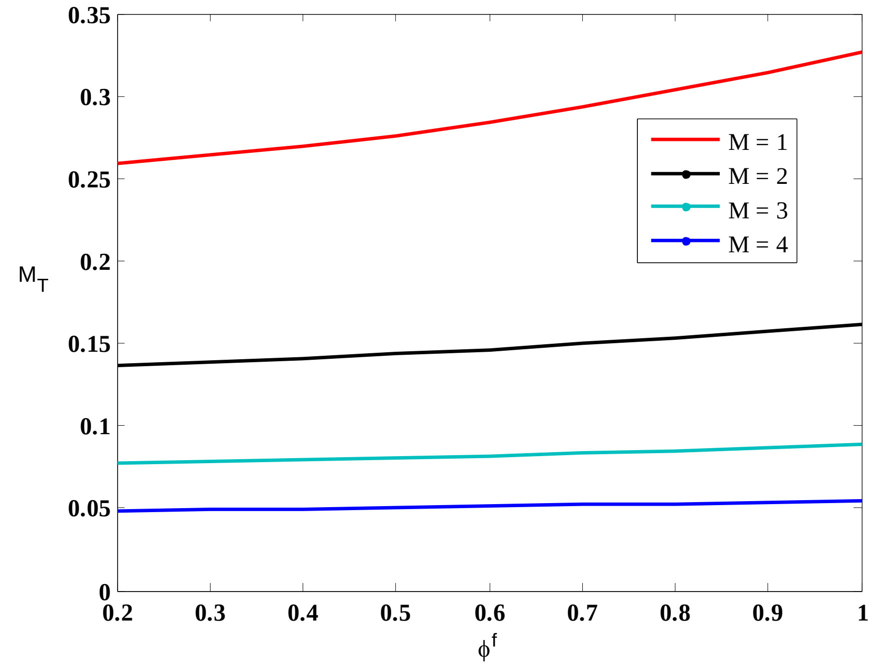

Figure 6: Mass flux M_T for different values of M .

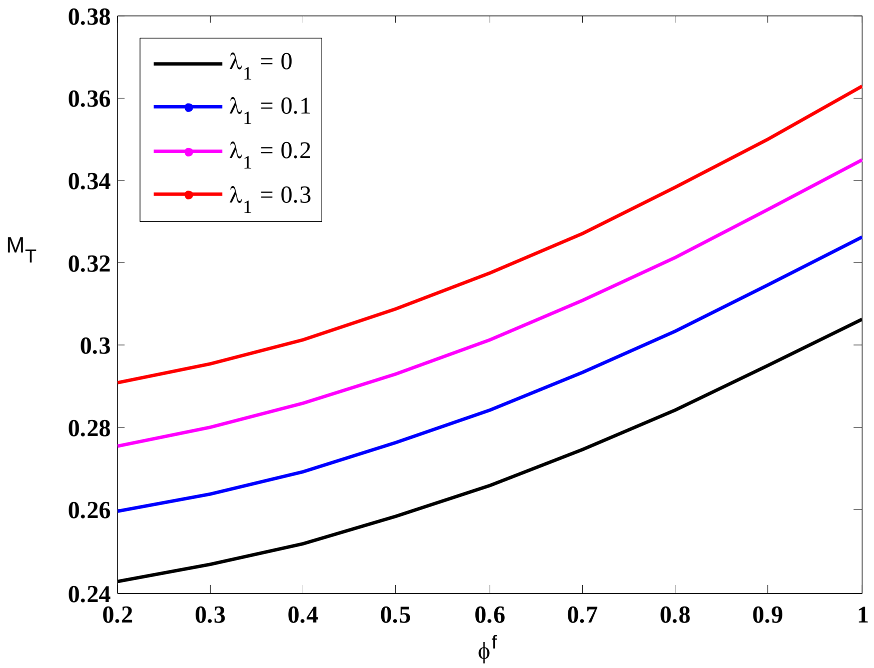

Figure 7: Mass flow rate M_T for different values λ_1 .

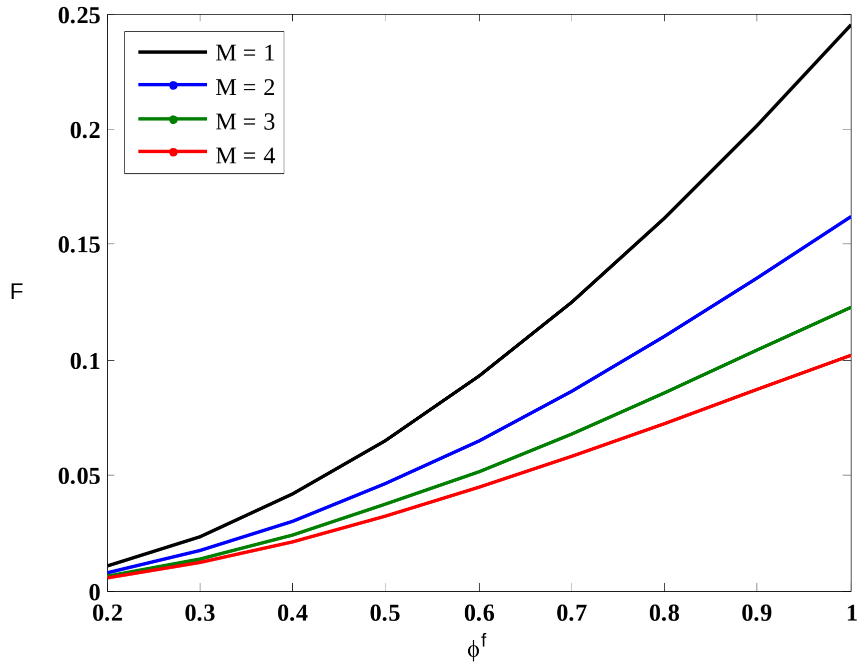

Figure 8: Fractional increase F for different values of M .

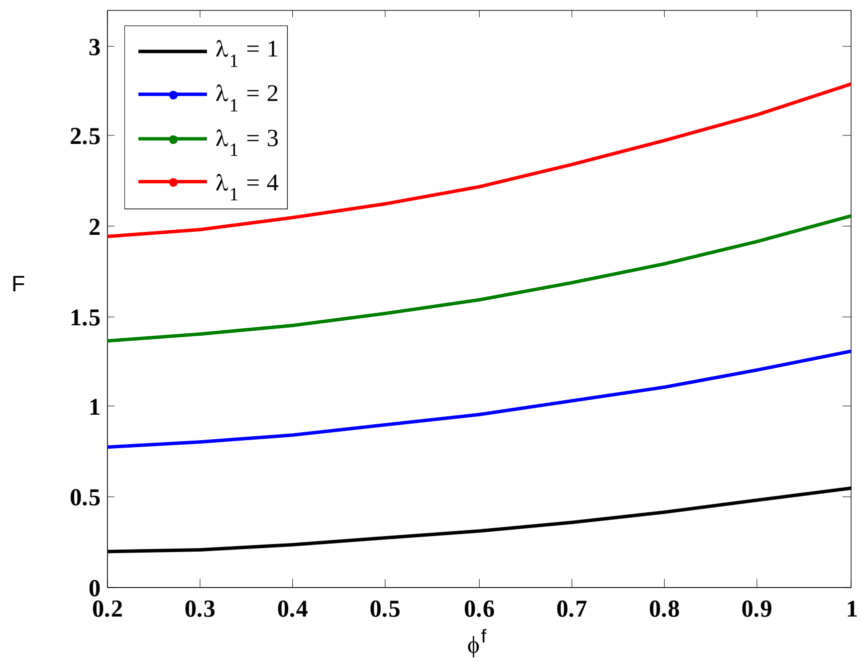

Figure 9: Fractional increase F for different values of λ_1 .

Table 1: Shear stress for different values of M and λ_1 at the porous boundary wall $y=0$.For fixed values of $\delta=2$, $\varepsilon=0.2$, $\eta=0.5$.

ϕ^f	$(\tau_1)_{y=0}$ for λ_1 = 0.1				$(\tau_1)_{y=0}$ for $M=1$			
	M = 1	M = 2	M = 3	M = 4	λ_1 = 0	λ_1 = 0.1	λ_1 = 0.2	λ_1 = 0.3
0.2	0.742285	0.458438	0.312334	0.233978	0.759030	0.742285	0.726438	0.711416
0.3	0.738939	0.453352	0.307086	0.228815	0.755846	0.738939	0.722945	0.707791
0.4	0.734300	0.446397	0.299993	0.221915	0.751428	0.734300	0.718106	0.702769
0.5	0.728410	0.437724	0.291285	0.213563	0.745814	0.728410	0.711966	0.696405
0.6	0.721323	0.427515	0.281222	0.204071	0.739052	0.721323	0.704587	0.688762
0.7	0.713102	0.415973	0.270082	0.193759	0.731199	0.713102	0.696038	0.679919
0.8	0.703820	0.403309	0.258141	0.182927	0.722319	0.703820	0.686398	0.669959
0.9	0.693554	0.389741	0.245661	0.171842	0.712482	0.693554	0.675752	0.658977
1.0	0.682387	0.375478	0.232880	0.160733	0.701762	0.682387	0.664191	0.647069

6. CONCLUSIONS:

The present study deals with MHD free surface flow of a Jeffrey fluid over a deformable porous layer. The results are analyzed for different values of the pertinent parameters, namely, Jeffrey parameter, volume fraction component. The findings of the problem find applications in understanding the blood (modelled as Jeffrey fluid) flow behavior near the tissue layer (modelled as a deformable porous layer). Some of the interesting findings are as follows:

1. The effect of increase in the volume fraction component ϕ^f is to enhance the fluid velocity between the parallel plates.
2. The effect of magnetic field is to reduce the fluid velocity in the free flow region whereas in the deformable porous layer, both the fluid velocity and solid displacement decreases with increasing magnetic field.
3. The flux in the free flow region increases with an increase in the Jeffrey parameter. Also opposite behavior is noticed in case of magnetic field.
4. The effect of increase in the magnetic field parameter and Jeffrey parameter is to reduce the shear stress at the deformable porous boundary.

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