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Graph critical with respect to Edge domination in Fuzzy graphs

S. Ramya[1] and S. Lavanya[2]

[1]Research scholar,
Research and development centre, Bharathiar University, Coimbatore,
Asst. Professor, R.M.K..Engineering College, R. S. M. Nagar, Kavaraipettai,
Tamilnadu 601206 India. ramyadheeru10@gmail. com
[2]Assistant Professor, Department Of Mathematics, Periyar Government arts college,
Cuddalore, Tamilnadu 607001 India. lavanyaprasad1@gmail. com

Abstract:

A subset X of E is called an edge dominating set of G if every edge not in X is adjacent to some edge in X. The edge domination number of G is the minimum cardinality taken over all edge dominating sets of G. In this paper we define edge domination in Fuzzy Graph and we analyze the concept of vertex and edge critical fuzzy graph with respect to edge domination and also Independent domination, and characterize the vertices or edges whose deletion affects the edge and independent domination number of a fuzzy graph.

Key Words: Fuzzy graph, Fuzzy domination, edge domination number, independent domination number.

Introduction:

Graph is a pictorial way of representation of things. Graph theory studies about the graphs which are mathematical structures used to model pair wise relationship between the objects. The vagueness in the real life situations served as a platform for the development of fuzzy set theory and in turn fuzzy graph theory. Rosenfeld in 1975 introduced the fuzzy graph theory as a generalization of Euler's graph theory. He obtained analogs of several graph theoretical concepts. Mordeson introduced the concept of fuzzy line graphs and he also discussed some operations on fuzzy graph[8]. The concept of domination in Fuzzy graph was introduced by Somasundaram and he obtained several bounds for domination number[4, 6]. Next, the concept of edge domination was introduced by Mitchell and Hedetniemi. Also S. Arumugam and S. velammal studied about the edge domination in graphs[5]. Critical graphs has served as one of the most important area of interest in the field of research in graph theory. D. K. Thakkar and J. C. Bosamiya, studied about Graph critical with respect to independent domination number [1]. In this paper we have made an attempt to investigate the critical concept with respect to the domination variants in fuzzy graphs. We define edge domination number of a Fuzzy graph and we analyze how the removal of a vertex or edge affects the edge domination number.

Preliminaries

Definition 1. 1:(4)

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy Graph G= (σ, μ) is a set with 2 functions σ : V -> [0, 1] and μ:E->[0, 1] such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y∊ V.

The order p and size q of a fuzzy graph G= (σ, μ) are defined to be $p=\sum_{x\epsilon V} \sigma(x)$ and $q = \sum_{xy\epsilon E} \mu(x,y)$.

Definition 1. 2: (4)

Degree of an edge uv of G is defined by deg e = degu+deg v -2.

Definition 1. 3: (4)

Let G= (σ, μ) be a fuzzy graph on V. let u, v ϵ V, we say that u dominates v in G if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. A subset S of V is called a dominating set in G if for each vertex u ϵ V-S, there exists a vertex v∊ S, such that u dominates v.

Definition 1. 4: (4)

A fuzzy dominating set S of a fuzzy graph G is called minimal fuzzy dominating set of G if for each vertex v∊ S, S-{v} is not a dominating set.

Definition 1. 5:(4)

The fuzzy domination number ϒ (G) is the minimum cardinality taken over all minimal fuzzy dominating set of G.

Definition 1. 6: (4)

Let G= (σ, μ) be a fuzzy graph on V. A subset S of V is said to be an independent set if $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all u, v ϵ S.

Definition 1. 7: (4)

A fuzzy independent set S of G is said to be maximal fuzzy independent set if there is no fuzzy independent set whose cardinality is greater than the cardinality of S.

Definition 1. 8: (4)

The maximum fuzzy cardinality of an independent set in G is called the independence number of G and it is denoted by $\beta_0(G)$.

Definition 1. 9: (4)
The smallest cardinality of all independent fuzzy dominating set of G is called independent fuzzy domination number of G and it is denoted by i(G).

Definiton 1. 10
Let G be a fuzzy graph the neighbourhood of an edge uv is a set of edges in ExE that are adjacent to uv.
Next in this section we define the edge domination number of a fuzzy graph.

Definition 2. 1:
Let G= (σ, μ) be a fuzzy graph on V. A subset S of V×V is called an edge dominating set of G if every edge not in S is incident to some edge in S.

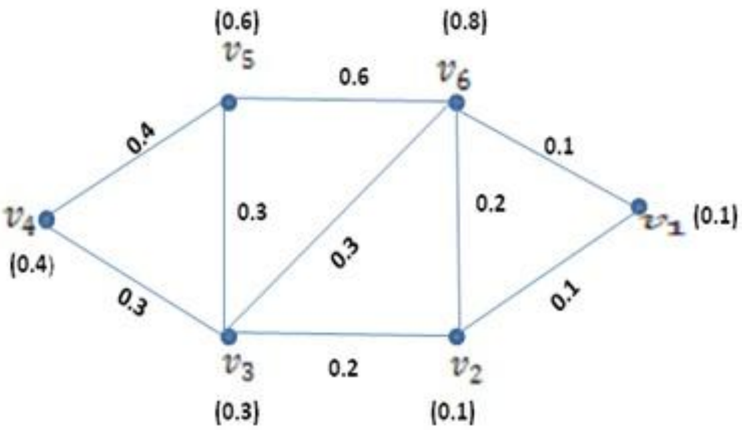
Definition 2. 2:
A subset S of V × V is said to be a minimal edge dominating set if no proper subset of S is an edge dominating set of G.

Definition 2. 3:
The minimum cardinality of an edge dominating set is called as the edge domination number of G and is denoted by ϒ' (G).

Theorem 2. 4:
An edge dominating set S is a minimal edge dominating set if for every edge e ϵ S, one of the following conditions holds
(i) N(e)∩S =φ
(ii) There exists an edge f ϵ E-S such that N(f) ∩S =$\{e\}$.

Proof:
Let S be a minimal edge dominating set of the fuzzy graph G and e ϵ S. Then $S_e = S - \{e\}$ is not an edge dominating set and hence there exists an edge fϵ E-S_e such that f is not adjacent to any element of S_e.
Thus if f=e, we get N(e)∩S =φ and if f≠e, we get N(f) ∩S =$\{e\}$.
We will now investigate the effect of the removal of a vertex of a fuzzy graph G on the edge domination number of G.

Example 2. 5:
Consider the fuzzy graph G = (σ, μ) given in figure1



We have edge dominating set : { v_1v_6, v_3v_5}
Edge domination number, ϒ' (G) = 0. 1+0. 3 = 0. 4
Removal of vertex v_1:

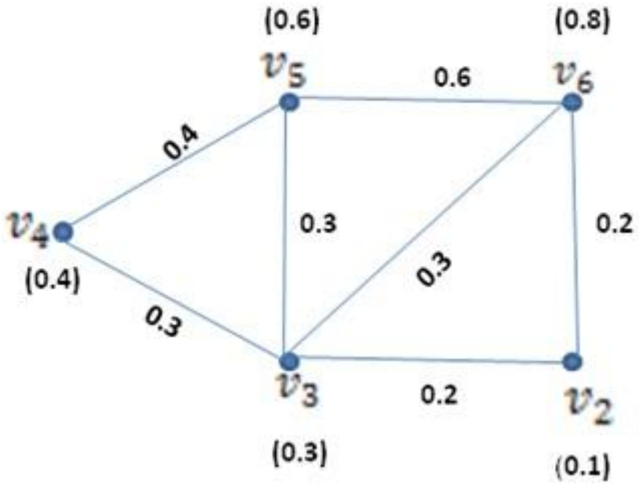


Figure 2

Here we have ϒ' (G-v_1) = μ(v_3v_5) + μ(v_2v_6) = 0. 3 +0. 2 = 0. 5
Therefore ϒ' (G-v_1) > ϒ' (G)
Further ϒ' (G-v_2) = 0. 4, ϒ' (G-v_3) = 0. 5, ϒ' (G-v_4) = 0. 3, ϒ' (G-v_5) =0. 3 and ϒ' (G-v_6) = 0. 4
Also consider the fuzzy graph given below in figure 3

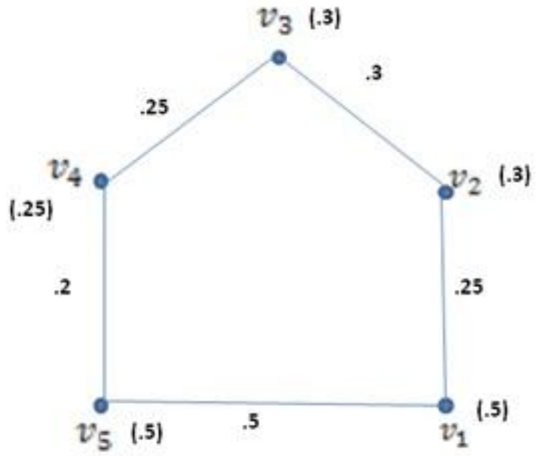


Figure 3

Here the edge domination number is given by 0. 45 and the removal of the vertex $v_3$decreases the edge domination number of the graph.
Thus the above examples shows that when we remove a vertex v from a fuzzy graph G its edge domination number may increase or decrease or remains unaltered.
We now partition the vertex set V into 3 subsets as follows

V_e^{0} = {v ϵ V/ ϒ ' (G-v) = ϒ ' (G)}
V_e^{+} = {v ϵ V/ ϒ ' (G-v) >ϒ ' (G)}
V_e^{-} = {v ϵ V/ ϒ ' (G-v) < ϒ ' (G)}

Theorem 2. 6:
Let G = (σ, µ) be a fuzzy graph on V. If a vertex v is in V^{+}, then v is an end vertex of some edge in every edge dominating set of G.

Proof:
Suppose there exists an edge dominating set S of G which does not contain an edge whose end vertex is v, then S will be an edge dominating set of G-v also.
So ϒ ' (G-v) ≤ $|S|$ = ϒ ' (G)
(ie) ϒ ' (G-v) ≤ ϒ ' (G)
This is a contradiction. thus v is an end vertex of some edge in every edge dominating set of G.
We now proceed to investigate the effect of removal of an edge in a fuzzy graph on its edge domination number.

Example 2. 7:
Consider the Fuzzy graph G given in Figure 1.
Here the removal of the edge v_4v_5 decreases the edge domination number of G.

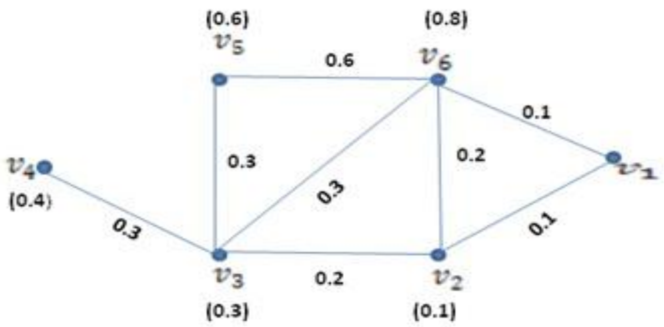


Figure 4

Here ϒ ' (G-v_4v_5) = 0. 3, whereas for all the other edges it is 0. 4
Consider the Fuzzy Graph given in figure 3.
Here the removal of the edge v_1v_2 increases the edge domination number.

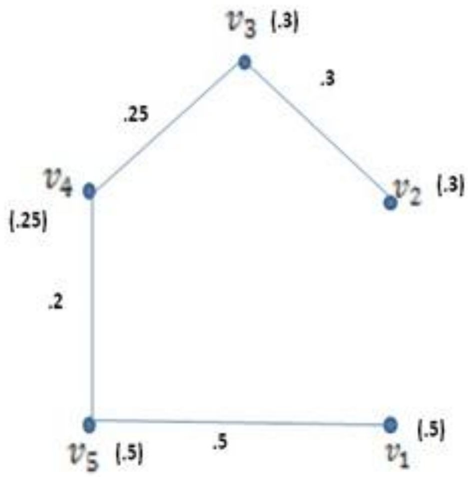


Figure 5

Here ϒ ' (G-v_1v_2) = 0. 5 > ϒ ' (G) = 0. 45, whereas for all the other edges it is 0. 45. Hence we conclude that the removal of an edge from a fuzzy graph G may increase or decrease or keeps unaltered its edge domination number.
Now we partition the edge set E into 3 subsets as follows:
E^0 = { uv ϵ E / ϒ ' (G-uv) = ϒ ' (G)}
E^+ = { uv ϵ E / ϒ ' (G-uv) > ϒ ' (G)}
E^- = { uv ϵ E / ϒ ' (G-uv) < ϒ ' (G)}

Theorem2. 8:
If an edge uv ϵ E^{+} then we have
(a) uv is not an isolated edge.
(b) uv is in every edge dominating set of G.

Proof:
(a) Suppose uv is an isolated edge of a fuzzy graph G. Let S be an edge dominating set of G. Then uvϵ S. So S-uv is an edge dominating set of G-uv.

(ie) ϒ' (G-uv) = ϒ' (S-uv)
$\leq |S|$
= ϒ' (G)

Therefore ϒ' (G-uv) $<$ ϒ' (G) which implies uv ϵ E^{-}, which is a contradiction because uv ϵ E^{+}. Thus uv is not an isolated edge.
(b)Suppose there is an edge dominating set S of G which does not contain the edge uv. Now, S is an edge dominating set of G-uv, so ϒ' (G-uv) $\leq |S|$ = ϒ' (G)

Therefore uv ϵ E^{-}, which is a contradiction. Thus the edge uv is in every edge dominating set of G.

Example 2. 9:
Now consider the following fuzzy graph G

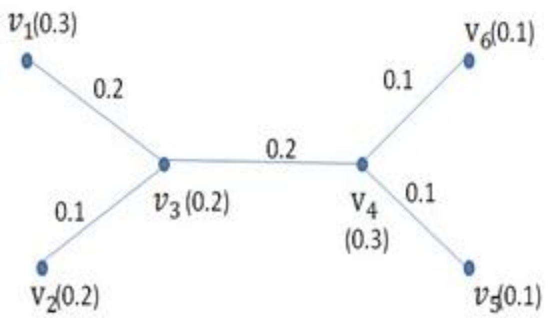


Figure :6

The Independent dominating set of the fuzzy graph given in figure 6 is $\{v_3, v_5, v_6\}$ and the independent domination number is i(G)= 0. 2+0. 1+0. 1 =0. 4
The removal of vertex v_3 results in

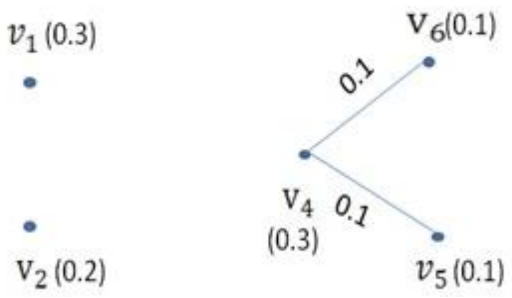


Figure: 7

Now the independent domination number of the fuzzy graph given in figure 2 is
i(G) = 0. 7
Here the removal of vertex v_3 has increased the independent domination number.
Similarly the removal of vertex v_5 and v_6 decreases the independent domination number of the fuzzy graph given in figure 6, whereas the removal of vertex $v_{1,}v_2$ or v_4 keeps unaltered the independent domination number.

We now introduce the following sets
$V_i^{\ 0}$ = {v ϵ V/ i (G-v) $=$ i (G)}
$V_i^{\ +}$ = {v ϵ V/ i (G-v) $>$i (G)}
$V_i^{\ -}$ = {v ϵ V/ i (G-v) $<$ i (G)}
These three sets are mutually disjoint and their union is V(G).

Theorem 2. 10:
If a vertex v belongs to $V_i^{\ +}$, then we have
(i) v is not an isolated vertex
(ii) v is in every i-set of G
(iii) No independent subset S of V(G)- N[v] with lSl≤i(G) can dominate G-{v}.

Proof:
(i) Suppose v is an isolated vertex. Let S be a minimal independent dominating set of G. Then we have v ϵ S.

Therefore S-{v} is an independent dominating set of G-{v}.
So i(G-v) = i(S-v)
< $|S|$
= i(G)
Therefore i(G-v) < i(G), which is a contradiction since v ϵ $V_i^{\ +}$. Therefore v is not an isolated vertex.

(ii) Suppose there is some i-set s of G which does not contain v. Now s is an independent dominating set of G-{v} also. So i(G-v) < i(G) which is a contradiction.

Therefore v is in every i-set of G.

(iii) Suppose there is an independent subset S of V(G)-N[v] with $|S| \leq i(G)$ which can dominate G-{v}. Then i(G-v) < i(G) so v does not belong to ${V_i}^+$ which is a contradiction.

Result 2. 11:
A vertex v ϵ ${V_i}^-$ iff there is a i-set S containing v such that S-{v} is an Independent dominating set in G-{v}.

Conclusion:
The concept of dominations in graphs has very rich theoretical development and applications. In this paper we have characterized vertices and edges which cause variation in the edge domination number and independent domination number of a fuzzy graph with examples. Graph critical concept is very useful in the field of networks. We are interested in exploring the variations caused by removal or addition of vertices or edges in other domination variants also.

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