

Intuitionistic Fuzzy Analytical Hierarchy Process with Fuzzy Delphi Method

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Abstract

Analytic hierarchy process (AHP) is an important theory in the decision making problem. Using AHP many complex decision problems are solved. Intuitionistic Fuzzy set is a very good output, over the uncertain and vague ones. In this work we are going to combine intuitionistic fuzzy analytic hierarchy process(IFAHp) with fuzzy Delphi method. The methods of fuzzy Delphi IFAHP are given in detail. Sample work is the Customers Requirement in the automobile sector in India.

AMS subject classification:

Keywords: Intuitionistic Fuzzy Analytical Hierarchy Process, Analytical Hierarchy process, customer requirement

1. Introduction

Fuzzy sets were introduced by Prof. Lotfi A.Zadeh of University of California at Berkeley in the year 1965. A fuzzy set is a set that is defined by a membership function. A membership function assigns to each element in the set under consideration a membership grade which is a value in the interval [0,1]. Fuzzy set introduces vagueness with the aim of reducing complexity by eliminating the sharp boundary dividing the members of the pair from non-members. This mapping associates each element in a set with its degree of membership. It can be expressed as a discrete value or as a continuous function. In fuzzy sets, each elements is mapped by membership function. The membership function are Triangular and trapezoidal membership functions are commonly used for defining continuous membership functions [1].The triangular fuzzy membership function is given by

$$\mu_A(x) = \begin{cases} \frac{(x - a_1)}{(a_m - a_2)} & : a_1 \leq x \leq a_m \\ \frac{(x - a_2)}{(a_m - a_2)} & : a_m \leq x \leq a_2 \end{cases}$$

And the trapezoidal fuzzy membership function and is given by

$$\mu_A(x) = \begin{cases} \frac{(x - a_1)}{(a_1^{(1)} - a_1)} & : a_1 \leq x \leq a_1^{(1)} \\ 1 & : a_1^{(1)} \leq x \leq a_2^{(1)} \\ \frac{(x - a_2)}{a_2^{(1)} - a_2} & : a_2^{(1)} \leq x \leq a_2 \end{cases}$$

1.1. Analytic hierarchy Process (AHP)

Analytic Hierarchy Process (AHP) is one of the best methods for the Multi criteria Decision making problem, which was originally developed by Satty [2]. The AHP can help to split complex problem into a multi level hierarchy structure, sub criteria and alternatives. Usage of hierarchical order helps to simplify the problem in to a condition which is more easily understood. The weights are calculated in each attribute and checked for consistency. Based on the weights of each attribute suitable decision can be taken by the decision maker.

1.2. Fuzzy Analytic hierarchy Process (FAHP)

The combination of Analytic Hierarchy Process and the fuzzy set theory is called Fuzzy Analytical Hierarchy Process (FAHP). It was proposed by Lahaven and Pedrycz (1983). In FAHP method the ratio of the fuzzy comparison are able to better accommodate vagueness than AHP values.

1.3. Intuitionistic fuzzy set (IFS)

$0 \leq \pi_A(x) \leq 1$ for each $x \in X$ $\mu_A(x) \in [0, 1]$ is the membership function of the fuzzy set $A^1 : \mu_{A^1}(x) \in [0, 1]$ is the membership of $x \in A^1$ an intuitionistic fuzzy set

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introduced by Atanassov [3]. The intuitionistic fuzzy set defined by

$$A = \{\langle x, \mu_x, \nu_x \rangle | x \in X\}, 0 \leq \mu_x + \nu_x \leq 1 \quad (1.1)$$

where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ s.t $\mu_A(x) \in [0, 1]$ denote the membership function and $\nu_A(x) \in [0, 1]$ denote the non-membership function. Obviously $A = \{\langle x, \mu_{A^1}(x), 1 - \mu_{A^1}(x) \rangle | x \in X\}$ and $\pi_A(x) = 1 - (\mu_x + \nu_x)$ is called the hesitation degree or degree of nondeterminacy of $x \in A$ or $x \notin A$. Szmida and Kacprzyk [1] point out that when calculating the distance between two IFSs we cannot omit $\pi_A(x)$. We consider that $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ is an intuitionistic fuzzy values where $\mu_\alpha \in [0, 1]$ and $\nu_\alpha \in [0, 1]$, $\mu_\alpha + \nu_\alpha \leq 1$. According to the Szmida and Kacprzyk [4] put forth a function in mathematical form

$$\rho(\alpha) = 0.5(1 + \pi_\alpha)(1 + \mu_\alpha) \quad (1.2)$$

The α means its contain all positive information included. Therefore intuitionistic fuzzy set mainly based on membership function and non membership function and hesitation degree.

Table 1: Comparison Scale [5]

Linguistic Value	Scale	Linguistic scale
9	0.9	Extreme Important
7	0.8	Very Strong Important
5	0.7	Strong Important
3	0.6	Moderately Important
1	0.5	Equal Preference
1/3	0.4	Moderately not Important
1/5	0.3	Strong not Important
1/7	0.2	Very strong not Important
1/9	0.1	Extreme not Important

1.4. Intuitionistic Relation

Let R be the relation in the intuitionistic values on the set $X = \{x_1, x_2 \dots x_n\}$ is represented by matrix $R = (M_i^k)_{n \times n}$, where $M_{ik} = \langle (x_i, x_k), \mu(x_i, x_k), \nu(x_i, x_k) \rangle$ $i, k = 1, 2, 3 \dots n$. Let Assume that $M_{ik} = (\mu_{ik}, \nu_{ik})$ and $\pi(x_i, x_k) = 1 - \mu(x_i, x_k) - \nu(x_i, x_k)$ is interpreted as an indeterminacy degree. The notion of intuitionistic fuzzy t -norm and t -conorm is as found in Deschrijver et al. [2]. The intuitionistic fuzzy triangular norms was studied by Xu [5]. He introduced the following operations

$$1. M_{ik} \bigoplus M_{lm} = (\mu_{ik} + \mu_{lm} - \mu_{ik}\mu_{lm}, \nu_{ik}\nu_{lm})$$

$$2. M_{ik} \bigotimes M_{lm} = (\mu_{ik}\mu_{lm}, \mu_{ik} + \mu_{lm} - \nu_{ik}\nu_{lm})$$

In this work we extend the FAHP into intuitionistic fuzzy AHP (IFAHP) with Fuzzy Delphi method, in which the predictions are represented by intuitionistic fuzzy values. Based on the above criteria we are going to apply DIFAHP in the customer requirement and finally we are to find the preference of customer requirement.

1.5. Fuzzy Delphi Method

Kaufman and Gupta [6] have studied about the Fuzzy Delphi Method. In 1993 ishikawa et al. too mention about the Fuzzy Delphi Method. The generalization of fuzzy Delphi method is as follows

1. Identify the experts based on the domain and make the experts panel members
2. using the experts opinion categorize the attributes. using the attributes make the questionnaires.
3. using the questionnaires gets the first set of the suggestion about the attributes.
4. From the attributes values compute the Mean. [7] Then deviation is calculated between mean and each expert opinion. [it is also a fuzzy number]. The deviation is sent to be each expert for reevaluation.
5. In the second round a new fuzzy number is received from the experts. Next, the same procedure is repeated (step-2) until two successive means become very close; else the Delphi expert will take the final decision.

2. Literary Survey

AHP approach for decision making was proposed by Saaty [2]. The Intuitionistic fuzzy sets has been introduced by atanassov [3]. Using the Intuitionistic fuzzy logic developed a intuitionistic fuzzy system to control the heater fans, in this work. the speed of the heater fan is calculated using intuitionistic fuzzy rules applied in an inference engine using defuzzification method by Muhamman akram et al. [8]. The Intuitionistic fuzzy sets are used in some Medical application by Eulalia szmidt et al. [9]. As generalization of fuzzy sets a new definition of distance between two intuitionistic fuzzy sets has been given by Atanassov by et al. [4]. Using the Intuitionistic fuzzy analytic hierarchy process the environmental decision in the best drilling fluid(mud) for drilling operation has been by Rehan sadiq et al. [10]. Determining the customer satisfaction in the automobile sector using the Intuitionistic fuzzy analytic hierarchy process has been studied by Rajaprakash et al. [11]. Development of hotel usage of atmospheres usage of Delphi fuzzy Analytical Hierarchy Process has been author studied in two phases the first one Delphi method and second by AHP, by Yen cheng chen et al. [12]. The selection of best DBMS among the several candidates in the Turikish National Identity Card Management project was done using the Fuzzy AHP by F.Ozgur Catak et al. [13]. Using the Fuzzy AHP evaluation of the E-commerce in order manage and determine the drawbacks and opportunities is studied by Feng Kong et al. [14]. The supplier selection problem under incomplete and

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uncertain information environment using TOPSIS Method under the interval valued of intuition fuzzy Numbers has been studied by Mohammad Izadikhah [15]. The prediction of highest and lowest temperature by BP neural networks training for abnormal weather alerts by using a fuzzy AHP and rough set. In this work the author compared the fuzzy AHP and rough set. Studied Dan Wang et al. [16]. Using the FAHP student's expectation in the present education system in Tamilnadu, India. In the work the authors have taken a sample work on Engineering education S. Rajaprapakash et al. [17]. Using the Fuzzy Delphi Method and Fuzzy analytic Hierarchy process is applied And determine the critical factors of the regenerative technologies and find the degree of each importance criterion as the measurable indices of the regenerative technologies by Yu-Lung Hsu et al. [18]. The study of human capital indicator and ranking by using IFAHP to evaluate the four main indicators of Human capital has been studied by Lazim Abdullah et al. Diagnosis progress in bacillus colonies identification in the Medical field using the intuitionistic fuzzy set theory studied by Hoda davarzani et al. [19]. Intuitionistic Fuzzy Delphi Method used as forecasting tool based on expert's suggestion. They used triangular fuzzy number are used and aggregation process based on the opinion of the expert Tapan Kumar et al. [20].

3. Methodology

1. Using the Delphi Method get the opinion from the experts panel. Here we have used 10 experts for the Delphi method.
2. After finalization of the expert suggestion the values are converted to Intuitionistic value based on the above comparison scale Table-4 and then Construction of the Comparison matrix is carried out.
3. In order to get a optimum solution, before deriving the priorities of the alternatives and criteria, we should check the consistency of the intuitionistic relation in the matrix which is formed earlier. According to Saaty [2] the AHP provided a CI (consistency Index) and CR (consistency Ratio) to measure the consistency of the comparison matrix. Its mathematical form is as follows.

$$CR = \frac{CI}{RI}$$

where

$$CI = \frac{(\lambda_{Max} - n)}{(n - 1)}$$

In satty method in AHP if the consistence index is less than 0.1 then that comparison matrix is consistent and that matrix is acceptable. Suppose it is not true then again it will go for except revaluation. So it will take lot of time. In this method to reevaluate the value of the except and to make it consistent. In our work we use the another method to check the consistency in the intuitionistic preference relation.

According to Xu et al. [21] the consistent interval fuzzy preference relation is as follows.

$R = (M_{ik})_{n \times n}$ with $M_{ik} = (\mu_{ik}, \nu_{ik})$ is multiplicative consistent if

$$\mu_{ik} = \begin{cases} 0 & \text{if } (\mu_{it}, \mu_{tk}) \in \{(0, 1), (1, 0)\} \\ \frac{\mu_{it}\mu_{tk}}{\mu_{it} + \mu_{tk} + (1 - \mu_{it})(1 - \mu_{tk})} & \text{otherwise} \end{cases} \quad (3.1)$$

$$\nu_{ik} = \begin{cases} 0 & \text{if } (\nu_{it}, \nu_{tk}) \in \{(0, 1), (1, 0)\} \\ \frac{\nu_{it}\nu_{tk}}{\nu_{it} + \nu_{tk} + (1 - \nu_{it})(1 - \nu_{tk})} & \text{otherwise} \end{cases} \quad (3.2)$$

Theorem 3.1. [21] In the fuzzy preference relation the following statement are equivalent:

$$b_{ik} = \frac{b_{ik}b_{tk}}{b_{ik}b_{tk} + (1 - b_{ik})(1 - b_{tk})} \quad i, t, k = 1, 2, 3, \dots \quad (3.3)$$

$$b_{ik} = \frac{\sqrt[n]{\prod_{s=1}^n b_{is}b_{sk}}}{\sqrt[n]{\prod_{s=1}^n b_{is}b_{sk} + \sqrt[n]{\prod_{s=1}^n b_{is}b_{sk}}}} \quad i, k = 1, 2, \dots, n \quad (3.4)$$

$$\bar{\mu}_{ik} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}}} \quad k > i + 1 \quad (3.5)$$

$$\bar{\nu}_{ik} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \nu_{it}\nu_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \nu_{it}\nu_{tk}}} \quad k > i + 1 \quad (3.6)$$

1. The distance between intuitionistic relation [4] is calculated using

$$d(M, \bar{M}) = \frac{1}{2(n-1)(n-2)} \sum_{t=1}^n \sum_{k=1}^n (|\bar{\mu}_{ik} - \mu_{ik}| + |\bar{\nu}_{ik} - \nu_{ik}| + |\bar{\pi}_{ik} - \pi_{ik}|) \quad (3.7)$$

2. The priority of the intuitionistic preference relation zeshuri Xu [21] is calculated by the following method.

$$W_i = \frac{\sum_{k=1}^n M_{ik}^1}{\sum_{i=1}^n \sum_{k=1}^n M_{ik}^1}$$

$$W_i = \left[\frac{\sum_{k=1}^n \mu_{ik}}{\sum_{i=1}^n \sum_{k=1}^n [1 - \nu_{ik}]}, 1 - \frac{\sum_{k=1}^n [1 - \nu_{ik}]}{\sum_{i=1}^n \sum_{k=1}^n \mu_{ik}} \right] \quad (3.8)$$

3. After finding the weights of the all level based on the weights ranking the weight by using the formula (1.1) then finding preference ranking. The flow diagram is provided in Fig. 1

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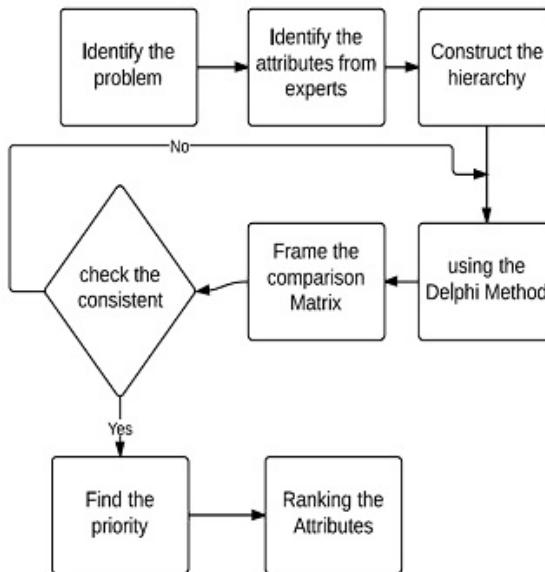


Figure 1: Flow Diagram

4. Illustrative work

To illustrate the above work, we worked with real time example of Customer Requirement in the automobile sector in the supplier park at Chennai, Tamilnadu, India. In this work the customer is a big car manufacturing company and the supplier is any who supply car parts to the customer. The illustrated problem is about finding the Customer Requirement from the supplier perspective.

4.1. Observation from the Experts

In the automobile sector the future technology is mainly based on factors like factors like Fuel efficiency, Emission reduction, Safety Durability, cost and innovative ideas. So every day the customer requirements in the automobile engineering are increasing. So finding the customer requirement in this sector is an uncertain element. In this work the customer requirement mainly has been classified into Quality, Cost, Delivery, and service at the first level.

4.1.1 Quality

The quality is mainly based on the policy of no line stoppage due to quality issue, Parts of the quality need to be meeting the design, otherwise Supplier request for Engineering change approval (SREA), Parts per million should be less than target, Emergency Response action (ERA) is if non conformation parts reached to the customer end it should be isolated immediately and submit the details of root cause and details of Corrective action and Preventive action taken (CA and PA) to eliminate the root cause. Q1 Certification is based on marks scored in customer audit and supplier performance, Process robust

index (PRI) it is based on design Failure mode effect analysis and process failure mode effective analysis (DMEA and PFMEA), yoke implemented (mistake proofing) as part of process control, Process capability or optimization it means no critical characteristics are identified in the part or product by customer for safety, fitment and functional requirement to be identified and Statistical process capability study should be conducted (C_p and C_{pk}) and value $C_p \geq 1.67$ (it indicates how much the designed process is capable of producing consistent part or dimensions with given parameters or repeatability.) and $C_{pk} \geq 1.33$ (process centering – how much the machine setting towards the nominal value), Infrastructure like Metrology lab, Chemical lab and other special test facilities, endurance lab etc, Qualified QA managers/engineers (six sigma black belt, master black belt etc.). Capability of analysis the problem both systematic and scientific method.

4.1.2 Cost

The cost is mainly based on the Raw material cost, Process cost (man hour rate and machine hour rate) is based on the Machine productivity, man productivity, Space productivity, asset productivity, asset utilization etc.), Administration costs based on the Design free it means that, In house capability and external support requirement etc, Tools, jigs and fixtures cost (in house development facilities like tool room, tool service center etc. with skilled people. This facilities will add value to supplier and customer with lesser development time and cost), Inventor carrying cost (ICC) it means that No. of parts to be produced inhouse / subcontract/imports, If supplier has facilities and capability to produce maximum parts inhouse it would be more advantage on cost reduction and inventories, number of localization projects for import substitute will be reviewed with supplier as cost reduction initiatives, Cost driver strategy is key for auto industries. Customer would like to review the supplier on various cost reduction projects initiatives. This would provide leverage to customer for year on year cost reduction to become competitive, Transportation cost depends on how much the distance of the supplier is closer to customer premises. This would reduce the inventory and transportation cost.

4.1.3 Delivery

The Delivery mainly is based on Implementation of MMOG (material management systems). Implementation of FMS (Flexible manufacturing system): supplier should be flexible of supplying different model with different quality as per customer demand on daily basis ,Manufacturing capacity. Supplier should have minimum of 15 to 20 percentage more manufacturing capacities than customer demand to take care market fluctuations, Implementation of KANBAN system / E-kanban for material and information flow across Supply Chain, Production Planning and control or SCM organization, hundred percentage on time delivery schedule to be met, No line stoppage due to non availability of parts, FIFO to be followed (First in first out), Safety stock at customer and supplier to be maintained.

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4.1.4 Service

The Service is mainly based on Quick response for any engineering changes in terms documents/tool/process modification, Quick response for any quality complaints or warranty, Collaborative design means that the process of joint development of part or product along with customers. Deputing a design engineer at customer's design center for developing collaborative design is in vague in Many Japanese/Korean companies.

In the level-1 CR (customer requirement) have four attribute G1,G2,G3,G4 represent Quality, Cost, Delivery and Service respectively. Using the Fuzzy Delphi Method we formed the expert's opinions as follows. The customer requirements hierarchy given in the Figure 2.

4.2. Customer Requirement in Level-1

In order to find the Customer Requirement in Level four attributes are available. Based on the experts opinion the first initial Table 2 formed

Table 2: Delphi 1

Experts	G1 to G2		G1 to G3		G1 to G4		G2 to G3		G2 to G4		G3 to G4	
1	0.5	0.4	0.4	0.2	0.6	0.3	0.4	0.3	0.5	0.3	0.6	0.4
2	0.6	0.3	0.5	0.3	0.5	0.4	0.3	0.4	0.7	0.3	0.6	0.4
3	0.7	0.3	0.3	0.2	0.5	0.3	0.5	0.3	0.5	0.5	0.5	0.6
4	0.3	0.6	0.6	0.3	0.6	0.3	0.4	0.4	0.5	0.4	0.7	0.4
5	0.4	0.5	0.2	0.3	0.3	0.6	0.5	0.5	0.6	0.4	0.5	0.4
6	0.5	0.4	0.3	0.4	0.5	0.5	0.6	0.4	0.3	0.5	0.6	0.5
7	0.6	0.3	0.4	0.2	0.5	0.4	0.7	0.3	0.5	0.4	0.5	0.6
8	0.7	0.2	0.3	0.3	0.6	0.5	0.3	0.4	0.6	0.4	0.6	0.4
9	0.5	0.4	0.2	0.5	0.4	0.4	0.5	0.5	0.7	0.4	0.4	0.5
10	0.4	0.5	0.7	0.3	0.3	0.6	0.5	0.2	0.6	0.4	0.5	0.6

The Mean values are calculated. The deviations of experts' opinion from the calculated Mean values are given below Table 3.

Here the Delphi expects not satisfied with deviation Table 3. Therefore opinion is sent back to the experts for one more opinion.

The deviation from the mean is calculated Table 5

Now the Delphi expert is satisfied with the above deviation Table (5). Based on the expert suggestion the first intuitionistic preference relation matrix M formed is shown below.

$$M = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.7) & (0.4, 0.5) & (0.4, 0.6) \\ (0.7, 0.3) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.6) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.5) \\ (0.6, 0.4) & (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

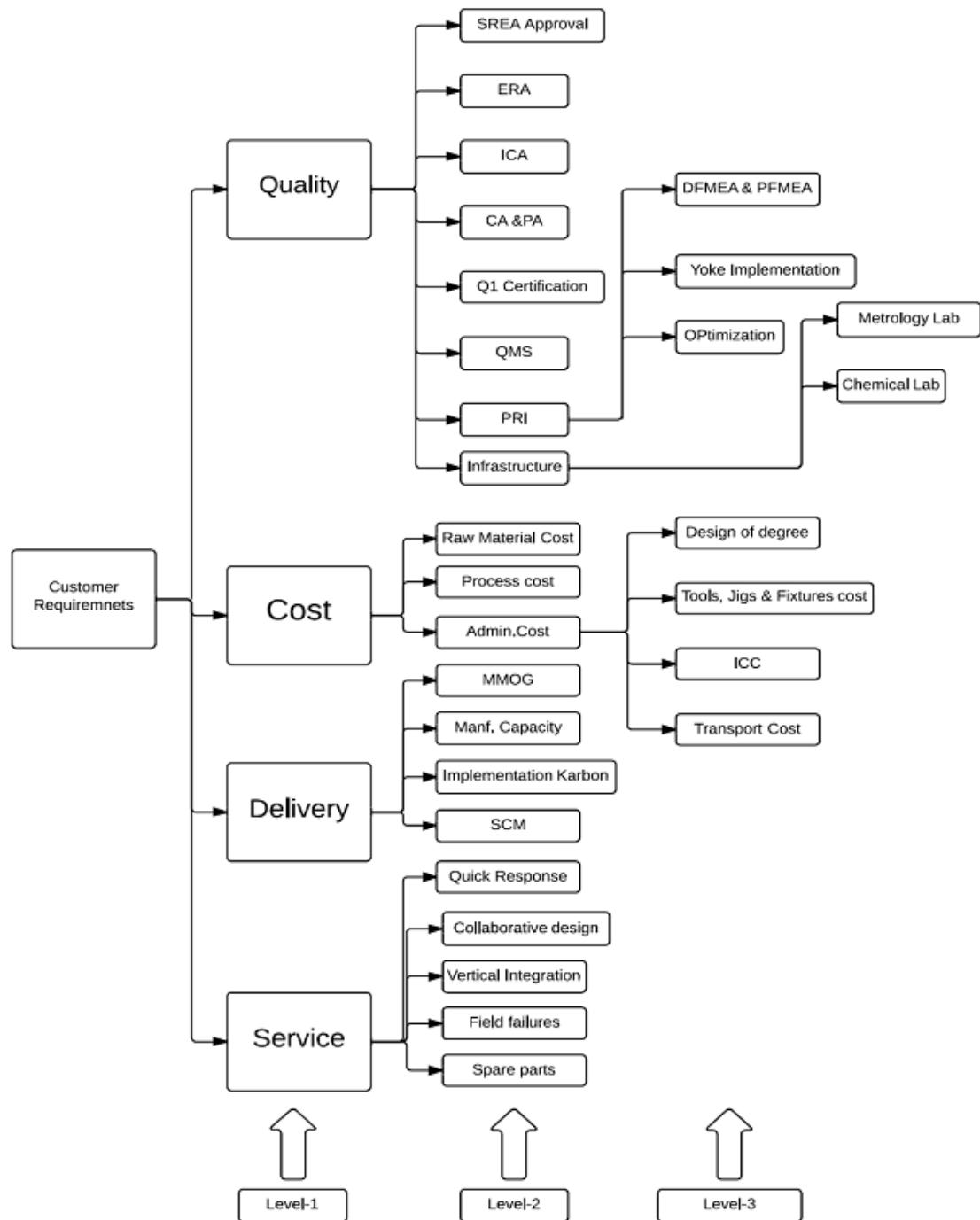


Figure 2: Hierarchy of Customer Requirements

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Table 3: Delphi2

Experts	G1 to G2		G1 to G3		G1 to G4		G2 to G3		G2 to G4		G3 to G4	
1	0.02	-0.01	-0.01	0.1	-0.12	0.13	0.07	0.07	0.05	0.1	-0.05	0.08
2	-0.08	0.09	-0.11	0	-0.02	0.03	0.17	-0.03	-0.15	0.1	-0.05	0.08
3	-0.18	0.09	0.09	0.1	-0.02	0.13	-0.03	0.07	0.05	-0.1	0.05	-0.12
4	0.22	-0.21	-0.21	0	-0.12	0.13	0.07	-0.03	0.05	0	-0.15	0.08
5	0.12	-0.11	0.19	0	0.18	-0.17	-0.03	-0.13	-0.05	0	0.05	0.08
6	0.02	-0.01	0.09	-0.1	-0.02	-0.07	-0.13	-0.03	0.25	-0.1	-0.05	-0.02
7	-0.08	0.09	-0.01	0.1	-0.02	0.03	-0.23	0.07	0.05	0	0.05	-0.12
8	-0.18	0.19	0.09	0	-0.12	-0.07	0.17	-0.03	-0.05	0	-0.05	0.08
9	0.12	-0.11	-0.31	0	0.18	-0.17	-0.03	0.17	-0.05	0	0.05	-0.12
10	0.12	-0.11	-0.31	0	0.18	-0.17	-0.03	0.17	-0.05	0	0.05	-0.12

Table 4: Delphi3

Experts	G1 to G2		G1 to G3		G1 to G4		G2 to G3		G2 to G4		G3 to G4	
1	0.7	0.3	0.4	0.4	0.6	0.3	0.4	0.4	0.52	0.4	0.6	0.3
2	0.6	0.4	0.5	0.5	0.6	0.4	0.4	0.4	0.6	0.4	0.5	0.4
3	0.8	0.2	0.5	0.2	0.8	0.2	0.5	0.4	0.7	0.3	0.4	0.3
4	0.8	0.2	0.6	0.4	0.8	0.2	0.4	0.4	0.5	0.4	0.4	0.4
5	0.7	0.3	0.5	0.4	0.5	0.6	0.5	0.5	0.5	0.5	0.5	0.5
6	0.7	0.2	0.3	0.4	0.5	0.5	0.6	0.4	0.6	0.4	0.5	0.4
7	0.6	0.4	0.4	0.4	0.5	0.4	0.7	0.3	0.6	0.4	0.5	0.3
8	0.8	0.2	0.5	0.5	0.6	0.4	0.4	0.4	0.5	0.4	0.5	0.4
9	0.8	0.2	0.6	0.4	0.5	0.4	0.6	0.4	0.8	0.5	0.5	0.5
10	0.5	0.5	0.7	0.3	0.5	0.5	0.5	0.4	0.6	0.3	0.5	0.2

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.7) & (0.4, 0.5) & (0.262, 0.651) \\ (0.6, 0.3) & (0.5, 0.5) & (0.4, 0.5) & (0.449, 0.5) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.5) \\ (0.651, 0.262) & (0.5, 0.449) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

Then using the equation (3.7) calculate the distance between intuitionistic relation is calculated $d(\bar{M}, M) = 0.09578$ which is less than τ . Here let we will fix the threshold value as $\tau = 0.1$.Therefore the above matrix is consistent.The next step is calculating the weight of all attributes using the equation (3.8). It is given in the Table (6) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (7) and the figure (3). we can see that the first preference is quality, second is delivery, third is

Table 5: Delphi4

Experts	G1 to G2		G1 to G3		G1 to G4		G2 to G3		G2 to G4		G3 to G4	
1	0	-0.01	0.1	-0.01	-0.01	0.09	0.1	0	0.072	0	-0.11	0.07
2	0.1	-0.11	0	-0.11	-0.01	-0.01	0.1	0	-0.01	0	-0.01	-0.03
3	-0.1	0.09	0	0.19	-0.21	0.19	0	0	-0.11	0.1	0.09	0.07
4	-0.1	0.09	-0.1	-0.01	-0.21	0.19	0.1	0	0.092	0	0.09	-0.03
5	0	-0.01	0	-0.01	0.09	-0.21	0	-0.1	0.092	-0.1	-0.01	-0.13
6	0	0.09	0.2	-0.01	0.09	-0.11	-0.1	0	-0.01	0	-0.01	-0.03
7	0.1	-0.11	0.1	-0.01	0.09	-0.01	-0.2	0.1	-0.01	0	-0.01	0.07
8	-0.1	0.09	0	-0.11	-0.01	-0.01	0.1	0	0.092	0	-0.01	-0.03
9	-0.1	0.09	-0.1	-0.01	0.09	-0.01	-0.1	0	-0.21	-0.1	-0.01	-0.13
10	0.2	-0.21	-0.2	0.09	0.09	-0.11	0	0	-0.01	0.1	-0.01	0.17

Table 6:

weight	μ	ν
W(G1)	0.173389	0.779159
W(G2)	0.231089	0.705247
W(G3)	0.225222	0.705247
W(G4)	0.255054	0.680088

Table 7:

Attribute	$\rho(\alpha)$	P
Quality(G1)	0.88958	1
Delivery(G3))	0.85299	2
Cost(G2)	0.852624	3
Service(G4)	0.840044	4

cost, and last one is service. Similarly we calculated the other levels.

5. Comparison study

Comparing the work IFAHP with Delphi system with IFAHP [4], We can see that our work has so many advantages over IFAHP method.

1. In the work of Zeshui Xu [4] IFAHP, work he used two algorithms . In the algorithm I the deviation is calculated using (7). Suppose the deviation $d(M, \bar{M})$ is too large then the author use algorithm-II. If still there is no consistency, he starts all over again.

In the present work we use algorithms-I [4] we calculate the deviation $d(M, \bar{M})$. The deviation is mostly less than the threshold value $\rho(\alpha)$ since we have done enough iteration work in the fuzzy Delphi method.

2. In zeshuri Xu work [4] has not given clear picture of how to go about it when algorithm-II fails.

In our present work we mention that in step-4 if the preference relation is inconsistent then go to the step-1 (Fuzzy Delphi Method).

3. In zeshuri Xu work [4] ranking the supplier in the last level that is in the alternative

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criteria only, But in our work we calculated the preference of each level so that it useful at each and every level of the work.

6. Empirical result

According to the suggestion given by the expert the customer requirement hierarchy is formed and in the Level-1 we have four attributes and in that based on the Table 1.3 and Diagram-I the first preference Quality, second is delivery, third cost and fourth is Service. In the level-2 the Quality have eight attributes based in the Table 2.3 and Diagram-II the first preference is ICA, second ERA, third CA and PA, forth is SREA, fifth Q1certification, sisth QMS, seventh PRI and eight Infracture. In the quality 7th attribute is process robust index (PRI) in that based on the Table 7.3 and the Diagram-7 the first preference is DFMEA, second Yoke implementation, third Optimization. In the Level-2 the cost have three attributes, based on the Table 3.3 and Diagram-3, the first preference is Process cost, second administrator cost, third raw material cost. In the administration cost have four attributes, are show in the Table 6.3 and Diagram-6, the first preference is design free, second tools, jigs and fixtures cost, third ICC and fourth is Transport cost. In the delivery have four attributes, based on the Table 5.3 and Diagram-5 the first preference is MMOG, second Manufacture capacity, third implementation of karbon and forth SCM. In the level-2 service have five attributes, based on Table 4.3 and Diagram-4 , the first preference is Quick response, second collaborative design, third vertical integration, forth field failures, and fifth spare parts. Therefore from the value $\rho(\alpha)$ and Tables and Diagrams, we can get the preference ranking the attribute of the automobile sector using the IFAHP fuzzy Delphi method.

7. Conclusion

In this work we extended the IFAHP with Delphi method. Since the Intuitionistic Fuzzy Set is a powerful tool for the vagueness and the uncertainty. In the fuzzy Delphi method questionnaires framed based on the suggestion and get the opinion from the experts in automobile sector. This survey was taken using the web site Surveymonkey.com (here the questions are framed and sent link to the experts) Here we have taken sample work which constitutes flow diagram, diagram, tables based on that we can get the preference of the attributes on each level of the customer requirement in the automobile sector. which is useful for the automobile sector and it may improve the our industrial standard and Indian economy.

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8. Appendix

8.1. Quality

$$M = \begin{pmatrix} (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.2, 0.5) & (0.4, 0.5) & (0.2, 0.4) & (0.4, 0.5) & (0.2, 0.6) \\ (0.4, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.4, 0.5) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.6) \\ (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.5) & (0.4, 0.5) & (0.5, 0.5) & (0.3, 0.6) \\ (0.5, 0.2) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.6) & (0.4, 0.5) & (0.5, 0.5) & (0.3, 0.6) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.4) & (0.6, 0.4) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.4, 0.5) \\ (0.4, 0.2) & (0.5, 0.5) & (0.5, 0.4) & (0.4, 0.6) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.4, 0.5) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.5) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.5, 0.4) \\ (0.6, 0.2) & (0.6, 0.4) & (0.6, 0.3) & (0.4, 0.5) & (0.5, 0.4) & (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.449489) & (0.449489, 0.449489) & (0.21393, 0.5) & (0.21393, 0.449489) & (0.19074, 0.485) & (0.037, 0.55) \\ (0.4, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.35247, 0.5) & (0.35247, 0.5) & (0.44948, 0.5) & (0.16, 0.55) \\ (0.449489, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.4, 0.5) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.5501) & (0.225, 0.55) \\ (0.449489, 0.449489) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.6) & (0.449489, 0.55051) & (0.4, 0.550) & (0.307, 0.5) \\ (0.449489, 0.449489) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.6) & (0.449489, 0.55051) & (0.4, 0.550) & (0.307, 0.5) \\ (0.5, 0.21393) & (0.5, 0.35247) & (0.5, 0.35247) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) \\ (0.49489, 0.2139387) & (0.5, 0.35247) & (0.5, 0.4) & (0.449489, 0.55051) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.449) \\ (0.485281, 0.190743) & (0.5, 0.449489) & (0.40, 0.55051) & (0.55051, 0.4) & (0.5, 0.5) & (0.5, 0.4) & (0.5, 0.5) & (0.5, 0.4) \\ (0.550510, 0.037425) & (0.55051, 0.16) & (0.55051, 0.22538) & (0.5, 0.30769) & (0.5, 0.5) & (0.44948, 0.5) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

Then using the equation (3.7) Calculate the distance between intuitionistic relation $d(\bar{M}, M) = 0.085004$ which is less than τ . Here we will fix the threshold value $\tau = 0.1$. Therefore the above matrix is consistent. The next step is calculating the weight of all attributes using the equation (3.8). From the Table (8) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (9) and Figure 4.

Table 8:

weight	μ	ν
W(Q1)	0.075613	0.853301
W(Q2)	0.084694	0.862566
W(Q3)	0.080557	0.864323
W(Q4)	0.076372	0.862566
W(Q5)	0.08222	0.84224
W(Q6)	0.073246	0.84224
W(Q7)	0.142811	0.839607
W(Q8)	0.1341	0.813152

Table 9:

Attributes	$\rho(\alpha)$	Preference
ICA (Q3)	0.932162	1
ERA (Q2)	0.931283	2
CA and PA (Q4)	0.931283	3
SREA (Q1)	0.92665	4
Q1 Certi (Q5)	0.92112	5
QMS(Q6)	0.92112	6
PRI (Q7)	0.919804	7
INFRA (Q8)	0.906576	8

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Table 10:

weight	μ	ν
W(C1)	0.271628	0.693298
W(C2)	0.347826	0.613636
wC3)	0.337067	0.647611

Table 11:

Attribute	$\rho(\alpha)$	P
Process cost	0.8466	1
Admin.Cost	0.8068	2
Raw Material Cost	0.8238	3

8.2. cost

The next attribute in the level 1 is Cost which is classified in to three attributes according the expert suggestion. The attributes are C1, C2, C3 represent Raw Material cost, Admin Cost, Process Cost respectively. Based on the expert suggestion the first intuitionistic preference relation matrix formed.

$$M = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.6) & (0.4, 0.6) \\ (0.6, 0.3) & (0.5, 0.5) & (0.5, 0.5) \\ (0.6, 0.4) & (0.5, 0.5) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.6) & (0.44949, 0.55051) \\ (0.6, 0.3) & (0.5, 0.5) & (0.5, 0.5) \\ (0.55051, 0.4494) & (0.5, 0.5) & (0.5, 0.5) \end{pmatrix}$$

Now using the equation (3.7) Calculate $d(\bar{M}, M) = 0.04949$ which is less than τ . Therefore the above relation is consistent, and weight of the above attributes calculated by the equation (3.8). Using the equation (1.2) generate Table 11 and the Figure 5 we can get the preference of the attributes.

8.3. service

In the service is categories into five parts Quick response (S1), Collaborative design (S2), Vertical Integration (S3), Field failures (S4), Spare Parts (S5). According the experts suggestion using the Table-c the intutionistic preference relation matrix (M) formed (Table 4).

$$M = \begin{pmatrix} (0.5, 0.5) & (0.2, 0.6) & (0.5, 0.5) & (0.5, 0.5) & (0.2, 0.7) \\ (0.6, 0.2) & (0.5, 0.5) & (0.4, 0.6) & (0.4, 0.6) & (0.3, 0.6) \\ (0.5, 0.5) & (0.6, 0.4) & (0.5, 0.5) & (0.3, 0.6) & (0.2, 0.7) \\ (0.5, 0.5) & (0.6, 0.4) & (0.6, 0.3) & (0.5, 0.5) & (0.4, 0.6) \\ (0.7, 0.2) & (0.6, 0.3) & (0.7, 0.2) & (0.6, 0.4) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.5, 0.5) & (0.2, 0.6) & (0.395644, 0.55051) & (0.423232, 0.537713) & (0.4232, 0.541) \\ ((0.6, 0.2) & (0.5, 0.5) & (0.4, 0.6) & (0.423232, 0.55051) & (0.4232, 0.5635) \\ (0.6, 0.2) & (0.6, 0.4) & (0.5, 0.5) & (0.3, 0.6) & (0.449, 0.525) \\ (0.55051, 0.395644) & (0.55051, 0.423232) & (0.6, 0.3) & (0.5, 0.5) & (0.4, 0.6) \\ (0.54196, 0.423232) & (0.563508, 0.423232) & (0.525063, 0.44949) & (0.6, 0.4) & (0.5, 0.5) \end{pmatrix}$$

Then using the equation (3.7) Calculate the distance between intuitionistic relation $d(\bar{M}, M) = 0.07398$ which is less than τ . Here the we will fix the threshold value $\tau = 0.1$. Therefore the above matrix is consistent. The next step is calculating the weight of all attributes using the equation (3.8). From the Table (12) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (13) and 9.

Table 12:

weight	μ	ν
Weight(S1)	0.177981	0.81194
Weight(S2)	0.176251	0.785744
weight(S3)	0.183286	0.770089
weight(S4)	0.201709	0.769576
Weight(S5)	0.186716	0.767677

Table 13:

Attributes	$\rho(\alpha)$	P
Quick response	0.905969	1
Collaborative design	0.892872	2
Vertical Integration	0.885044	3
Field failures	0.884788	4
Spare Parts	0.883836	5

8.4. Delivery

The delivery is categories into four areas MMOG (D1), Manufacturing capacity (D2), Implementation of Karbon (D3), SCM (D4) Table C the intuitionistic preference relation matrix (M) formed (Table 5).

$$M = \begin{pmatrix} (0.6, 0.44) & (0.44, 0.6) & (0.4, 0.5) & (0.4, 0.6) \\ (0.6, 0.44) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.6) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.5) \\ (0.6, 0.4) & (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.6, 0.44) & (0.44, 0.6) & (0.4, 0.5) & (0.4, 0.6) \\ (0.6, 0.44) & (0.5, 0.5) & (0.4, 0.5) & (0.4, 0.6) \\ (0.5, 0.4) & (0.5, 0.4) & (0.5, 0.5) & (0.4, 0.5) \\ (0.6, 0.4) & (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.5) \end{pmatrix}$$

Then using the equation (3.7) Calculate the distance between intuitionistic relation $d(\bar{M}, M) = 0.058179$ which is less than τ . Here the we will fix the threshold value $\tau = 0.1$. Therefore the above matrix is consistent. The next step is calculating the weight of all attributes using the equation (3.8). From the Table (15) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (16) and Figure 6.

Table 14: weight and Preference

Table 15:

weight	μ	ν
Weight(D1)	0.209053	0.75891
Weight(D2)	0.230844	0.732986
weight(D3)	0.224984	0.714839
weight(D4)	0.248666	0.698631

Table 16:

Attributes	$\rho(\alpha)$	P
MMOG(D1)	0.879455	1
Manf. Capacity(D2)	0.866493	2
Implementation of karbon (D3)	0.85742	3
hline SCM(D4)	0.849316	4

8.5. Administration Cost

The delivery is categories into four areas Design free(A1), Tools, Jigs and Fixtures cost(A2), ICC(A3), Transport cost(A4). Using the Table C the intuitionistic preference relation matrix (M) formed (Table 6).

$$M = \begin{pmatrix} (0.5, 0.5) & (0.4, 0.6) & (0.44, 0.55) & (0.4, 0.6) \\ (0.6, 0.4) & (0.50, 0.5) & (0.44, 0.55) & (0.4, 0.55) \\ (0.55, 0.44) & (0.55, 0.44) & (0.5, 0.5) & (0.3, 0.7) \\ (0.6, 0.4) & (0.55, 0.4) & (0.7, 0.3) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M})

$$\bar{M} = \begin{pmatrix} ((0.5, 0.5)) & (0.4, 0.6) & (0.44, 0.55) & (0.278946, 0.695731) \\ (0.6, 0.4) & (0.5, 0.5) & (0.44, 0.55) & (0.469891, 0.525063) \\ (0.55, 0.44) & (0.55, 0.44) & (0.5, 0.5) & (0.4, 0.5) \\ (0.695731, 0.278946) & (0.525063, 0.469891) & (0.5, 0.4) & (0.50, 0.5) \end{pmatrix}$$

Then using the equation (3.7) Calculate the distance between intuitionistic relation $d(\bar{M}, M) = 0.03039$ which is less than τ . Here the we will fix the threshold value $\tau = 0.1$. Therefore the above matrix is consistent. The next step is calculating the weight of all attributes using the equation (3.8). From the Table (17) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (18) and Figure 8.

Table 17:

weight	μ	ν
Weight(A1)	0.198635	0.789255
Weight(A2)	0.246601	0.742034
weight(A3)	0.245388	0.729924
weight(A4)	0.272478	0.700475

Table 18:

Attributes	$\rho(\alpha)$	P
Design fee(A1)	0.894628	1
Tools, Jigs and Fixtures cost(A2)	0.871017	2
ICC(A3)	0.864962	3
Transport cost(A4)	0.850237	4

8.6. process robust index

In the Quality the 7th attribute process robust index(PRI)classified into three sub attributes in the level-3. The attributes are design Failure mode effect analysis and process failure mode effect analysis (DMEA and PFMEA), yoke implemented, Optimization.

$$M = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.6) & (0.2, 0.8) \\ (0.6, 0.3) & (0.5, 0.5) & (0.3, 0.7) \\ (0.8, 0.2) & (0.7, 0.3) & (0.5, 0.5) \end{pmatrix}$$

To check the consistence preference relation using the above formula (3.5) and (3.6) we can get the multiplicative fuzzy relation Matrix (\bar{M}).

$$\bar{M} = \begin{pmatrix} (0.5, 0.5) & (0.3, 0.6) & (0.246606, 0.753394) \\ (0.6, 0.3) & (0.5, 0.5) & (0.3, 0.7) \\ (0.753394, 0.246606) & (0.7, 0.3) & (0.5, 0.5) \end{pmatrix}$$

Then using the equation (3.7) Calculate the distance between intuitionistic relation $d(\bar{M}, M) = 0.085004$ which is less than τ . Here the we will fix the threshold value $\tau = 0.1$. Therefore the above matrix is consistent. The next step is calculating the weight of all attributes using the equation (3.8). From the Table (19) and using (1.2) we will get the preference (P) of all the attributes. It is given in the Table (20) and Figure 7.

Table 19:

weight	μ	ν
Weight(A1)	0.227523	0.739408
Weight(A2)	0.304348	0.659091
weight(A3)	0.424651	0.556047

Table 20:

Attributes	$\rho(\alpha)$	P
DFMEA	0.869704	1
YOKE	0.829545	2
OPTIMIZATION	0.778023	3

8.7. Figures



Figure 3: customer Requirement

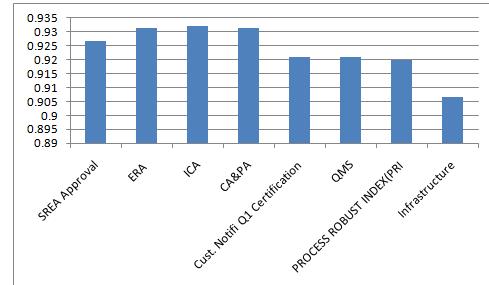


Figure 4: Quality

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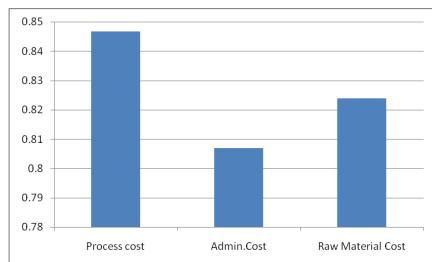


Figure 5: Cost

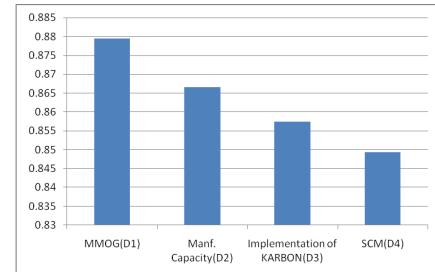


Figure 6: delivery

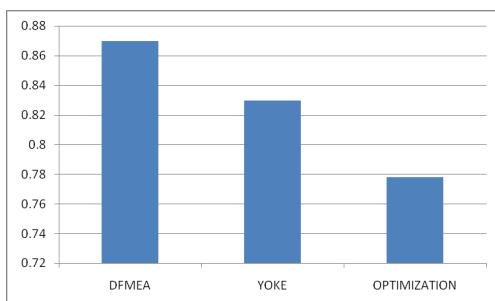


Figure 7: process

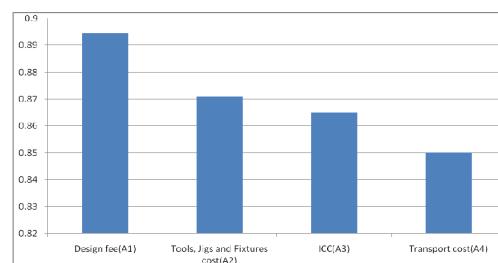


Figure 8: Adminstration

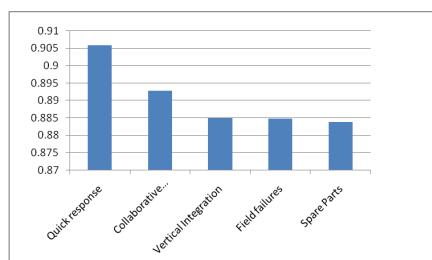


Figure 9: service

