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# Fuzzy Geometric Programming (FGP) and Its Application In Riser Design Problem

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## Abstract

The riser design plays an important role in the quality and cost of a metal casting. Due to the lack of fixed theoretical procedure to follow, the design process is carried out on a trial-and-error basis. The casting design optimization problem is characterized by multiple design variables, conflicting objectives and a complex search space making it unsuitable for sensitivity-based optimization. In this paper Riser design problem is introduced in fuzzy environment. This fuzzy model is solved by fuzzy geometric programming technique. Here fuzzy geometric programming is discussed through fuzzy decision making of Bellman and Zadeh we get the optimal solution by using three different operators, max additive operator, max product operator, max-min operator. Numerical example is taken to illustrate the fuzzy model.

**Index Terms:** optimization, fuzzy geometric programming, duality theorem, riser design problem, Chvorinov’s rule.

## Introduction

Geometric programming is an effective method to solve a nonlinear programming problem. It has certain advantages over the other optimization methods. Here, the advantages are that it is usually much simpler to work with the dual than primal. Degree difficulty plays an important role for solving a nonlinear programming problem by geometric programming method. Since late 1960, geometric programming has been known and used in various fields . Duffin and Petersen and Zener (1966) [12] discussed thebasic theories of geometric programming. There are many references on application and the method of geometric programming. in the papers like Eckar(1980), Beightler(1979) [3], Zener(1971), Jung and klain(2001) developed single item inventory problems and solved by geometric programming method. In the last 20 yrs fuzzy geometric programming has received rapid development in the theory and application. In 2002 ,B.Y.Cao [8] published the first monograph of fuzzy geometric programming as applied optimization series (vol 76), fuzzy geometric programming by Kluwer academy publishing (the present spinger), the book gives a detailed exposition to theory and application of fuzzy geometric programming. R. C. Creese [10, 11] used geometric programming to optimize riser design problem. Here werepresent an approach for solving geometric programming problem under fuzzy environment. Based on fuzzy decision making of Bellman and Zadeh [4] we get the optimal solution by using max additive operator, max product operator, max-min operator.

## Fuzzy Geometric Programming (FGP)

$\widetilde{Minimize}$ $g_0$ (t) ………….. (1)

Subject to $g_j$ (t) $\lesssim$ $b_j$ j=1,2,…………..m

t ≥ 0

the linear membership functions can be represented by

$$\mu_j(g_j(t)) = \begin{cases} 1 & if\ g_j(t) \le g_j^0 \\ \frac{g_j' - g_j(t)}{g_j' - g_j^0} & if\ g_j^0 \le g_j(t) \le g_j' \\ 0 & if\ g_j(t) \ge g_j^0 \end{cases}$$

for j= 0,1,2,3,…………m.

Based on fuzzy decision making of bellman and zadeh (1972), we may write

1) $\mu_D(t^*) = \max(\min \mu_j(g_j(t)))$ (max-min operator) subject to $\mu_j(g_j(t)) = \frac{g_j' - g_j(t)}{g_j' - g_j^0}$ (j= 0,1,2,3,…………m.)
   t > 0
2) $\mu_D(t^*) = \max(\sum_{j=0}^{m} \lambda_j \mu_j(g_j(t)))$(max-additive operator) subject to $\mu_j(g_j(t)) = \frac{g_j' - g_j(t)}{g_j' - g_j^0}$ (j= 0,1,2,3,…………m.)
   t > 0
3) $\mu_D(t^*) = \max(\prod_{j=0}^{m} \left(\mu_j(g_j(t))\right)^{\lambda_j})$ (max-product operator) subject to $\mu_j(g_j(t)) = \frac{g_j' - g_j(t)}{g_j' - g_j^0}$ (j= 0,1,2,3,…………m.)
   t > 0

Here for $\lambda_j$ (j=0,1,2,…………m) are numerical weights considered by a decision making unit . for normalized weights $\sum_{j=0}^{m} \lambda_j = 1$

For equal importance of objective and constraint goals, $\lambda_j = 1$

## Riser Design Problem

The function of a riser is to supply additional molten metal to a casting to ensure a shrinkage porosity free casting. Shrinkage porosity occurs because of the increase in density from the liquid to solid state of metals. To be effective a riser must solidify after casting and contain sufficient metal to feed the casting. Casting solidification time is predicted from Chvorinov's rule. Chvorinov's rule provides guidance on why risers are typically cylindrical. The longest solidification time for a given volume is the one where the shape of the part has the minimum surface area. From a practical standpoint cylinder has least surface area for its volume and is easiest to make. Since the riser should solidify after the casting, we want it's solidification time to be longer than the casting.

A cylindrical side riser which consists of a cylinder of height H and diameter D. The theoritical basis for riser design is Chvorinov's rule, which is $t = k\,(V/SA)^2$ .

Where t = solidification time (minutes/seconds)

K = solidification constant for molding material (minutes/in$^2$ or seconds/cm$^2$)

V = riser volume (in$^3$ or cm$^3$)

SA = cooling surface area of the riser.

The objective is to design the smallest riser such that tR ≥ tC

Where $t_R$ = solidification time of the riser.

$T_C$ = solidification time of the casting.

$K_R\,(V_R/SA_R)^2 \geq K_C\,(V_C/SA_C)^2$

The riser and the casting are assumed to be molded in the same material, so that $K_R$ and $K_C$ are equal. So $(V_R/SA_R) \geq (V_C/SA_C)$ .

The casting has a specified volume and surface area , so $V_C/SA_C$ = Y = constant, which is called the casting modulus .

$(V_R/SA_R) \geq Y$ , $V_R = n\,D^2H/4$ , $SA_R = n\,DH + 2\,n\,D^2/4$

$(n\,D^2H/4)/(n\,DH + 2\,n\,D^2/4) = (DH)/(4H+2D) \geq Y$

Or, $4YD^{-1} + 2YH^{-1} \leq 1$

Primal side cylindrical riser design problem can be stated as :

Minimize $V = n\,D^2H/4$

Subject to $4YD^{-1} + 2YH^{-1} \leq 1$.

## Numerical Example

$\widetilde{Minimize}$ $g_0 = n\,D^2H/4$ ………………………….. (2)

Subject to $(V_R/SA_R)$ Y

Or, $(n\,D^2H/4)/(n\,DH + 2\,n\,D^2/4)$ Y

Or, (DH)/4( H + $\frac{D}{2}$) Y

Here linear membership functions for the fuzzy objective and constraint goals are

$$\mu_{g_0} = \begin{cases} 1 & if\ g_0 \leq 225 \\ \frac{240-g_0}{15} & if\ 225 \leq g_0 \leq 240 \\ 0 & if\ g_0 \geq 240 \end{cases}$$

$$\mu_{g_1} = \begin{cases} 0 & if\ g_1 \le 1 \\ \frac{g_1 - 1}{0.2} & if\ 1 \le g_1 \le 1.2 \\ 1 & if\ g_1 \ge 1.2 \end{cases}$$

$\mu_{g_1} = \frac{\frac{DH}{4(H+\frac{D}{2})} - 1}{0.2} = \frac{DH}{0.8t}$ - 5, (H+D/2 ≤ t)

**1. Based on max-additive operator FGP (2) reduces to**

Max z = $-\frac{n\mathrm{D}^2\mathrm{H}}{60} + \frac{DH}{0.8t}$

s.t. $Ht^{-1} + \frac{D}{2}t^{-1} \le 1$

or, Min z = $\frac{n\mathrm{D}^2\mathrm{H}}{60} - \frac{DH}{0.8t}$ ………………….. (3)

s.t. $Ht^{-1} + \frac{D}{2}t^{-1} \le 1$

The dual prpblem of this GP (3)

Max d(w) =

$\xi_0\left\{\left(\frac{n}{60w_{01}}\right)^{w_{01}}\left(\frac{1}{0.8w_{02}}\right)^{-w_{02}}\left(\frac{1}{w_{03}}\right)^{w_{03}}\left(\frac{1}{2w_{04}}\right)^{w_{04}}(w_{03}+w_{04})^{w_{03}+w_{04}}\right\}^{\xi_0}$ …(4)

Subject to $w_{01} - w_{02} = -1$…………(4.1) (Here $\xi_0$ =-1 )

$2w_{01} - w_{02} + w_{04} = 0$ …………(4.2)

$w_{01} - w_{02} + w_{03} = 0$ …………….(4.3)

$w_{02} - w_{03} - w_{04} = 0$ …………….(4.4)

$W_{01}$ = ½ , $w_{02}$ = 3/2 , $w_{03}$ = 1 , $w_{04}$ = ½

Primal-dual variable relations are

$\frac{n\mathrm{D}^2\mathrm{H}}{60} = w_{01}\, d(w)$ ………………(4.5)

$-\frac{DH}{0.8t} = -w_{02}\, d(w)$ ……………….(4.6)

$Ht^{-1} = \frac{w_{03}}{w_{03}+w_{04}}$ ………………..(4.7)

$\frac{D}{2}t^{-1} = \frac{w_{04}}{w_{03}+w_{04}}$ ………………..(4.8)

From (4..7 ) and (4.8) $\frac{H}{t} = \frac{2}{3}$ , $\frac{D}{2t} = \frac{1}{3}$

From (4.5) and (4.6) $\frac{\frac{n\mathrm{D}^2\mathrm{H}}{60}}{\frac{DH}{0.8t}} = \frac{w_{01}}{w_{02}} = \frac{1}{3}$

Or, $\frac{n}{60}\frac{2t}{3}\frac{0.8t}{10} = \frac{1}{3}$ 0r, $t^* = 3.45$

So, the optimal solution is $H^* = D^* = 2.30$

2. **Based on the max-product operator FGP (2 )reduces to**

Max g $=\left(\frac{240-g_0}{15}\right)\left(\frac{g_1-1}{0.2}\right) = 80g_1 + \frac{1}{3}g_0 - \frac{g_0 g_1}{3} - 80$

$=\frac{20DH}{H+\frac{D}{2}} + \frac{1}{12}n\mathrm{D}^2\mathrm{H} - \frac{n\mathrm{D}^2\mathrm{H}}{48}\frac{DH}{H+\frac{D}{2}} - 80$

Min $\frac{nD^3H^2}{48}t^{-1} - 20DHt^{-1} - \frac{1}{12}n\mathrm{D}^2\mathrm{H}$ ……………… (5)

Such that $Ht^{-1} + \frac{D}{2}t^{-1} \leq 1$

D, H, t > 0

Here D.D =5-3-1 =1

The dual prpblem of this GP (5)

Max d(w) = $\xi_0\left\{\left(\frac{n}{48w_{01}}\right)^{w_{01}}\left(\frac{20}{w_{02}}\right)^{-w_{02}}\left(\frac{n}{12w_{03}}\right)^{-w_{03}}\left(\frac{1}{w_{11}}\right)^{w_{11}}\left(\frac{1}{2w_{12}}\right)^{w_{12}}(w_{11}+w_{12})^{w_{11}+w_{12}}\right\}^{\xi_0}$ ……………………….(6)

Subject to $w_{01} - w_{02} - w_{03} = -1$ …………(6.1) (Here $\xi_0$ =-1 )

$3w_{01} - w_{02} - 2w_{03} + w_{12} = 0$ …………(6.2)

$2w_{01} - w_{02} - w_{03} + w_{11} = 0$ …………….(6.3)

$-w_{01} + w_{02} - w_{11} - w_{12} = 0$ …………….(6.4)

$W_{01} = 1-2w_{12}$ , $w_{02} = 1+w_{12}$ , $w_{03} = 1-3w_{12}$ , $w_{11} = 2w_{12}$

Primal-dual variable relations are

$\frac{n\mathrm{D}^3\mathrm{H}^2t^{-1}}{48} = w_{01}$ d() ……………(6.5)

$-\frac{20DH}{t} = -w_{02}$ d(w) ……………(6.6)

$-\frac{1}{12}n\mathrm{D}^2\mathrm{H} = -w_{03}$ d(w) ……………..(6.7)

$Ht^{-1} = \frac{w_{11}}{w_{11}+w_{12}}$ ……………(6.8)

$\frac{D}{2}t^{-1} = \frac{w_{12}}{w_{11}+w_{12}}$ ..………….(6.9)

From (6) ln(d(w)) = ln(-1) –

$$\left[(1-2w_{12})ln\left(\frac{n}{48(1-2w_{12})}\right) - (1+w_{12})ln\left(\frac{20}{1+w_{12}}\right) - (1-3w_{12})ln\left(\frac{n}{12(1-3w_{12})}\right) + 2w_{12}ln\left(\frac{1}{2w_{12}}\right) + w_{12}ln\left(\frac{1}{2w_{12}}\right) + 3w_{12}ln(3w_{12})\right]$$

$\frac{\partial[\ln(d(w))]}{\partial w_{12}} = 0$

$\Rightarrow$ -2 $ln\left(\frac{n}{48(1-2w_{12})}\right)$ - $ln\left(\frac{20}{1+w_{12}}\right) + 3\ ln\left(\frac{n}{12(1-3w_{12})}\right)$ +3 $ln\left(\frac{1}{2w_{12}}\right) + 3ln(3w_{12})$ = 0

$W_{12}$ = 0.272 , $w_{11}$ = 0.544 , $w_{03}$ = 0.184 , $w_{02}$ = 1.272 , $w_{01}$ = 0.456
From (6.5) and (6.6) t= 9.57 ,H= 4.66 ,D= 6.38
So, the optimal solution is $H^* = D^*$ =6.38

**3. Based on the max-min operator FGP (2 )reduces to**

Max Min $\left\{\mu_{g_0}, \mu_{g_1}\right\}$

Max $\left(\frac{240-g_0}{15}\right)$

Subject to $\frac{240-g_0}{15} \leq \frac{g_1-1}{0.2}$

Max (16 - $\frac{n D^2 H}{60}$)

Subject to $\frac{1}{1260}\ nD^2H + \frac{5}{84}\frac{DH}{H+\frac{D}{2}} \geq 1$

Min $\frac{nD^2H}{60}$

Subject to $\frac{84}{5}\ D^{-1}H^{-1}t - \frac{n}{75}Dt \leq 1$ (Here $H+\frac{D}{2} \leq t$)

$Ht^{-1} + \frac{D}{2}t^{-1} \leq 1$

D, H, t > 0 ……………….. (7)

Here D.D = 5-3-1=1

The dual prpblem of this GP (7)

$$\text{Max}\quad d(w) = \xi_0\left\{\left(\frac{n}{60w_{01}}\right)^{w_{01}}\left(\frac{84}{5w_{11}}\right)^{w_{11}}\left(\frac{n}{75w_{12}}\right)^{-w_{12}}\left(\frac{1}{w_{13}}\right)^{w_{13}}\left(\frac{1}{2w_{14}}\right)^{w_{14}}(w_{11}-w_{12})^{w_{11}-w_{12}}(w_{13}+w_{14})^{w_{13}+w_{14}}\right\}^{\xi_0} \quad \text{…………..} \quad (8)$$

Subject to $w_{01} = 1$ …………(8.1) (Here $\xi_0$ =1 )

$2w_{01} - w_{11} - w_{12} + w_{14} = 0$ …………(8.2)

$w_{01} - w_{11} + w_{13} = 0$…………(8.3)

$w_{11} - w_{12} - w_{13} - w_{14} = 0$…………(8.4)

$W_{01}$ = 1 , $w_{12} = \frac{3-w_{11}}{2}$ , $w_{13} = w_{11} - 1$ , $w_{14} = \frac{w_{11}-1}{2}$

Primal-dual variable relations are

$\frac{nD^2H}{60} = w_{01}\, d(w)$ ……………(8.5)

$\frac{84}{5}\ D^{-1}H^{-1}t = \frac{w_{11}}{w_{11}+w_{12}}$ ……………(8.6)

$\frac{n}{75}D\ t = \frac{w_{12}}{w_{11}+w_{12}}$ …………(8.7)

$Ht^{-1} = \frac{w_{13}}{w_{13}+w_{14}}$ …………(8.8)

$\frac{D}{2}t^{-1} = \frac{w_{14}}{w_{13}+w_{14}}$ .............(8.9)

From (8) $\ln(d(w)) = \ln(\frac{n}{60}) + w_{11}ln\left(\frac{84}{5w_{11}}\right) - \left(\frac{3-w_{11}}{2}\right)ln\left(\frac{2n}{75\,(3-w_{11})}\right) + (w_{11}-1)ln\left(\frac{1}{w_{11}-1}\right) + \frac{w_{11}-1}{2}\ln\left(\frac{1}{w_{11}-1}\right) + 2\left(\frac{-3+3w_{11}}{2}\right)ln\left(\frac{-3+3w_{11}}{2}\right)$

$\frac{\partial[\ln(d(w))]}{\partial w_{11}} = 0$

$\Rightarrow ln\left(\frac{84}{5w_{11}}\right) + \frac{1}{2}\,ln\left(\frac{2n}{75(3-w_{11})}\right) + \frac{3}{2}\ln\left(\frac{1}{w_{11}-1}\right) + 3ln\left(\frac{3w_{11}-3}{2}\right) = 0$

$W_{01} = 1$ , $w_{11} = 1.21$ , $w_{12} = 0.895$ , $w_{13} = 0.21$ , $w_{14} = 0.105$

D= 6.66 , H= 6.66

So, the optimal solution is $H^* = D^* = 6.66$

**Table 1:**

| Optimization method | Optimal dual variables | Optimal primal variables | Optimal objective value | | Aspiration level |
|---|---|---|---|---|---|
| | | | $g^*_0$ | $g^*_1$ | $\mu_{g_0}$ , $\mu_{g_1}$ |
| Max-additive Operator | $W_{01}^*=0.5$<br>$W_{02}^*=1.5$<br>$W_{03}^*=1$<br>$W_{04}^*=0.5$ | $H^*=2.30$<br>$D^*=2.30$<br>$H^*+D^*=4.6$ | 9.55 | 0.38 | $\mu_{g_0}=1$<br>$\mu_{g_1}=0$ |
| Max-product Operator | $W_{01}^*=0.428$<br>$W_{02}^*=1.286$<br>$W_{03}^*=0.142$<br>$W_{11}^*=0.572$<br>$W_{12}^*=0.286$ | $H^*=6.38$<br>$D^*=6.38$<br>$H^*+D^*=12.76$ | 204.04 | 1.06 | $\mu_{g_0}=1$<br>$\mu_{g_1}=0.3$ |
| Max-min operator | $W_{01}^*=1$<br>$W_{11}^*=1.21$<br>$W_{12}^*=0.895$<br>$W_{13}^*=0.21$<br>$W_{14}^*=0.105$ | $H^*=6.66$<br>$D^*=6.66$<br>$H^*+D^*=13.32$ | 232.10 | 1.11 | $\alpha = \mu_{g_0} = \mu_{g_1}=0.52$ |

Comparison of optimal solution in different optimization method

## Conclusion

Here fuzzy geometric programming is illustrated through fuzzy decision- making process . Three different operators namely ,max-min, max-additive, max-product are considered for this fuzzy geometric programming. We compare the optimal solution in different optimization method. There are more several operators based on different t-norms. This fuzzy model may developed using these operators. Max additive operator method gives the better result to optimize the riser design problem.This fuzzy model can be solved easily through fuzzy geometric programming than other fuzzy non-linear programming method.

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