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Special Pythagorean Triangles and 3-Digit Dhuruva Number

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Abstract

Pythagorean triangles, each with a leg represented by a 3-digit Dhuruva number are obtained. A few interesting results are given.

Keywords: Pythagorean triangles, 3-digit Dhuruva numbers

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Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited.Thus the objective of this paper is to find out the special Pythagorean triangles in connection with 3 digit Dhuruva number 495.

In this communication we have presented Pythagorean triangles each with a leg represented by 3-digit Dhuruva number 495. Also, a few special Pythagorean triangles in connection with 495 are obtained.

Basic Definitons

Definition 2.1:
The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2:
Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

Definition 2.3: Dhuruva numbers
The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

Method of Analysis

In this section, we exhibit Pythagorean triangles, each with a leg represented by the 3-digit Dhuruva number 495 and denote this number by N.

To start with, it is noted that the leg y can not be represented by N as y is even and N is odd. Also z can not be written as sum of two squares. Since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P = p_1^{r_1} p_2^{r_2} \ldots\ldots p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 \pmod 4$ then r_i is even. A prime $p \equiv 1 \pmod 4$ can be written as $p = a^2 + b^2$

Now, consider x=N => $m^2 - n^2 = 495$

which is a binary quadratic Diophantine equation. Solving the above equation for m, n we get 6 integer solutions and thus, we have 6Pythagorean triangles, each having the leg $x$to be represented by the three digit Dhuruva number $N = 495$ as shown in the table below:

S. No	m	n	x	y	z	P	A
1	248	247	495	122512	122513	245520	3021720
2	52	47	495	4888	4913	10296	1209780
3	84	81	495	13608	13617	27720	3367980
4	32	23	495	1472	1553	3520	364320
5	24	9	495	432	657	1584	106920
6	28	17	495	952	1073	2520	235620

Note that there are 4 primitive and 2 non-primitive triangles. Also the relation $\frac{4A}{P} - y + z$ represent a 3-digit Dhuruva number for each of the above Pythagorean triangles, where A and P represents the area and perimeter of the Pythagorean triangle.

In a similar manner, it is seen that there are 12 Pythagorean triangles where –in, each of the following expressions $\frac{2A}{P}$, $\frac{1}{2}(y+x-z)$ represent 495 as shown in the table below.

S. No	m	n	x	y	z	A	P	$\frac{2A}{P}$	$\frac{1}{2}$(y+x-z)
1	*104*	5	10791	1040	10841	5611320	22672	495	495
2	*64*	9	4015	1152	4177	2312640	9344	495	495
3	*56*	11	3015	1232	3257	1857240	7504	495	495
4	*168*	3	28215	1008	28238	14220360	57456	495	495
5	*48*	15	2079	1440	2529	1496880	6048	495	495
6	*48*	33	1215	3162	3393	1924560	7776	495	495
7	*56*	45	1111	5040	5161	2799720	11312	495	495
8	*64*	55	1071	7040	7121	3769920	15232	495	495
9	*104*	99	1015	20592	20617	10450440	42224	495	495
10	*168*	165	999	55440	55449	27692280	111888	495	495
11	*496*	1	246015	992	246017	122023440	493024	495	495
12	*496*	495	991	491040	491041	23310320	983072	495	495

Note that there are 8 primitive and 4 non-primitive triangles.

Also, it is observed that there are 6 Pythagorean triangles where in each, the expressions $x - \frac{2A}{P}$, $\frac{1}{2}$(z+x-y) is represented by 495 as shown in the table below:

S. No	m	n	x	y	z	A	P
1	55	46	909	5060	5141	2299770	11110
2	99	94	965	18612	18637	8980290	38214
3	33	18	765	1188	1413	454410	3366
4	45	34	869	3060	3181	1329570	7110
5	165	162	981	53460	53469	26222130	107910
6	495	1	989	489060	489061	241840170	979110

Note that there are 4 primitive and 2 non-primitive triangles.

Conclusion

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

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