Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 3 (2015), pp. 1595-1602

http://www.ripublication.com

Sensitivity Analysis of Mathematical Model of Coriolis Mass Flow Meter

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Abstract

Coriolis mass flow meters (CMFs) are increasingly used in many process industries with the feature of directly measuring the mass flow rate. Though it is considered as an accurate system, it is generally affected by external factors such as external temperature, pressure, vibrations etc. The influence of these factors causes tremendous changes in the mass flow rate measurement since the process is prone to change even by a small deviation. In this paper, the effect of external temperature over the meter factor or the calibration factor ‘k’ is discussed in detail. The influence of variation in external factors over the system is studied and the corrective measures required are also discussed.

Keywords: mass flow rate, meter factor, temperature compensation, young’s modulus

Introduction

Mass flow measurement plays an important role in the material balance, billing and custody transfer operations throughout the process industries. The Coriolis Mass Flow meter (CMF) remains the only instrument that directly measures the mass flow rate. This being the pre-eminent flow measurement in the processing plants, the accuracy and reliability of mass flow measurement is indispensable [8]. Usually, any constant present in the mathematical expression is not considered while analysing the system properties with respect to changes in the external factors. Similarly, the mathematical expression of mass flow rate of the CMF [1] also involves a constant called Meter Factor ‘k’ which plays a greater role in the sensitivity of the meter. The Meter Factor is calculated by the process called ‘Wet Calibration’. In this technique the dimensions of the meter such as radii of the flow tubes, thermal coefficient of expansion, length of the flow tubes etc are taken into consideration for the calculation of the meter factor. We know that the mechanical structures and characteristics of flow tubes are easily prone to variations with changes in the external factors like pressure [9], temperature, vibrations [5] etc. Though the meter factor is considered as a constant usually, it depends on two major properties called Young's Moduli (E) and Area moment of Inertia (I) which in turn varies with the change in temperature and diameter of the flow tube respectively [4]. These characteristics restrict the reliability of Coriolis mass flow meters at adverse conditions [7]. The effect of external temperature over the meter factor is discussed nearly 20 years back [10]. An attempt to make the real time system behave similar to the ideal mathematical model strictly requires the constant also to be in consideration [11]. The deviation in actual value due to the meter factor must also be taken into account when the mathematical model is expected to behave like an ideal system. The range by which the accuracy of mass flow rate gets affected with respect to the meter factor is studied and its corrective measures are discussed.

Objectives

The volumetric flow meters or the variable area flow meters (like orifice, venturi meters) gives how much volume of fluid is passing through the given point. It does not give the real mass of the fluid. In case of the coriolis mass flow meter it measures directly and continuously the accurate value of mass flow rate. Despite being relatively expensive, the Coriolis meters have high reliability (due to the absence of moving parts in the flow path). It gives multiple measured values like mass flow, density, volume flow and temperature. Many industrial processes are widely using the Coriolis mass flow meter for its higher accuracy. Thus obtaining the sensitivity analysis of the CMF model and making it act similar to the original system will be captivating [3]. Here the mathematical model of the Twin U tube Coriolis mass flow meter is considered and steps are taken towards making the real time system behave similar to the ideal model.

Mathematical Model of CMF

Modelling of CMF has been proposed by many researchers which involves traditional differential equations that use the mass, stiffness and damping matrices. Modelling by finite element analysis is also described in many papers. A direct mathematical expression for the Coriolis mass flow meter was formulated by M. Kazahaya recently. Coriolis mass flow meters are available in various configurations depending on the shape and number of flow tubes present. They may be single straight tube CMF, single U tube CMF, S shaped tube CMF, twin U tube CMF etc. The flow tubes may be of straight or bent form, and some configurations can also be self draining when placed vertically [12]. When the configuration consists of two parallel flow tubes, the fluid flow is divided into two streams by a splitter near the inlet and then it is recombined at the exit. In a single tube configuration (or in two tubes joined in series), there is no splitting of flow inside the meter. Here a Coriolis mass flow meter with Twin U shaped configuration is considered.

Sensitivity analysis of CMF is going to be done with the help of this expression. The mass flow rate of the Coriolis mass flow meter as given in [1] is

$$\mathrm{M} = \frac{4\mathrm{EI}}{\pi l^3} \mathrm{x} \frac{1}{\mathrm{f}} \tan\varphi \quad (1)$$

$$\mathrm{M} = \frac{\mathrm{k}}{\mathrm{f}} \tan\varphi \quad (2)$$

Where, M is the mass flow rate (kg/sec), f is the frequency (Hz) and ϕ is the phase shift in degrees. The constant K in the above expression is the main parameter to be considered. K is a constant and it is provided by the manufacturers. It is calibrated using wet calibration depending on the materials used to design the Coriolis mass flow meters and particularly its flow tubes.

Analysis of K Factor

Mathematically the K factor is given as,

$$\mathrm{k} = \frac{4\mathrm{EI}}{\pi l^3} \quad (3)$$

Where, E is the Young's modulus (Pa), I is the Area moment of Inertia ($\mathrm{kg.m^2}$) and l= Length of the tube in meters.

From the above equation, it can be seen that the meter factor depends on the E and I. The young's moduli (E) is a temperature dependent factor and the area moment of inertia varies with radius of the flow tubes.

Effect of Temperature

Young's modulus which is defined as a ratio of stress to strain is a temperature dependent property since the stress and strain varies directly with temperature. When steel elements and structures are subjected to high temperature, they progressively lose their stiffness and carrying capacity as young's modulus and elasticity limit are decreasing. The strength and stiffness degrade with temperature and this deterioration has to be properly accounted. It is also influenced by the composition steel.

The effect of temperature over the young's modulus is given by the expression

$$E=206\{1+ (+ \alpha)*(T-20)\} \quad (4)$$

where, ε is the temperature coefficient of young's moduli, α is the coefficient of thermal expansion and T is the temperature(°C). Equation (4) depicts the relation between the temperature and the young's moduli.

Results and Discussions

The deviation of meter factor with respect to temperature changes are studied with the help of the simulation result. As explained earlier, any changes in temperature changes the young's modulus and thus varies the meter factor.

Eefect of T on K

For a set of values [2], considering k as constant, the simulation is done and the effect of temperature over k is also simulated. Equation (3) can be written as,

$$k = \frac{4E}{\pi l^3} x \frac{\pi}{4} (r_1^4 - r_2^4) \tag{5}$$

$$k = \frac{E}{l^3} (r_1^4 - r_2^4) \tag{6}$$

Now, $k_0$represents all the variables that are obtained at 20ºC. It is given in equation ( )

$$k_0 = \frac{E_0}{l_0^3} (r_{1o}^4 - r_{2o}^4) \tag{7}$$

Where,$E = E_0[1 + (T - 20]$, $r_{1=}r_{1o}[1 + \alpha(T - 20)]$ and $r_{2=}r_{2o}[1 + \alpha(T - 20)]$. Therefore k becomes,

$$k = k_0[1 + (+ \alpha)(T + 20)] \tag{8}$$

Fig (1) shows that the variation of k with increase in temperature from 0 to 200 ºC. It can be seen that the value changes from 0.009 to 0.014. Though the drift may appear to be a small value, its effect over the mass flow rate will be high when there is a situation when there should not be any sacrifice in the accuracy of the output.

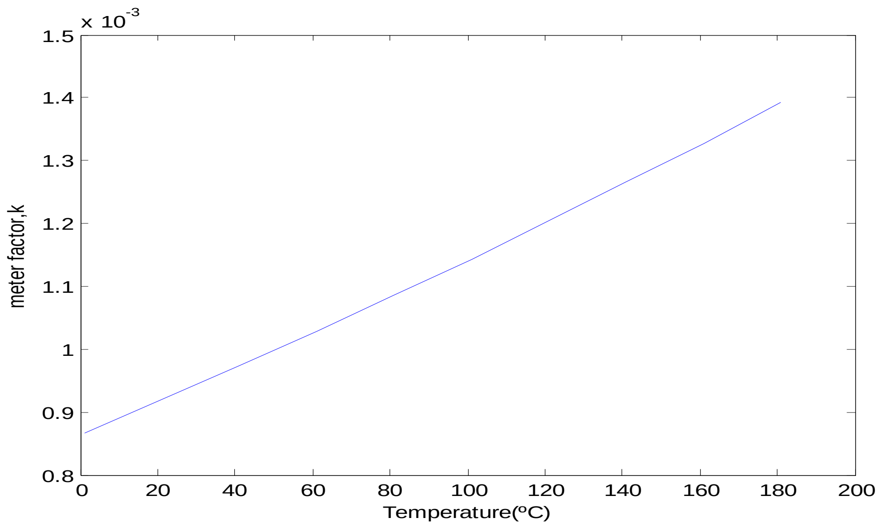

Figure 1: Temperature Vs Meter factor

Effect of T on Mass Flow Rate

It is evident that the meter factor gets varied with the change in temperature. As a result, the mass flow rate of the Coriolis mass flow meter also gets varied. Fig (2) depicts the changes in the mass flow rate with respect to the meter factor k.

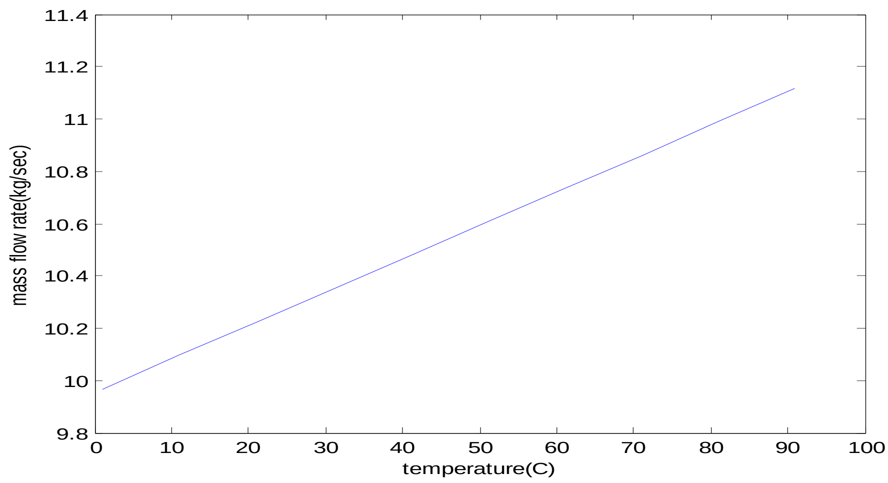

Figure 2: Temperature Vs Mass flow rate

For a particular mass flow rate of value 10.04 kg/sec, variation in temperature causes the flow rate to change from 10.04 to 11.2 kg/sec.

A set of values of mass flow rate obtained from [2], are used for analysis of the temperature effect over the mass flow rate. The simulation response of the mass flow rate depicts the values of mass flow rate tend to change drastically when there is a temperature hike in the process. Fig (3) illustrates the plot of original values and deviated values. The solid line represents the original set of mass flow rate and the starred line depicts the deviated value.

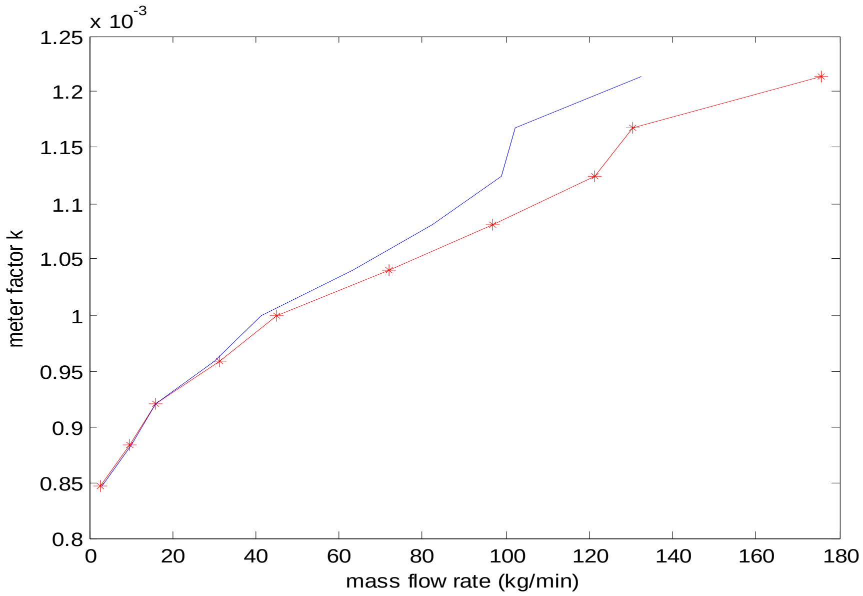

Figure 3: Effect of Meter Factor Changes on Mass Flow Rate

Corrective Measures Taken

From equation (8), it can be seen that there is a direct relation between the meter factor and the temperature. Elimination of the effect of temperature over the performance of the Coriolis mass flow meter can be obtained by varying the meter factor term i.e. a compensation factor has to be introduced into the meter factor term. Therefore, the compensation factor introduced is shown in equation (9).

$$k = k_0[1 + (+ \alpha)(T + 20)] - (T/x) \quad (9)$$

Where, x = (1000*e^6). Fig (4) shows the reduction in the deviation of the mass flow rate after the introduction of the compensation factor. Here the starred line represents the mass flow rate after the reduction in deviation. Fig (5) illustrates the three dimensional view of variation of the mass flow rate, temperature and its corresponding error percentage.

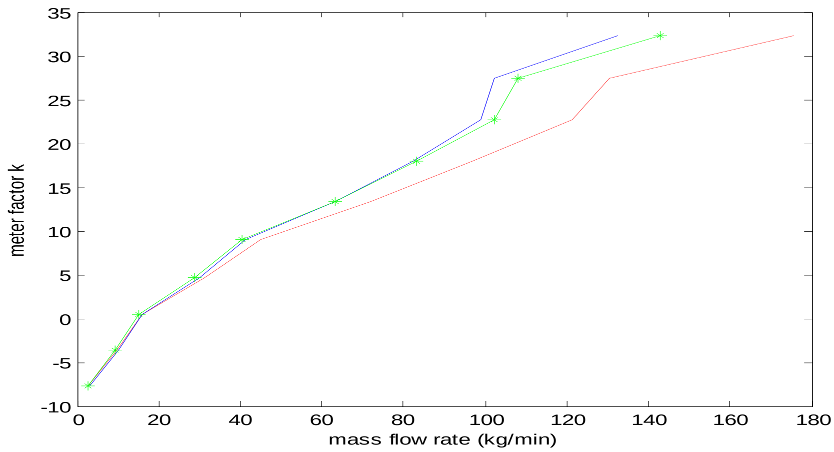

Figure 4: Corrected Mass flow rate

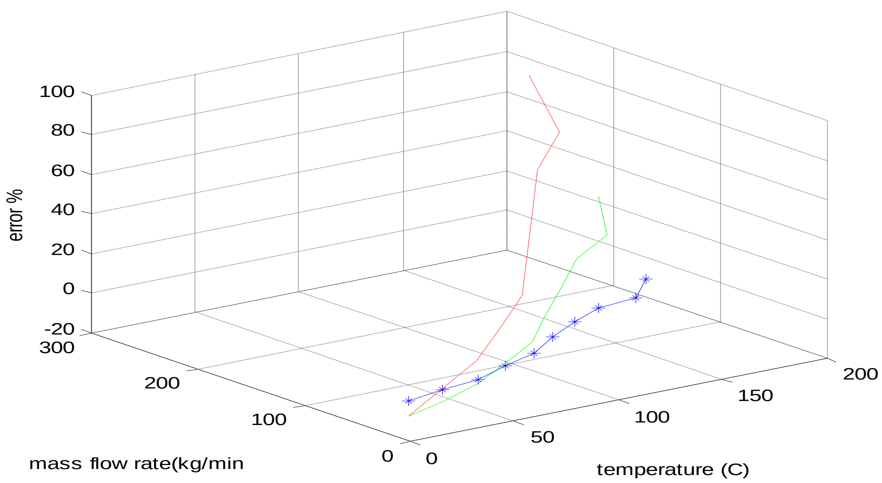

Figure 5: Error Vs mass flow rate Vs Temperature

Conclusion

Practically, no real time system is ideal. This is due to the effect of the external factors that are making the system deviate from its original value. Compensating the influence of those factors on the system can be achieved by modifying the mathematical model of the system. Similarly, in order to make the model of the Coriolis mass flow meter resistant to the temperature changes, compensation factor is introduced. The effect of temperature over the system is mitigated and the results are simulated.

References

[1] M. Kazahaya, 2011. A Mathematical Model and Error Analysis of Coriolis Mass Flowmeters. IEEE Trans on Instrumentation and Measurement., Vol.60 (4), pp. 1163–1174.

[2] Tu, Yaqing, et al, 2014. CMF Signal Processing Method Based on Feedback Corrected ANF and Hilbert Transformation. Measurement Science Review., 14.1 (2014): 41-47

[3] D. Wiklund and M. Peluso, 2002. Quantifying and specifying the dynamic response of flowmeters. ISA 2002 Technical Conference, Chicago, IL, USA.

[4] A. Mehendale, 2008. Coriolis mass flow rate meters for low flows. Ph.D. dissertation, University of Twente, Enschede.

[5] R. Cheesewright, C. Clark and D. Bisset, 1999. Understanding the experimental response of Coriolis massflow meters to flow pulsations. Flow Measurement and Instrumentation.,10, 207–215.

[6] G. Sultan and J. Hemp, 1989. Modelling of the Coriolis mass flowmeter. J.Sound Vib., 132 (3) 473–489.

[7] H. Raszillier and F. Durst, 1991. Coriolis effect in mass flow metering. Arch Appl. Mech., vol. 61, no. 3, pp. 192–214

[8] Bobovnik G, Kutin B and Bajsic I, 2004. The effect of flow conditions on the sensitivity of the Coriolis flowmeter. Flow Meas Instrum., 15:69–76.

[9] Cascetta F, 1996. Effect of fluid pressure on Coriolis mass flowmeter's performance. ISA Transactions., 35(4): 365-370.

[10] Paton R,1998. Calibration techniques for Coriolis mass flowmeters. Proceedings of the 9th International Conference on Flow Measurement (FLOMEKO '98), Lund, Sweden.

[11] J. Hemp, 2002. Calculation of the sensitivity of a straight tube Coriolis mass flowmeter with free ends. Flow Measurement and Instrumentation., 12 (2002) 411-420.

[12] H. Raszillier and V. Raszillier, 1991. Dimensional and symmetry analysis of Coriolis mass flowmeters. Flow Measurement and Instrumentation., 2, 180-184