Optimization of Fuzzy Partial Back Ordering Economic Order Quantity Model on The Boundaries of The Fill Rate

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Abstract

This article investigates an inventory model with partial backordering and correlated demand caused by cross-selling on the boundaries of the fill rate in a fuzzy situation by employing the type of fuzzy numbers which are triangular. The objective is to determine the optimal order lotsize to maximize the total profit. We first propose a model with a fuzzy positive integer (K) on the boundaries of the fill rate (F = 0 and F = 1). For each case, we employ the signed distance, a ranking method for fuzzy numbers, to find the estimate of total profit per unit time in the fuzzy sense and then derive the corresponding optimal order cycle. Numerical examples are provided to illustrate the results of proposed models.

Keywords: Inventory, Fuzzy Set, EOQ, Partial Backordering, Correlated Demand, Signed Distance, Cross-selling.

Introduction

In this paper the authors address a two-item inventory system where the demand of minor item is correlated to that of a major item because of cross-selling. It is extended to fit a more practical case where the order cycle of the major item is an integer multiple of that of the minor item. Montgomery et al (1973) presented the earliest study on EOQ with partial backordering. After that, Rosenberg (1979), Park (1982), Pentico and Drake (2009) proposed some similar EOQ models with partial backordering. Leung (2008) use a different method. ie. the complete square method to solve the model of Montgomery et al (1973).

Liu and Yuan (2000) proposed an inventory model of the coordinated replenishment with correlated demand, in which the correlation among orders of different items were considered by the can-order policy. This paper also considers the correlated demand that is caused by cross-selling. It implies the very frequent

phenomenon in retail shops or super markets that some items are always purchased together by customers due to their unknown interior associations (Anand et al., 1997 ; Kleinberg et al., 1998). In the proposed model, one of the two items is the major item that typically drives the profit, the demand of which is independent and can be partially backordered.

Hung-Chichang (2004) Chang et al (1998) presented a fuzzy model for inventory with backorder, where the backorder quantity was fuzzified as the triangular fuzzy number. Lee and Yao (1998) and Lin and Yao (2000) discussed the production inventory problems, where Lee and Yao (1998) fuzzified the demand quantity and production quantity per day, and Lin and Yao (2000) fuzzified the production quantity per cycle, treating all as the triangular fuzzy numbers. The model is extended to be more practical by considering that the order cycle of the minor item is an integer multiple of the order cycle of the major item.

In Section 2, some basic concepts of fuzzy sets, fuzzy numbers and signed distance method are introduced. In Section 3 fuzzy EOQ on the boundaries of the fill rate with F = 1 is discussed. Section 4 presents, fuzzy EOQ on the boundaries of the fill rate with F = 0 is given. Section 5 provides Numerical examples to illustrate the results of the proposed models. Finally the conclusions are given in Section 6.

Preliminaries

Before presenting the fuzzy inventory models, we introduce some definitions and properties about fuzzy numbers with relevant operations.

Definition 1

For $0 \leq \alpha \leq 1$, the fuzzy set $\tilde{a}_z$ defined on $R = (-\infty, \infty)$ is called an $\square$-level fuzzy point if the membership function of $\tilde{a}_z$ is given by

$$\mu_{\tilde{a}_z}(x) = \begin{cases} \alpha, & x = a, \\ 0, & x \neq a. \end{cases} \tag{3.1}$$

Definition 2

The fuzzy set $\tilde{A} = (a, b, c)$, where $a < b < c$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a)/(b - a), & a \leq x \leq b, \\ (c - x)/(c - b), & b \leq x \leq c, \\ 0, & \text{otherwise.} \end{cases} \qquad \ldots (3.2)$$

Remark 1

1. When $\square = 1$, the membership function of the 1-level fuzzy point $\tilde{a}_1$ becomes the characteristic function, ie., $\mu_{\tilde{a}_1}(x) = 1$ if $x = a$ and $\mu_{\tilde{a}_1}(x) = 0$ if $x \neq a$. In this case, the real number $a \in R$ is the same as the fuzzy point $\tilde{a}_1$ except for their representations.

2. If c = b = a, then the triangular fuzzy number $\tilde{A} = (a, b, c)$ is identical to the 1-level fuzzy point $\tilde{a}_1$.

Definition 3

For $0 \leq \alpha \leq 1$, the fuzzy set $[a_\square, b_\square]$ defined on R is called an $\square$-level fuzzy interval if the membership function of $[a_\square, b_\square]$ given by

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases} \qquad \ldots (3.3)$$

Definition 4

Let $\tilde{B}$ be a fuzzy set on R, and $0 \leq \alpha \leq 1$. The $\square$-cut B(a) of $\tilde{B}$ consists of points x such that $\mu_{\tilde{B}}(x) \geq \alpha$, that is, $B(\square) = \{ x | \mu_{\tilde{B}}(x) \geq \alpha \}$.

Decomposition Principle

Let $\tilde{B}$ be a fuzzy set on R and $0 \leq \alpha \leq 1$. Suppose the $\square$-cut of $\tilde{B}$ to be closed interval $[B_L(\square), B_U(\square)]$, that is, $B(\square) = [B_L(\square), B_U(\square)]$. Then, we have (see, [16])

$$\tilde{B} = \bigcup_{0 \leq \alpha \leq 1} \alpha B(\alpha) \qquad \ldots (3.4)$$

or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \leq \alpha \leq 1} \alpha C_{B(\alpha)}(x), \qquad \ldots (3.5)$$

where

1. $\square B(\square)$ is a fuzzy set with membership function
$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

2. $C_{B(\square)}(x)$ is a characteristic function $B(\square)$, that is
$$C_{B(\alpha)}(x) = \begin{cases} 1, & x \in B(\alpha) \\ 0, & x \notin B(\alpha) \end{cases}$$

Remark 2

From the Decomposition Principle and (5), we obtain

$$\tilde{B} = \bigcup_{0 \leq \alpha \leq 1} \alpha B(\alpha) = \bigcup_{0 \leq \alpha \leq 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \qquad \ldots (3.6)$$

or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \leq \alpha \leq 1} \alpha C_{B(\alpha)}(x) = \bigvee_{0 \leq \alpha \leq 1} \mu_{[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]}(x) \qquad \ldots (3.7)$$

For any a, b, c, d, k $\in$ R, a < b, and c < d, the interval operations are as follows [16] :

1. [a, b](+)[c, d] = [a + c, b + d]
2. [a, b](-)[c, d] = [a - d, b - c]   . . . (3.8)
3. $k(.)[a, b] = \begin{cases} ka, kb, & k > 0, \\ kb, ka, & k < 0, \end{cases}$
4. further, for a > 0 and c > 0,
5. [a, b](.)[c, d] = [ac, bd]
6. $[a, b](\div)[c, d] = \left[\dfrac{a}{d}, \dfrac{b}{c} \right]$

Next, as in Yao and Wu [15], we introduce the concept of the signed distance of fuzzy set. We first consider the signed distance on R.

Definition 5

For any a and $0 \in$ R, define the signed distance from a to 0 as $d_0(a, 0) = a$. If a > 0, the distance from a to 0 is $a = d_0(a, 0)$; if a < 0, the distance from a to 0 is $-a = -d_0(a, 0)$. Hence, $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R with which the $\square$-cut $B(\square) = [B_L(\square), B_U(\square)]$ exists for every $\square \in [0, 1]$, and both $B_L(\square)$ and $B_U(\square)$ are continuous functions on $\square \in [0, 1]$. Then, for any $\tilde{B} \in \Omega$, from (8) we have

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \qquad \qquad \qquad . . . (3.9)$$

From Definition 5, the signed distance of two end points, $B_L(\square)$ and $B_U(\square)$, of the $\square$-cut $B(\square) = [B_L(\square), B_U(\square)]$ of $\tilde{B}$ to the origin 0 is $d_0(B_L(\square), 0) = B_L(\square)$ and $d_0(B_U(\square), 0) = B_U(\square)$, respectively. The average, $(B_L(\square) + B_U(\square))/2$.

In addition, for every $\square \in [0, 1]$, there is a one-to-one mapping between the α-level fuzzy interval $[B_L(\square), B_U(\square)]$ and the real interval $[B_L(\square), B_U(\square)]$, that is, the following correspondence one-to-one mapping.

$$[B_L(\square)_1, B_U(\square)_1] \leftrightarrow [B_L(\square), B_U(\square)] \qquad \qquad . . . (3.10)$$

Also, the l-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0. Hence, the signed distance of $[B_L(\square)_1, B_U(\square)_1]$ to $\tilde{0}_1$ can be defined as $d([B_L(\square)_1, B_U(\square)_1], \tilde{0}_1) = d([B_L(\square), B_U(\square)]0) = ((B_L(\square) + B_U(\square))/2)$. Moreover, $\tilde{B} \in \Omega$, since the above function is continuous on $0 \le \square \le 1$, we can use the integration to obtain the mean value of the signed distance as follows :

$$\int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1) \, dx = \frac{1}{2} \int_0^1 (B_L(\alpha) + B_U(\alpha)) \, dx. \qquad . . . (3.11)$$

Then, from (11) and (13), we have the following definition.

Definition 6

For $\tilde{B} \in \Omega$, define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ (ie. y-axis) as

$$d(\tilde{B}, \tilde{0}_1) = \int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx = \frac{1}{2}\int_0^1 (B_L(\alpha) + B_U(\alpha))\, dx. \qquad \ldots (3.12)$$

According to Definition 6, we obtain the following property.

Property 1

For the triangular fuzzy number $\tilde{A} = (a, b, c)$, the α-cut of $\tilde{A}$ is $[A_L(\square), A_U(\square)], \alpha \in [0, 1]$, where $A_L(\square) = a + (b - a)\alpha$ and $A_U(\square) = c - (c - b)\alpha$. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is

$$d\ \tilde{A}, \tilde{0}_1\ = \frac{1}{4}(a + 2b + c) \qquad \ldots (3.13)$$

Furthermore, for two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$, where $\tilde{B} = \bigcup_{0 \le \alpha \le 1} B_L(\alpha)_\alpha, B_U(\alpha)_\alpha$

and $\tilde{G} = \bigcup_{0 \le \alpha \le 1} G_L(\alpha)_\alpha, G_U(\alpha)_\alpha$, and $k \in R$, using (10) and (12), we have

1. $\tilde{B}\ (+)\ \tilde{G} = \bigcup_{0 \le \alpha \le 1} B_L(\alpha)_\alpha + G_L(\alpha)_\alpha, B_U(\alpha)_\alpha + G_U(\alpha)_\alpha,$

2. $\tilde{B}\ (\text{-})\ \tilde{G} = \bigcup_{0 \le \alpha \le 1} B_L(\alpha)_\alpha - G_L(\alpha)_\alpha, B_U(\alpha)_\alpha - G_U(\alpha)_\alpha,$ $\qquad \ldots (3.14)$

3. $\tilde{k}_1\ (\square)\ \tilde{B} = \begin{cases} \bigcup_{0 \le \alpha \le 1} (kB_L(\alpha))_\alpha, (kB_U(\alpha))_\alpha , & k > 0, \\ \bigcup_{0 \le \alpha \le 1} (kB_U(\alpha))_\alpha, (kB_L(\alpha))_\alpha , & k > 0, \\ \tilde{0}_1, & k = 0. \end{cases}$

From the above and Definition 6, we obtain the following property.

Property 2

For two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$ and $k \in R$,

$$d(\tilde{B}, \tilde{0}_1) = \int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx = \frac{1}{2}\int_0^1 (B_L(\alpha) + B_U(\alpha))\, dx.$$

1. $d(\tilde{B}\ (+)\ \tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) + d(\tilde{G}, \tilde{0}_1)$
2. $d(\tilde{B}\ (\text{-})\ \tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) - d(\tilde{G}, \tilde{0}_1)$ $\qquad \ldots (3.15)$
3. $d(\tilde{k}_1(\square)\tilde{B}, \tilde{0}_1) = kd(\tilde{B}, \tilde{0}_1)$

EOQ ON THE BOUNDARIES OF THE FILL RATE WHEN F = 1

Supposing there is a single item called the major item, the notations in the inventory model are as follows.

Parameters and Variables of The Major Item

$D \rightarrow$ demand rate of the major item, the dimensions of which are units/unit time.

$A \rightarrow$ the ordering cost for placing and receiving an order, the dimensions of which are $ / order.

$C_o \rightarrow$ the opportunity cost of unit lost sale, including the lost profit and any goodwill loss, the dimensions of which are $ / unit.

$C_h \rightarrow$ the cost to hold one unit item for one unit time, the dimensions of which are $ / unit time / unit.

$C_b \rightarrow$ the cost to keep one unit backordered for one unit time, the dimensions of which are $ / unit time / unit.

$\square \rightarrow$ the backordering rate

$T \rightarrow$ the order cycle

$F \rightarrow$ the fill rate
(ie) the percentage of demand that is filled from the shelf stock.

$K \rightarrow$ positive integer.

The total cost per unit time as

$$H_K(K, T) = \frac{A}{T} + \frac{A_1}{KT} + \frac{(C_h D + KC_{h1}D_1)}{2} \cdot T \qquad \ldots (3.1)$$

Taking the first partial derivative of H(K, T) with respect to T and setting it equal to zero gives the optimal time length (Ren-quan Zhang, Kou Kaku, Yi-Yong Xiao, 2011) of the replenishment cycle under given K as

$$T_{HK}(K) = \sqrt{\frac{2\left(A + \frac{A_1}{K}\right)}{C_h D + KC_{h1}D_1}} \qquad \ldots (3.2)$$

We fuzzify K to be a triangular fuzzy number $\tilde{K} = (K - \Delta_1, K, K + \Delta_2)$ where $0 < \Delta_1 < K$ and $0 < \Delta_2 \le 1 - K$, Δ_1 and Δ_2 are determined by the decision makers. In this case, the total profit per unit time is a fuzzy value also, which is expressed as

$$\tilde{H}_K(K, T) = \frac{A}{T} + \frac{A_1}{\tilde{K}T} + \frac{C_h DT}{2} + \frac{\tilde{K}C_{h1}D_1}{2}T$$

Now we defuzzify $\tilde{H}_K(K, T)$ using the signed distance method. The signed distance of $\tilde{H}_K(K, T)$ to $\tilde{0}_1$ is given by

$$d\left(\tilde{H}_K(K, T), \tilde{0}_1\right) = \frac{A}{T} + \frac{A_1}{T} d\left(\frac{1}{\tilde{K}}, \tilde{0}_1\right) + \frac{C_h D}{2}T + \frac{C_{h1}D_1}{2}T \, d\left(\tilde{K}, \tilde{0}_1\right)$$

where $d\left(\tilde{K}, \tilde{0}_1\right)$ and $d\left(\frac{1}{\tilde{K}}, \tilde{0}_1\right)$ are measured as follows.

The signed distance of fuzzy number $\tilde{K}$ to $\tilde{0}_1$ is

$$d\left(\tilde{K}, \tilde{0}_1\right) = \frac{1}{4}\left[(K - \Delta_1) + 2K + (K + \Delta_2)\right] = K + \frac{1}{4}(\Delta_2 - \Delta_1)$$

Also, the left and right end points of the $\square$-cut ($0 \leq \alpha \leq 1$) of $\tilde{K}$ are

$K_L(\alpha) = (K - \Delta_1) + \Delta_1\alpha > 0$ and $K_U(\square) = (K + \Delta_2) - \Delta_2\alpha > 0$ respectively.

Since $0 < K_L(\square) < K_U(\square)$. The left and right end points of the $\square$-cut ($0 \leq \alpha \leq 1$) of $\frac{1}{\tilde{K}}$ are

$$\left(\frac{1}{K}\right)_L (\alpha) = \frac{1}{K_U(\alpha)} = \frac{1}{(K + \Delta_2) - \Delta_2\alpha} \quad \text{and}$$

$$\left(\frac{1}{K}\right)_U (\alpha) = \frac{1}{K_L(\alpha)} = \frac{1}{(K - \Delta_1) + \Delta_1\alpha} \quad \text{respectively.}$$

Then the signed distance of $\frac{1}{\tilde{K}}$ to $\tilde{0}_1$ is

$$d\left(\frac{1}{\tilde{K}}, \tilde{0}_1\right) = \frac{1}{2}\int_0^1 \left[\left(\frac{1}{K}\right)_L (\alpha) + \left(\frac{1}{K}\right)_U (\alpha)\right]$$

$$= \frac{1}{2}\left(\frac{1}{\Delta_1}\ln\frac{K}{K - \Delta_1} - \frac{1}{\Delta_2}\ln\frac{K}{K + \Delta_2}\right)$$

which is positive since $\Delta_1 > 0$, $\Delta_2 > 0$.

$$\ln\left(\frac{K}{K - \Delta_1}\right) > 0 \quad \text{and} \quad \ln\left(\frac{K}{K + \Delta_2}\right) < 0$$

we have

$$H_K^*(K, T) = d\left(H_K^*(K, T), \tilde{0}_1\right)$$

$$= \frac{A}{T} + \frac{A_1}{2T}\left(\frac{1}{\Delta_1}\ln\frac{K}{K - \Delta_1} - \frac{1}{\Delta_2}\ln\frac{K}{K + \Delta_2}\right)$$

$$+ \frac{C_h D}{2}T + \frac{C_{h1}D_1}{2}T\left(K + \frac{\Delta_2 - \Delta_1}{4}\right) \qquad \dots (3.3)$$

$H_K^*(K, T)$ is regarded as the estimate of the total profit per unit time in the fuzzy sense. The objective of the problem is to find the optimal order cycle T^*. Such that $H_K^*(K, T)$ has a maximum value. We take the fist partial derivative of $H_K^*(K, T)$ with respect to T and obtain $\dfrac{\partial H_K^*(K, T)}{\partial T} = 0$.

Because $\dfrac{\partial^2 H_K^*(K, T)}{\partial T^2} < 0.$

Therefore $H_K(K, T)$ is concave in T and hence solving for T we obtain the optimal order cycle.

$$T^* = \sqrt{\dfrac{2A + A_1\left(\dfrac{1}{\Delta_1}\ln\dfrac{K}{K - \Delta_1} - \dfrac{1}{\Delta_2}\ln\dfrac{K}{K + \Delta_2}\right)}{C_hD + C_{h1}D_1\left(K + \dfrac{\Delta_2 - \Delta_1}{4}\right)}} \qquad \ldots (3.4)$$

EOQ on The Boundaries of The Fill Rate When F = 0

The total cost function is

$$Q_K(K, T) = \dfrac{A}{T} + \dfrac{A_1}{KT} + (C_0 + \lambda C_{01})D(1 - \beta) + \dfrac{\beta C_b + [(d - \lambda)K + \lambda\beta(K-1)]Ch_1}{2}DT \ldots (4.1)$$

Taking the first partial derivative of $Q_K(K, T)$ with respect to T and setting it equal to zero gives the optimal time length of the replenishment cycle under given K as

$$Q_K(K, T) = \sqrt{\dfrac{2\left(A + \dfrac{A_1}{K}\right)}{DCh_1[(d - \lambda)K + \lambda\beta(K-1)] + \beta C_b D}} \qquad \ldots (4.2)$$

We fuzzify K to be a triangular fuzzy number $\tilde{K} = (K - \Delta_1, K, K + \Delta_2)$ where $0 < \Delta_1 < K$ and $0 < \Delta_2 \leq 1 - K$, Δ_1 and Δ_2 are determined by the decision makers. In this case, the total profit per unit time is a fuzzy value also which is expressed as

$$\tilde{Q}_K(K, T) = \dfrac{A}{T} + \dfrac{A_1}{KT} + (C_0 + \lambda C_{01})D(1 - \beta) + \dfrac{\beta C_b D}{2}T + \dfrac{K[(d - \lambda) + \lambda\beta]}{2}C_{h1}DT - \dfrac{\lambda\beta}{2}C_{h1}DT$$

Now we defuzzify $\tilde{Q}_K(K, T)$ using the signed distance method. The signed distance of $\tilde{Q}_K(K, T)$ to $\tilde{0}_1$ is given by

$$d\left(\tilde{Q}_K(K, T), \tilde{0}_1\right) = \dfrac{A}{T} + \dfrac{A_1}{KT}\, d\left(\dfrac{1}{\tilde{K}}, \tilde{0}_1\right) + (C_0 + \lambda C_{01})D(1 - \beta) + \dfrac{\beta C_b D}{2}T$$

$$+ \dfrac{[(d - \lambda) + \lambda\beta]}{2}C_{h1}DT \cdot d\left(\tilde{K}, \tilde{0}_1\right) - \dfrac{\lambda\beta}{2}C_{h1}DT$$

where $d\left(\tilde{K}, \tilde{0}_1\right)$ and $d\left(\dfrac{1}{\tilde{K}}, \tilde{0}_1\right)$ are measured as follows.

The signed distance of fuzzy number $\tilde{K}$ to $\tilde{0}_1$ is

$$d\left(\tilde{K}, \tilde{0}_1\right) = \frac{1}{4}\left[(K - \Delta_1) + 2K + (K + \Delta_2)\right]$$

$$= K + \frac{1}{4}(\Delta_2 - \Delta_1)$$

Also, the left and right end points of the $\Box$-cut $(0 \leq \alpha \leq 1)$ of $\tilde{K}$ are
$K_L(\alpha) = (K - \Delta_1) + \Delta_1\alpha > 0$ and
$K_U(\Box) = (K + \Delta_2) - \Delta_2\alpha > 0$ respectively.
Since $0 < K_L(\Box) < K_U(\Box)$. The left and right end points of the $\Box$-cut $(0 \leq \alpha \leq 1)$ of $\frac{1}{\tilde{K}}$ are

$$\left(\frac{1}{K}\right)_L(\alpha) \text{ and } \frac{1}{K_U(\alpha)} \text{ respectively}$$

$$d\left(\frac{1}{\tilde{K}}, \tilde{0}_1\right) = \frac{1}{2}\left(\frac{1}{\Delta_1}\ln\frac{K}{K - \Delta_1} - \frac{1}{\Delta_2}\ln\frac{K}{K + \Delta_2}\right)$$

$$\tilde{Q}_K^*\left(K, T\right) = d\left(Q_K^*(K, T), \tilde{0}_1\right)$$

$$= \frac{A}{T} + \frac{A_1}{T}\frac{1}{2}\left(\frac{1}{\Delta_1}\ln\frac{K}{K - \Delta_1} - \frac{1}{\Delta_2}\ln\frac{K}{K + \Delta_2}\right) + \frac{BC_bDT}{2}$$

$$+ \frac{[(d - \lambda) + \lambda\beta]}{2}C_{h_1}DT\left(K + \frac{\Delta_2 - \Delta_1}{4}\right) + (C_0 + \lambda C_{01})D(1 - \beta) + \frac{\lambda\beta C_{h_1}DT}{2} \quad \ldots (4.3)$$

We take the first order partial derivative of $\tilde{Q}_K^*\left(K, T\right)$ with respect to T and obtain

$$\frac{\partial Q_K^*(K, T)}{\partial T} = 0.$$

We obtain the optimal order cycle

$$T^* = \sqrt{\frac{2A + A_1\left(\frac{1}{\Delta_1}\ln\frac{K}{K - \Delta_1} - \frac{1}{\Delta_2}\ln\frac{K}{K + \Delta_2}\right)}{[(d - \lambda) + \lambda\beta]C_{h1}D\left(K + \frac{\Delta_2 - \Delta_1}{4}\right) + \beta C_bD - \lambda\beta C_{h1}D}} \quad \ldots (4.4)$$

Numerical Examples

To illustrate the results of the proposed models (3 and 4) we consider an inventory system with the data.

Demand rate $D = 2000$ units / year ; $D_1 = 3000$ units / year ;

Ordering cost $A = 500$ units / year   ; $A_1 = 400$ units / order ;

Opportunity cost $C_0 = 10$ / unit / year   ; $C_{01} = 4$ / unit / year
Holding cost $C_h = 20$ / unit / year   ; $C_{h1} = 5$ / unit / year
Keeping cost $C_b = 5$ / unit / year
Backordering rate $\square = 0.8$
    $\Delta_1 = 0.0005\, \Delta_2 = 0.02$
    when $F = 1$
    $T^* = 0.29$; $H_K(K, T) = 12056.76$
    when $F = 0$
    $T^* = 0.62$; $\tilde{Q}_K^*\left(K, T\right) = 12325.80$

Conclusion

This paper proposed two fuzzy models for an inventory problem with boundaries of the fill rate. In the first model, the fill rate is equal to one and positive integer is represented by a fuzzy number while the order cycle is treated as a fixed constant. In the second model, the fill rate is equal to zero. For each fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of total profit per unit time in the fuzzy sense, and then the corresponding optimal order lotsize is derived to maximize the total profit. Numerical examples are carried out to investigate the behavior of our proposed models and the results are compared with those obtained from the crisp model.

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