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Analysis Of The Transient Solution Of M/G/1/K Queueing System Subject To Catastrophe

M. Reni Sagayaraj[1], S. Anand Gnana Selvam[2], R. Reynald Susainathan[3]

[1, 2]Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur - 635601, Vellore District. Tamil Nadu, S. India.
[3]Reckitt Benckiser, India Limited (Gurgaon), New Delhi. India.
e-mail: reni. sagaya@gmail. com

ABSTRACT

we consider a single non-Markovian queueing system which plays a vital role in modeling real life phenomena. In this model the customer arrival pattern id Poisson process where as the service time follows general distribution. In addition to a Poisson stream of positive arrivals, we assume that there is also a Poisson stream of negative arrivals, which are called catastrophes. These catastrophes may occur at any time of instant and leads to annihilation of all customers in the system. Also catastrophes make system to be inactive. In this paper we have obtained the time dependent solution of our model.

Key words- M/G/I/k model, Catastrophes, Transient Solution, Steady State Analysis.

I. INTRODUCTION

The determination of transient solution is very much essential in analyzing the behavior of the system. There are methods that have been derived for obtaining transient solution of queues. Generating method by Bailey [2], combinatorial methods by Chambernowne [3], difference equation by Conolly [4] and continued fraction method which could be found in Jones and Thron[8]. One of the feature related with queueing model, which has been widely studied in the literature is queueing system or from another service failure, for example if a job is infected with a virus arrives, it transmit the virus to other processor and inactivate them in computer networks with a virus by queueing networks with catastrophes which has been studied by Chao[5]. Jain and Kumar[7] introduced the concept of restoration in catastrophic queues. Jain and Kumar[6] studied M/G/1 queue in presence of catastrophes. Krishnakumar [10] studied the transient solution M/M/1 queue with catastrophes, failures and repairs.

Thangaraj [18] studied the transient analysis of M/M/1 feedback queue with catastrophes using continued fraction methods.

The rest of the paper has been organized as follows: in section 2, the mathematical description of our model has been found, in section 3, the transient solution of the system has been derived, in section 4, the steady state analysis has been discussed.

II. MATHEMATICAL DESCRIPTION OF THE MODEL

The following assumptions were assumed:

1. Customers arrive at the system one by one in accordance with Poisson process with arrival rate λ(>0) and they are provided service one by one on a FCFS.
2. There is a single server which provides service following a general (arbitrary) distribution with distribution function B(v) and density function b(v). Let u(x) dx be the conditional probability density function of service completion during the interval (x, x+dx) given that the elapsed service time is x, so that

a. $\mu(x) = \frac{b(x)}{(1-B(x)}$

b. And therefore

c. $b(v) = \mu(v)e^{-\int_0^v \mu(x)dx}$

3. The catastrophes may occur at the service facility, when it is not empty according to Poisson stream with mean rate ξ. The catastrophes annihilate all the customers in the system instantaneously and after the system has been repaired, the system starts to work.
4. During the repair process no customer is allowed. The repair time follows exponential distribution with mean time is $1/\gamma$
5. The various stochastic processes involved are independent to each other.

III. TRANSIENT STATE SOLLUTION OF THE QUEUEING MODEL

i) P_n (x, t) = probability that at time t, there are n customers in the queue excluding one customer being served and the elapsed service time of this customer is x.

ii) P_{00} (t) = the probability that at time‘t’ there are no customer in the system without the occurrence of catastrophe.

iii) Q_{00} (t) = the probability that at time‘t’ there are no customer in the system with the occurrence of catastrophe.

The differential-difference equations governing the queueing model are

$$\frac{\partial}{\partial t} p_n(x,t) + \frac{\partial}{\partial x} p_n(x,t) + (\lambda + \mu(x) + \xi) p_n(x,t) = \lambda p_{n-1}(x,t) n \geq 1 \qquad (3)$$

$$\frac{\partial}{\partial t} p_0(x,t) + \frac{\partial}{\partial x} p_0(x,t) + (\lambda + \mu(x) + \xi) p_0(x,t) = 0 \quad (4)$$

$$\frac{d}{dt} p_{00}(t) = -\lambda p_{00}(t) + \int_0^\infty p_0(x,t)\mu(x)dx + \gamma Q_{00}(t) \quad (5)$$

$$\frac{d}{dt} Q_{00}(t) = -\gamma Q_{00}(t) - \xi P_{00}(t) + \xi \quad (6)$$

These equation are to be solved with the following boundary conditions

$$P_n(0,t) = \int_0^\infty P_{n+1}(x,t)\mu(x)dx \qquad n \ge 1 \quad (7)$$

$$P_0(0,t) = \int_0^\infty P_1(x,t)\mu(x)dx + \lambda P_{00}(t) \quad (8)$$

Define Laplace transfom

$$\overline{f}(s) = \int_0^\infty f(t)e^{-st}dt \quad (9)$$

Take a Laplace transforms for the equations (3)-(9)

$$\frac{\partial}{\partial x} \overline{p_n}(x,s) + (s + \lambda + \mu(x) + \xi)\overline{p_n}(x,s) = \lambda \overline{p_{n-1}}(x,s), n \ge 1 \quad (10)$$

$$\frac{\partial}{\partial x} \overline{p_0}(x,s) + (s + \lambda + \mu(x) + \xi)\overline{p_0}(x,s) = 0 \quad (11)$$

$$(s+\lambda)\overline{P}_{00}(s) = 1 + \int_0^\infty \overline{P}_0(x,s)\mu(x)dx + \gamma \overline{Q}_{00}(s) \quad (12)$$

$$(s+\lambda)\overline{Q}_{00}(s) = \frac{\xi}{s} - \xi \overline{P}_{00}(s) \quad (13)$$

$$\overline{P}_n(0,s) = \int_0^\infty \overline{P}_{n+1}(x,s)\mu(x)dx, n \ge 1 \quad (14)$$

$$\overline{P}_0(0,s) = \int_0^\infty \overline{P}_1(x,s)\mu(x)dx + \lambda \overline{P}_{00}(s) \quad (15)$$

Define probability generating function as follows

$$P_q(x,z,t) = \sum_{n=0}^\infty P(x,t)z^n$$

$$P_q(z,t) = \sum_{n=0}^\infty P(t)z^n \quad (16)$$

Which are convergent inside the circle given by $|z| \le 1$Multiply (10) by z^n and add with (11) we get

$$\frac{\partial}{\partial x} \overline{p_q}(x,z,s) + (s + \lambda + -\lambda z + \mu(x) + \xi)\overline{p_q}(x,z,s) = 0 \quad (17)$$

For the boundary conditions multiply (14) by z^{n+1} and multiply (15) by Z

Adding them and using equation (12) we get

$$z\overline{p_q}(0,z,s) = (1 - s\overline{P}_{00}(s)) + \lambda(z-1)\overline{P}_{00}(s) + \gamma\overline{Q}_{00}(s) + \int_0^\infty \overline{p_q}(x,z,s)\mu(x)dx \quad (18)$$

Integrating (17)from 0 to x yields

$$\overline{p_q}(x,z,s) = \overline{p_q}(0,z,s)e^{-(s+\lambda-\lambda z+\xi)x-\int_0^x \mu(t)dt} \quad (19)$$

Where $\overline{p_q}(0,z,s)$ is given by (18)

Again integrating equation (19) by parts with respect to x

$$\overline{p_q}(z,s) = \overline{p_q}(0,z,s)\left[\frac{1-\overline{B}(s+\lambda-\lambda z+\xi)}{(s+\lambda-\lambda z+\xi)}\right] \quad (20)$$

Where $\overline{B}(s+\lambda-\lambda z+\xi)$ is given by

$\int_0^\infty e^{-(s+\lambda-\lambda z+\xi)x}dB(x)$ is the Laplace –Stieltjes transform of the service timeB(x).

Now multiplying both sides of equation (19) by $\mu(x)$ and integrating over x we get

$$\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx = \overline{P}_q(0,z,s)\overline{B}(s+\lambda-\lambda z+\xi) \quad (21)$$

Using (21) in (18)

$$\overline{P}_q(0,z,s) = \frac{(1-sP_{00}(s)) + \lambda(z-1)P_{00}(s) + \gamma Q_{00}(s)}{z - \overline{B}(s+\lambda(1-z)+\xi)} \quad (22)$$

Using (22)in (20)

$$\overline{P}_q(0,z,s) = \frac{(1-sP_{00}(s)) + \lambda(z-1)P_{00}(s) + \gamma Q_{00}(s)}{z - \overline{B}(s+\lambda(1-z)+\xi)}\left[\frac{1-\overline{B}(s+\lambda-\lambda z+\xi)}{(s+\lambda-\lambda z+\xi)}\right] \quad (23)$$

IV. Steady state Solution of the model

In this section we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities we suppress the argument "t" wherever it appears in the time dependent solution analysis by using the well known "Tauberian " Property

$$Lt_{s\to 0} s\overline{f}(s) = Lt_{t\to\infty} f(t) \quad (24)$$

Mulptiplying both sides of the equation (23) by S takingthe limit s to 0 and applying the propery from equation(24) we get,

$$P_q(z) = \left[\frac{\lambda(z-1)P_{00} + \gamma Q_{00}}{z - \overline{B}(\lambda(1-z)+\xi)}\right]\left[\frac{1-\overline{B}(\lambda-\lambda z+\xi)}{(\lambda-\lambda z+\xi)}\right] \quad (25)$$

Let $W_q(z)$ be denote the probability generating function of the queue size irrespective of the state of the system.

$$W_q(z) = P_q(z) + P_{00} + Q_{00} \tag{26}$$

$$W_q(z) = \left[\frac{\lambda(z-1)P_{00} + \gamma Q_{00}}{z - \overline{B}(\lambda(1-z)+\xi)}\right]\left[\frac{1-\overline{B}(\lambda - \lambda z + \xi)}{(\lambda - \lambda z + \xi)}\right] + P_{00} + Q_{00} \tag{27}$$

$$Q_{00} = \frac{\xi(1-P_{00})}{\gamma} \tag{28}$$

Now P_{00} can be obtained by using the normalizing condition fro

$$\left|W(z)\right|_{z=1} = 1$$

$$P_{00} = \frac{\left[\lambda\overline{B}(\xi)+1\right] + \lambda\xi\overline{B}(\xi)\left[\dfrac{\lambda\overline{B}(\xi)+1}{\lambda(1-\overline{B}(\xi) - \xi\overline{B}(\xi) - \xi}\right] - \dfrac{\xi}{\gamma}}{\left[1 - \dfrac{\xi}{\gamma}\right]} \tag{29}$$

Thus the steady state solution of our model coincides with the steady state solution of the model discussed by Rakesh kumar [17].

CONCLUSION

We have proposed a single server subject to catastrophes which leads to server failure. Transient solution for single server queue with impatience and catastrophes is derived. This model can applied in computer and communication networks. We have the probability generating function of transient solutions explicitly and along with this the steady state has also been analyzed. In some real life public service systems when potential customers leave the system one by one when the system is down. For handling such problems transient solutions portray the system better than a steady state solution.

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