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Topological Properties of Hexagonal Cage Networks

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Abstract

Meshes and tori are widely used topologies for Network on chip (*NoC*). In this paper a new planar architecture called Hexagonal cage network *HXCa*(*n*) with two layers derived using two hexagonal meshes of same dimension. And a Hamiltonian cycle is shown in *HXCa*(4). In the last section a new operation called "*Boundary vertex connection*"(*BVC*) is introduced and conjectured that *BVC* of a 2-connected plane graph is Hamiltonian. For an ordered set $M=\{m_1,m_2,m_3,\ldots,m_p\}$ of vertices in a connected graph G and a vertex u of G, the code of u with respect to M is the *p-dimensional distance vector* $C_M(u) = (d(v, m_1), d(v, m_2), d(v, m_3),\ldots,d(v, m_p))$. The set M is called the resolving set for G if $d(x, m) \neq d(y, m)$ for x, y in $V \setminus M$ and m in M. A resolving set of minimum cardinality is called a minimum resolving set or a metric basis for G. The cardinality of the metric basis is called the metric dimension of G and is denoted by $dim(G)$. In this paper the metric dimension problem is investigated for *HXCa*(*n*) Finding a metric basis and Hamiltonian cycle in a arbitrary graph is *NP* hard problem.

Index Terms- Hexagonal cage networks, Hamiltonian cycle, Interconnection networks, Metric basis, Metric dimension.

2010 Mathematics subject Classification: 05C12

I INTRODUCTION

Interconnection networks are becoming increasingly pervasive in many different applications, with the operational costs and topological properties of these networks depending considerably on the application. Some applications of Interconnection networks are studied in depth for decades. In a computer network, computing nodes connect to each other through an interconnection network. The nodes perform the computation by passing messages to other nodes through the network using a standard message passing mechanism such as the message passing interface[1]. Therefore the topology of the network which identifies how the nodes are connected to each other is of very much important in terms of the network performance. Meshes, Honeycomb, Spidergon, Spin, Ring, Octagon, Fat tree, Butter fat tree, Hexagonal networks and Tori are well known topologies often used in Network on Chip *NoC*. Three dimensional *IC* is an important focus of researchers. The 3*D*-*NoC* is a cutting edge technology by integrating *NoC* system in 3*D* fashion containing multiple layers of a active devices. 3*D* *NoC* have the potential to enhance system performance. In 3*D*-*NoC*, package density can be increased significantly, power can be reduced via shorter communication links and immune to noise[2].

1.3 NETWORK ON CHIP

The three dimensional Honeycomb topology was studied by Alexander et al[3]. They have illustrated that the honeycomb topology is an advantageous design alternative in terms of network cost. Network cost is one of the most important parameter that reflects both network performance and implementation cost. They have explored the *NoC* related topological properties of both honeycomb mesh and torus topologies in to rectangular brick shapes. They have demonstrated that honeycomb topologies are feasible to be implemented with rectangular devices. They proposed a 3*D*-Honeycomb topology for new 3*D*-*IC*. The well controlled and regular network structure allows the *NoC* to reduce the design complexity for communication scheme while enhance the system predictability and reliability[3].They have extended the honeycomb topology in to 3*D* paradigm which is non planar topology. The topological properties of Honeycomb cage network is studied in[4]. In this paper we have derived a new planar graph called Hexagonal Cage network $HXCa(n)$ which can be used in *NoC* after studying its advantages and properties. One of the main plus point for $HXCa(n)$ is that it can be used in 2*D* as well 3*D* environment.

1.1 BRIEF SURVEY ON METRIC DIMENSION

The metric dimension problem was first investigated by Harary and Melter[5]. They gave a characterization for the metric dimension of trees. The metric dimension problem for grid graphs was studied by Melter and Tomescu[6].Result of Melter and Tomescu have generalized by Khuller et al. They have proved that the metric dimension of d dimensional grid graph is d[7]. The metric dimension problem is *NP* complete for any arbitrary graph[8].This problem is also *NP* complete for bipartite graph[9]. Slater[10] and Later[11] called metric basis as reference set. Slater called the number of elements in a reference set of the graph as location number of the graph. He described the application of metric basis in sonar and loran stations. Chartand et al have used the term minimum resolving set instead of metric basis[12]. The metric dimension problem has been studied for trees and grid graphs[7], Petersen graphs[13], Honeycomb networks, Hexagonal networks[14], Circulent and Harary graphs[15], Enhanced Hyper cubes[16], Silicate stars[17], Triangular oxide networks[18] and Star of David networks[19]. The application of metric dimension to problems of Robot navigation, pattern recognition[7], Network discovery and verification[20], Joins in graphs[21], and coin weighing problems[22].

1.2 OVERVIEW

The contribution of this paper is as follows. In the second section, a drawing algorithm is given, and topological properties of Hexagonal Cage network are compared with other well known interconnection networks. In the third section, the metric dimension of n dimensional Hexagonal Cage network with two layers is found to be 4. In the section 4 a new operation called "Boundary vertex connection" (BVC) is introduced for the at least 2connected plane graph and a conjecture is given.

II CAGE NETWORKS

2.1 DRAWING ALGORITHM FOR HEXAGONAL CAGE NETWORKS $HXCa(n)$

Step-1: Consider two hexagonal networks $HX_1(n)$ and $HX_2(n)$ of dimension n.

Step-2: Connect every boundary vertices of $HX_1(n)$ to its mirror image vertices in $HX_2(n)$ by an edge. Resulting graph is called Hexagonal cage network with two layers.

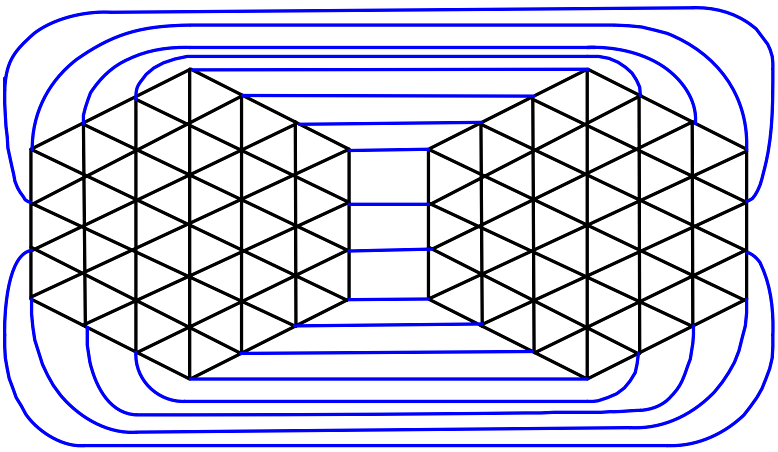

Figure: 1: Planner embedding of *HXCa*(4)

In Table-2, Few results on Mesh, Honeycomb networks, Hexagonal networks and their tori, are referred from[23] to compare with results of Cage networks.

TABLE 1 comparison of Hexagonal and Hexagonal cage networks

Network	Vertices	Edges	Faces
$HX(n)$	$3n^2-3n+1$	$9n^2-15n+6$	$6n^2-12n+7$
$HXCa(n)$	$6n^2-6n+2$	$18n^2-24n+6$	$12n^2-18n+6$

TABLE 2 Comparison of network cost based on number of nodes

Network	Degree	Diameter	Communication Cost	Bisection width
Honeycomb networks	3	$1.63\sqrt{N}$	$4.89\sqrt{N}$	$0.81649\sqrt{N}$
Honeycomb Cage[4]	3	$1.1547\sqrt{N}$	$3.46\sqrt{N}$	$1.1547\sqrt{N}$
Mesh connected computers	4	$2\sqrt{N}$	$8\sqrt{N}$	$\sqrt{N}$
Torus	4	$\sqrt{N}$	$4\sqrt{N}$	$2\sqrt{N}$
Hexagonal torus	6	$0.58\sqrt{N}$	$3.46\sqrt{N}$	$4.61\sqrt{N}$
3D-Hexagonal cage	6	$0.8164\sqrt{N}$	$4.89\sqrt{N}$	$2.449\sqrt{N}$
Hexagonal mesh	6	$1.16\sqrt{N}$	$6.93\sqrt{N}$	$2.31\sqrt{N}$

III THE METRIC DIMENSION OF HEXAGONAL CAGE NETWORKS WITH 2 LAYERS

The metric dimension problem of Hexagonal network and Honeycomb network was studied by Paul et al[14]. They proved that metric dimension of Honeycomb and Hexagonal networks is 3. Finding the metric dimension of Hexagonal topology of higher dimension is an open problem given in[14].

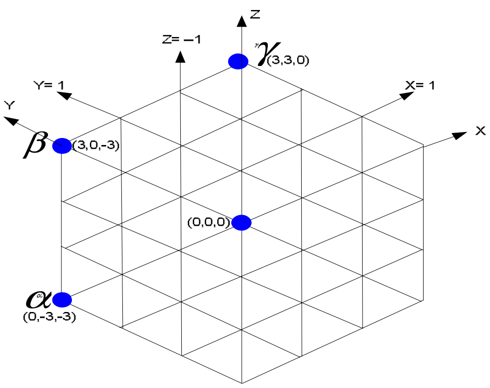

Figure 2: Coordinate of vertices in *HX*(4)

Here $\{\alpha,\beta,\gamma\}$ is a metric basis for $HX(4)$ as in[14] and $\{\alpha,\beta,\gamma,\gamma'\}$ is a metric basis for Hexagonal Cage network of dimension 4. If $v=(x,y,z,0)$ is a vertex in $HX_1(n)$ then its image in $HX_2(n)$ is $v'=(x',y',z',1)=(x,y,z,1)$. We solve the metric dimension problem for Hexagonal Cage network and prove that the metric dimension of Hexagonal cage networks is 4. First we estimate the lower bound for the metric dimension of Hexagonal Cage networks $HXCa(n)$. To prove the result, we use the following result of Khuller et al[7].

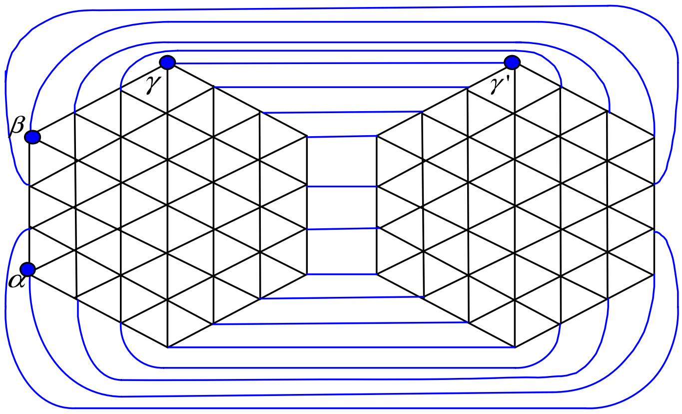

Figure 3: ***HXCa*** **(4) with a metric basis** $\{\alpha, \beta, \gamma, \gamma'\}$

Theorem 1:[7] Let $G = (V, E)$ be a graph with metric dimension 2 and let $\{a, b\} \subset V$ be a metric basis in G. The following are true: There is a unique shortest path P between a and b.

1. The degree of a and b are at most 3.
2. Every other node on P has degree at most 5.

Lemma 1: Let G be Hexagonal Cage network of dimension n. Then $dim(G) > 2$.

Proof: Let a and b be any two vertices of G. The set $\{a, b\}$ cannot be a metric basis for G, as the degree of a and b are at least 4. Hence the metric dimension of G is strictly greater than two□.

Lemma 2: Let G be Hexagonal Cage networks $HXCa(n)$, $n > 2$. Then $dim\ (G) > 3$.

To prove lemma 2, we need the concept of neighborhood of a vertex of *HX(n)*[14]. Let V be the vertex set of Hexagonal Cage network $HXCa(n)$.

An r – Neighborhood of v is defined by $N_r(v) = \{u \in V : d(u,v) = r\}$. Stojmenovic[23] proposed a coordinate system for honeycomb network. This was adapted by Nocetti et al.[24] to assign coordinates to the vertices in the hexagonal networks. In this scheme, three axes X, Y, and Z parallel to 3 edge directions and at mutual angel 120 degree between any two of them are introduced. Paul et al called lines parallel to the coordinate axes as X- lines, Y- lines, and Z- lines. Here $X = h$ and $X = -k$ are two X- lines on either side of the X axis. Any vertex of the first layer $HX_1(n)$ is assigned coordinates $(x, y, z, 0)$ in the above scheme. We denote P_X, a segment of X line consisting of points $(x, y, z, 0)$ with the fixed x coordinate. That is, $P_X = \{(x_0, y, z, 0) \ / \ y_1 \le y \le y_2, z_1 \le z \le z_2\}$. Similarly $P_Y = \{(x, y_0, z, 0) \ / \ x_1 \le x \le x_2, z_1 \le z \le z_2\}$ and $P_Z = \{(x, y, z_0, 0) \ / \ x_1 \le x \le x_2, y_1 \le y \le y_2\}$. Also, we denote P'_X, a segment of X ’ line in second layer of Hexagonal Cage network $HX_2(n)$ consisting of points

$(x', y', z', 1) = (x, y, z, 1)$ with fixed x coordinate that is $P'_X = \{(x_0, y, z, 1) \,/\, y_1 \le y \le y_2, z_1 \le z \le z_2\}$, $P'_Y = \{ (x, y_0, z, 1) \,/\, x_1 \le x \le x_2, z_1 \le z \le z_2\}$ and $P'_Z = \{ (x, y, z_0, 1) \,/\, x_1 \le x \le x_2, y_1 \le y \le y_2\}$ [14].

Theorem 2:[14]In any $HX(n)$, we have $N_r(\alpha) = P_Y o\, P_Z$, $N_r(\beta) = P_X o\, P_Z$, $N_r(\gamma) = P_X o\, P_Y$.
The following theorem 3 is straight forward from theorem 2.

Theorem 3: In any second layer $HX_2(n)$, $N_r(\alpha') = P'_Y o\, P'_Z$, $N_r(\beta') = P'_X o\, P'_Z$ and $N_r(\gamma') = P'_X o\, P'_Y$ □

A metric basis for $HX(n)$ $\{\alpha, \beta, \gamma\}$ [14]. Let us fix the landmarks α, β and γ in $HX_1(n)$ as in figure 2 which was studied in[14]. Also let us fix the fourth landmark γ' which is the image of γ. Let us prove that $\{\alpha, \beta, \gamma, \gamma'\}$ is a metric basis for Hexagonal Cage network.

Lemma 3: In Hexagonal Cage network, the neighbor hood of α, β and γ is defined as $N_r(\alpha) = P_Y o\, P_Z \cup N_{r-1}(\alpha')$, $N_r(\beta) = P_X o\, P_Z \cup N_{r-1}(\beta')$, respectively.

Proof: From the structure of Hexagonal Cage network of first layer and second layer that $N_r(\alpha)$ is composed of a Y- line segment followed by a Z-line segment. More precisely, for $1 \le r \le n$-1, we have

$$P_Y = \{ (r-i,\ -(n-1)+r,\ -(n-1)+i,\ 0), 0 \le i \le r\} \text{ and}$$

$$P_Z = \{ (\ -j,\ -(n-1)+r-j,\ -(n-1)+r,\ 0), 1 \le j \le r\}$$

For $n \le r \le 2n$-2,

$$P_Y = \{ ((n-1)-i,\ -(n-1-r),\ -(2n-2-r)+i, 0), 0 \le i \le n-1\}$$

$$P_Z = \{ (-j,\ -(n-1-r)-j,\ -(n-1-r), 0), 1 \le j \le n-1\}.$$

$N_r(\alpha)$ in Hexagonal Cage network first layer is $P_Y o\, P_Z$ [14] and also α has neighbors at the distance r in the second layer. And it is denoted by $N_{r-1}(\alpha')$.
Therefore for any r, $N_r(\alpha) = P_Y o\, P_Z \cup N_{r-1}(\alpha')$. Similarly $N_r(\beta) = P_X o\, P_Z \cup N_{r-1}(\beta')$ and $N_r(\gamma) = P_X o\, P_Y \cup N_{r-1}(\gamma')$ can be proved.□

Lemma 4: For any r_1 and r_2, $N_{r_1}(\alpha) \cap N_{r_2}(\beta) \cap N_{r_3}(\gamma)$ is either empty or singleton or two segments of Z line.

Proof: By lemma 3, $N_r(\alpha) = P_Y o\, P_Z \cup N_{r-1}(\alpha')$, $N_r(\beta) = P_X o\, P_Z \cup N_{r-1}(\beta')$ and

$N_r(\gamma) = P_X o P_Y \cup N_{r-1}(\gamma)$.Thus, for any r_1 and r_2, $N_{r1}(\alpha) \cap N_{r2}(\beta) \cap N_{r3}(\gamma)$ is either empty or singleton or one or two segments of *Z* line.□

In particular, for $r = n+1$, the intersection set $I = N_r(\alpha) \cap N_r(\beta) \cap N_r(\gamma)$ = $\{P(-n+1, -n+1, 0,0), Q(-n+2, -n+2, 0, 1)\}$ and the vertices *P* and *Q* are at distance $(n+1)$ from α, β and γ respectively. Other possibilities are ruled out by the symmetrical nature of Hexagonal networks. Hence the metric dimension of Hexagonal Cage network is strictly greater than 3 hence the lemma 2.□

Now we shall prove $\{\alpha, \beta, \gamma, \gamma'\}$ is a metric basis for *HXCa*(*n*).

Lemma 5: Let *u* is a vertex in $HX_1(n)$ with unique distance vector (d_1, d_2, d_3), where d_1, d_2, d_3 are the distances from *u* to *α*, *β*, and *γ* respectively, then its image *u*' has unique distance vector (d_1+1, d_2+1, d_3+1).

Proof: A shortest path from *u*' to *α* contains a shortest path from *u*' to *α*' and an edge *αα*'. That is $d(u', \alpha) = d(u', \alpha')+1 = d_1+1$.Similarly $d(u', \beta) = d(u', \beta')+1 = d_2+1$ and $d(u', \gamma) = d(u', \gamma')+1 = d_3+1$, Hence *u'* has unique vector of distance (d_1+1, d_2+1, d_3+1).□

Theorem 4: The metric dimension of Hexagonal Cage network *HXCa*(*n*) is 4.

Proof: Let *u* and *v* be any two distinct vertices in *HXCa*(*n*)**.**

Case 1: If *u* and *v* are in $HX_1(n)$ then $\{\alpha, \beta, \gamma\}$ resolves [14].

Case 2: If *u* and *v* are in $HX_2(n)$ then $\{\alpha, \beta, \gamma\}$ resolves, because by the above lemma 5, its vectors of distances are also unique.

Case 3: If $u(x_1, x_2, x_3, 0)$ is in $HX_1(n)$, $v(x_4, x_5, x_6, 1)$ is in $HX_2(n)$ and if $\{\alpha, \beta, \gamma\}$ does not resolve *u* and *v*, then γ' must resolve, that is $d(u, \gamma') \neq d(v, \gamma')$.

Proof for Case 3: Suppose that

$$d(u, \alpha) = d(v, \alpha) \quad (1)$$

$$d(u, \beta) = d(v, \beta) \quad (2)$$

$$d(u, \gamma) = d(v, \gamma) \quad (3)$$

Suppose that γ' does not resolve *u* and *v* then

$$d(u, \gamma') = d(v, \gamma') \quad (4)$$

Lemma 6: I = $N_{r1}(\gamma) \cap N_{r2}(\gamma') = \emptyset$ if $r_1 = r_2$

Proof for lemma 6: By the definition of $N_{r_1}(\gamma)$ and $N_{r_2}(\gamma')$ for $r_1 = r_2$, lemma 6 is true. □

Lemma 7: I = $N_{r1}(\gamma) \cap N_{r2}(\gamma') = P_X\, o\, P_Y$ if $r_1 < r_2$
And I = $N_{r1}(\gamma) \cap N_{r2}(\gamma') = P'_X\, o\, P'_Y$ if $r_1 > r_2$

Proof for lemma 7: We have

I $= N_{r_1}(\gamma) \cap N_{r_2}(\gamma') = N_{r_1}(\gamma) - N_{r_1-1}(\gamma')$ *if* $r_1 < r_2$

$= \{P_X\, o\, P_Y \cup N_{r_1-1}(\gamma')\} - N_{r_1-1}(\gamma')$

$= P_X\, o\, P_Y$ whenever $r_1 < r_2$

And I $= N_{r_1}(\gamma) \cap N_{r_2}(\gamma') = N_{r_2}(\gamma') - N_{r_2-1}(\gamma)$ if $r_2 < r_1$

$= \{P'_X\, o\, P'_Y \cup N_{r_2-1}(\gamma)\} - N_{r_2-1}(\gamma)$

$= P'_X\, o\, P'_Y$ whenever $r_1 > r_2$ □

Equation (3) implies that *u* and *v* are in $N_{r_1}(\gamma)$ and equation (4) implies that *u* and *v* are in $N_{r_2}(\gamma')$. This implies that *u* and *v* are in I $= N_{r_1}(\gamma) \cap N_{r_2}(\gamma')$. If $r_1 = r_2$ then lemma 6 implies that I = $\emptyset$ a null set which is a contradiction because the set I cannot be empty as it contains *u* and *v*. If $r_1 < r_2$ then lemma7 implies that I = $P_X\, o\, P_Y$, which means that *u* and *v* are in $HX_1(n)$, a contradiction. If $r_1 > r_2$ then lemma7 implies that I = $P'_X\, o\, P'_Y$, which means that *u* and *v* are in $HX_2(n)$, which is a contradiction ,because we have assumed that *u* is in $HX_1(n)$ and *v* is in $HX_2(n)$. Therefore equation (3) and (4) cannot be true simultaneously. If γ does not resolves *u* and *v*, then γ' must resolve *u* and *v*. Therefore $\{\alpha, \beta, \gamma, \gamma'\}$ resolves Hexagonal Cage Network. Hence the metric dimension of Hexagonal Cage network is 4. □

IV BOUNDARY VERTEX CONNECTION (*BVC*) OF A PLANE GRAPH

Consider a plane graph $G(V, E)$ and its mirror image say G'. Let $\{v_1, v_2, v_3, \ldots, v_k\}$ and $\{u_1, u_2, u_3, \ldots, u_k\}$ are the set of vertices on boundary in *G* and *G*' respectively. Let us connect the pair of vertices $(v_1, u_1), (u_2, v_2), (u_3, v_3), \ldots \text{and} (u_k, v_k)$ by an edge without any edge crossing. The resulting graph is called *BVC* of the graph *G*.

Examples :

1. *BVC* of $G = HX(4)$ is Hexagonal Cage network $HXCa(4)$.See figure 3.

2. *BVC* of $G = Q_1$ is Q_2 (Two dimensional Hypercube)

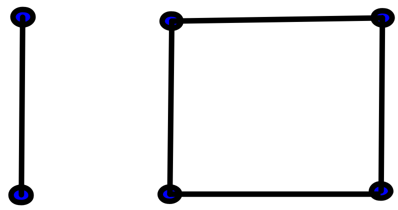

Figure 4: Q_1 and Q_2

3. *BVC* of $G = Q_2$ is Q_3 (Three dimensional Hypercube)

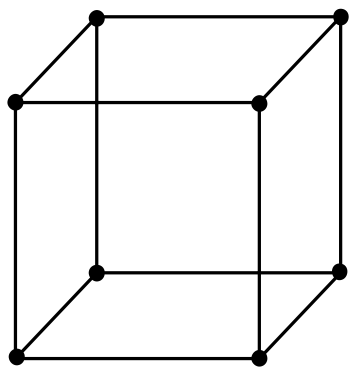

Figure 5: Q_3

4. *BVC* of a star graph with 4 vertices contains only Hamiltonian path and other three examples given above are containing Hamiltonian cycles.

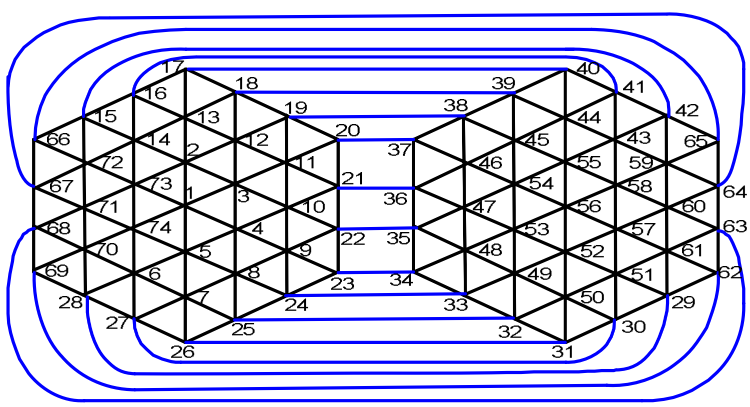

Figure 6: Hamiltonian cycle in *HXCa*(4)

Trace a path from vertex along 1, 2, 3,….,73, and 74, which is Hamiltonian path and there is an edge connecting the vertices 74 and 1 we get a Hamiltonian cycle. In the same way we can find a Hamiltonian cycle in Honeycomb Cage networks[4].

Conjecture: Boundary vertex connection of at least two connected plane graph is Hamiltonian.

CONCLUSION

In this paper we have introduced Hexagonal Cage network with two layers and solved the metric dimension problem. And *Boundary Vertex Connection* of a plane graph is introduced and one Hamiltonian cycle is shown in *HXCa*(4), Also we have given a conjecture. The metric dimension problem for Hexagonal cage network more than 2 layers is under investigation.

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