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Brinkman Flow Past An Impervious Spheroid Under Stokesian Assumption

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ABSTRACT

In this paper, we study the Brinkman flow, under Stokesian assumption, past an impervious prolate spheroid and obtain the expressions for the velocity and pressure fields in terms of Legendre functions, Associated Legendre functions, prolate radial and angular spheroidal wave functions. We further obtain an expression for the drag experienced by the spheroid and numerically study its variation with respect to the flow parameters and display the results through graphs.

Key words: Prolate spheoid, porous medium, Stokesian assumption, Brinkman model, velocity, pressure, drag

INTRODUCTION

The study of the isothermal flow of an incompressible viscous fluid about an impermeable body immersed there in has been a matter of concern for researches ever since Stokes initiated the discussion for the steady motion of a sphere in an incompressible viscous fluid [1]. The determination of the flow variables requires the solution of the Navier-Stokes equations and the equation of continuity subject to the appropriate boundary conditions and the regularity conditions in the flow regime. However, this job is not easy as the Navier-Stokes equations are nonlinear. In view of this, researches tried to study the related problems by linearizing the equations of motion under some simplifying assumptions. For some more details the reader can look into [2]. When the body dimension / characteristic flow velocity is suitably small or kinematic viscosity is suitably large, there will be negligible effect on the non linear terms in the Navier-Stokes equations near the body in comparison with the linear terms. In view of this, in such a case, the non-linear terms in the equations may be neglected and this results in a linearized version of the Navier-Stokes equations. Sir George Stokes seems to be the first to have proposed this argument [1]. With this argument he treated the motion of a sphere in a viscous liquid by omitting the non linear terms in the Navier-Stokes equations ever since, any problem of fluid flow studied under this assumption with the omission of nonlinear terms in comparison with the viscous terms is referred to a Stokes flow problem.

Payne and Pell in their highly significant paper [2] discussed the Stokes flow in a viscous liquid past a class of axially symmetric bodies with a uniform streaming at infinity parallel to the axis of symmetry. Several researches have worked the problem of Stokes flow past axisymmetric bodies taking the fluid to be a viscous fluid or a Non-Newtonian fluid. The problems, in general, are confined to an infinite regime of the fluid where in a body like sphere or spheroid or approximate sphere or a pair of spheres is immersed. To the extent the authors have surveyed, there seems to be no work related to a solid body immersed in a porous medium where there is flow past the body with uniform streaming at infinity.

In this paper we propose to study the flow of a viscous liquid past an impervious prolate spheroid immersed in a porous medium. One famous model describing such a flow situation is the so called Brinkmann model. For further details the reader is referred to the classic book **"**Convection in porous media**"** by Nield and Bejan [3]. We assume that a prolate spheroid is immersed in a porous medium where the viscous flow field is governed by Brinkmann equation and obtain the expression for the velocity and pressure fields and the drag experienced by the spheroid. The flow field variables are obtained in terms of Legendre functions, Associated Legendre functions, prolate radial and angular spheroidal wave functions. We have studied the variation in drag numerically and presented the variation through graphs.

MATHEMATICAL FORMULATION:

Consider an infinite expanse of a porous medium around an impervious prolate spheroid. Let there be a uniform viscous fluid flow in the porous region past the spheroid with velocity U far away from the body, along of the axis of symmetry of the spheroid. We assume that the flow is governed by Brinkman equations. Let us choose the centre of the spheroid as the origin of an orthogonal Prolate spheroidal coordinate system (ξ,η,ϕ) with and $(\bar{e}_\xi,\bar{e}_\eta,\bar{e}_\phi)$ as the unit base vectors. Let (h_1,h_2,h_3) be the scale factors of the system. Let the spheroid be given by $\xi=\xi_0$. In view of the axial symmetry of the flow field, the flow variables will all be independent of the azimuthal coordinate φ.

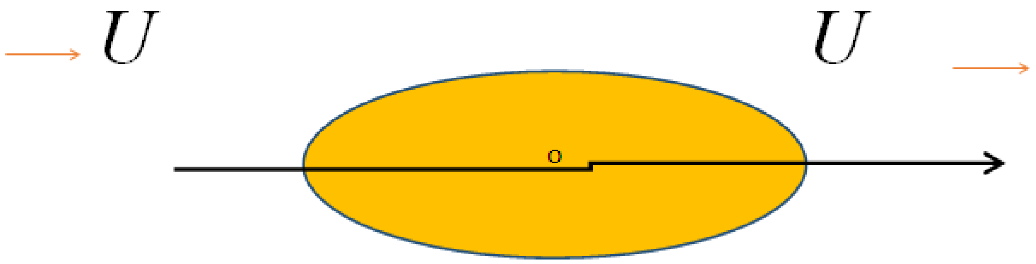

Fig-1. Schematic diagram of the flow

Let the velocity at any point of the flow be given by

$$\overline{q} = u(\xi,\ \eta)\overline{e_\xi} + v(\xi,\ \eta)\overline{e_\eta} \quad . \tag{1}$$

Ignoring the body force and the dispersion effects, the flow is assumed to be governed by the Brinkman equations:

$$\nabla p + \mu\, curl\, curl\, \overline{q} + \frac{\mu}{k}\overline{q} = 0 \tag{2}$$

and the usual continuity equation

$$\nabla \bullet \overline{q} = 0 \tag{3}$$

where μ is the coefficient of viscosity, k is the permeability of the porous medium, and $p(\xi,\eta)$ is the fluid pressure.

Equation (3) allows us to introduce the Stokes stream function $\psi(\xi,\eta)$ through

$$h_2 h_3 u = -\frac{\partial \psi}{\partial \eta},\ h_1 h_3 v = \frac{\partial \psi}{\partial \xi} \tag{4}$$

Taking curl of (2) and using (4) we get

$$-\frac{\mu}{h_3}E^4\psi + \frac{\mu}{k}\frac{1}{h_3}E^2\psi = 0 \tag{5}$$

where E^2 is the Stokes stream function operator given by

$$E^2 = \frac{h_3}{h_1 h_2}\left(\frac{\partial}{\partial\xi}\left(\frac{h_2}{h_1 h_3}\frac{\partial}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\frac{h_1}{h_2 h_3}\frac{\partial}{\partial\eta}\right)\right) \tag{6}$$

The equation (5) leads to

$$E^4\psi - \frac{1}{k}E^2\psi = 0 \tag{7}$$

which can be written as

$$E^2\left(E^2 - \frac{\alpha^2}{c^2}\right)\psi = 0 \tag{8}$$

with

$$\frac{\alpha^2}{c^2} = \frac{1}{k} \tag{9}$$

In view of the linearity and commutative nature of the operators E^2 and $\left(E^2 - \frac{\alpha^2}{c^2}\right)$ solution of (8) can be obtained by superposing the solutions of

$$E^2\psi = 0 \tag{10}$$

$$\left(E^2 - \frac{\alpha^2}{c^2}\right)\psi = 0 \tag{11}$$

Introduce $s = \cosh\xi$ and $t = \cos\eta$. (12)

We note that

$$h_1 = h_2 = c\sqrt{(s^2 - \mathrm{t}^2)},\ \ h_3 = c\sqrt{(s^2-1)(1-\mathrm{t}^2)}$$

and

$$E^2 = \frac{1}{c^2(s^2-t^2)}\left((s^2-1)\frac{\partial^2}{\partial s^2} + (1-t^2)\frac{\partial^2}{\partial t^2}\right) \tag{13}$$

Solution of $E^2\psi = 0$

General solution of $E^2\psi = 0$ is given by

$$\psi_1 = Uc^2(s^2-1)(1-t^2)\left(\frac{-1}{2} + \sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s) P'_{n+1}(t)\right) \tag{14}$$

in view of the uniform flow far away from the body and regularity of solution on the axis of symmetry. Here $P'_n(t)$ and $Q'_n(s)$ are Legendre functions of first and second kind respectively.

Solution of (11) $\left(E^2 - \frac{\alpha^2}{c^2}\right)\psi_2 = 0$

The expression of ψ_2, suitable for our problem is given by

$$\psi_2 = Uc\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} c_n R^{(3)}_{1n}(\mathrm{i}\alpha, s) S^{(1)}_{1n}(\mathrm{i}\alpha, t) \tag{15}$$

Here $S^1{}_{1n}(\mathrm{i}\alpha, t)$ and $R^{(3)}{}_{1n}(\mathrm{i}\alpha, s)$ are Prolate angular and radial spheroidal wave functions as in Lakshmana Rao and Iyengar [4] and Abramowitz and Stegun [5].

The solution ψ of (8) is given by

$$\psi = \psi_1 + \psi_2 \tag{16}$$

where ψ_1, ψ_2 are given in (14) and (15)

Thus stream function $\psi(s,t)$ is given by

$$\psi = -\frac{1}{2}Uc^2(\mathrm{s}^2-1)(1-t^2) + Uc^2(\mathrm{s}^2-1)(1-t^2)\sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(\mathrm{s}) P'_{n+1}(t) +$$

$$Uc^2\sqrt{(\mathrm{s}^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R^{(3)}_{1n}(\mathrm{i}\alpha, \tau) S^{(1)}_{1n}(\mathrm{i}\alpha, t) \tag{17}$$

Here

$$R_{1n}^{(3)}(\mathrm{i}\alpha,s)=\left(\mathrm{i}^{n+2}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)\mathrm{d}_r^{1n}(i\alpha)\right)^{-1}\left(\frac{2}{\pi\alpha}\frac{s^2-1}{r^3}\right)^{\frac{1}{2}}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)\mathrm{d}_r^{1n}(i\alpha)K_{r+\frac{3}{2}}(\alpha s) \tag{18}$$

$$S_{1n}^{(1)}(\mathrm{i}\alpha,\mathrm{t})=\sum_{r=0,1}^{\infty}{}'\mathrm{d}_r^{1n}(i\alpha)\,\mathrm{P}_{r+1}^{(1)}(t) \tag{19}$$

as in [3,4]. In equation (17) $\{A_n\}$ and $\{C_n\}$ are infinite sequences of arbitrary constants to be determined using the boundary conditions.

Boundary conditions on s=s$_0$:

The boundary of the spheroid is taken to be given by s = s_0. Employing the no slip condition on the impervious boundary $s = s_0$ we get

$\psi = 0$, $\psi_s = 0$ on $s = s_0$

Let us non dimensionalize the variables ψ and p using

$$\psi=\left(Uc^2\right)\psi^* \quad \text{and} \quad p=\frac{\mu U}{c}p^* \tag{20}$$

and later drop '$*$', we get

$$\psi=\frac{-1}{2}(s^2-1)(1-t^2)+(s^2-1)(1-t^2)\sum_{n=0}^{\infty}A_{n+1}Q_{n+1}'(s)P_{n+1}'(t)$$

$$+\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(\mathrm{i}\alpha,s)S_{1n}^{(1)}(\mathrm{i}\alpha,\mathrm{t}) \tag{21}$$

in non dimensional form As on s = s_0 , $\psi = 0$, we get

$$(s_0{}^2-1)(1-t^2)\sum_{n=0}^{\infty}A_{n+1}Q_{n+1}'(s_0)P_{n+1}'(t)+\sqrt{(s_0{}^2-1)(1-t^2)}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(\mathrm{i}\alpha,s)S_{1n}^{(1)}(\mathrm{i}\alpha,\mathrm{t})=\frac{1}{2}(s_0^2-1)(1-t^2) \tag{22}$$

Dividing by $\sqrt{(s_0{}^2-1)(1-t^2)}$, we get

$$\sqrt{({s_0}^2-1)}\sum_{n=0}^{\infty}A_{n+1}Q'_{n+1}(s_o)P_{n+1}^{(1)}(t)+\sum_{n=1}^{\infty}C_n R_{1n}^{(3)}(\mathrm{i}\alpha,s_0)S_{1n}^{(1)}(\mathrm{i}\alpha,\mathrm{t})=\frac{1}{2}\sqrt{({s_0}^2-1)(1-t^2)} \tag{23}$$

Further $\psi_s=0$ on $s=s_0$. This gives after dividing by $\sqrt{(1-t^2)}$

$$0=-\mathrm{s}_0\,P_1^{(1)}(t)+\sum_{n=0}^{\infty}A_{n+1}\,(n+1)(n+2)Q_{n+1}(s_0)\,P_{n+1}^{(1)}(t)+$$

$$\sum_{n=1}^{\infty}c_n\left((s^2-1)\frac{d}{ds}R_{1n}^{(3)}(\mathrm{i}\alpha,s)+sR_{1n}^{(3)}(\mathrm{i}\alpha,s)\right)_{s=s_0}S_{1n}^{(1)}(\mathrm{i}\alpha,\mathrm{t})\,/\,\sqrt{(s_o^2-1)} \tag{24}$$

(23) And (24) respectively are the consequences of $\psi=0,\psi_s=0\ on\ s=s_0$
We further note that

$$\int_{-1}^{1}P_{n+1}^{(1)}(t)P_{m+1}^{(1)}(t)\,\mathrm{dt}\quad=0\ \ if\ m\neq n$$

$$=\frac{2}{2n+3}(n+1)(n+2)\ \text{ if } m=n \tag{25}$$

$$\int_{-1}^{1}P_{n+1}^{(1)}(t)S_{1m}^{(1)}(\mathrm{i}\alpha,\mathrm{t})\,\mathrm{dt}=d_n^{1m}(i\alpha)\frac{2(n+1)(n+2)}{2n+3} \tag{26}$$

Multiplying (24) by $p_{n+1}^{(1)}(t)$ and integrating with respect to t, from -1 to 1, after simplification we get

$$A_{n+1}\sqrt{{s_0}^2-1}\,Q'_{n+1}(s_o)+\sum_{m=1}^{\infty}C_m\,R_{1m}^{(3)}(\mathrm{i}\alpha,s_0)d_n^{1m}(i\alpha)=\frac{1}{2}\sqrt{({s_0}^2-1)}\,\delta_{n0} \tag{27}$$

Multiplying (25) with $P_{n+1}^{(1)}(t)$ and integrating with respect to t, from -1 to 1, after simplification, we get

$$A_{n+1}(n+1)(n+2)Q_{n+1}(s_0)+\sum_{m=1}^{\infty}C_m\frac{1}{\sqrt{({s_0}^2-1)}}\left((s^2-1)\frac{d}{ds}R_{1m}^{(3)}(\mathrm{i}\alpha,s)+sR_{1m}^{(3)}(\mathrm{i}\alpha,s)\right)_{s=s_0}d_n^{1m}(i\alpha)=s_0\ \delta_{n0} \tag{28}$$

Determination of arbitrary constants:-

The equations (27) and (28) can be rewritten as

$$A_{n+1}\sqrt{(s_0{}^2-1)}\,Q'_{n+1}(s_o)+\sum_{m=1}^{\infty}C_m R^{(3)}_{1m}(\mathrm{i}\alpha,s_0)d_n^{1m}(i\alpha)=\frac{1}{2}\sqrt{s_0{}^2-1}\,\delta_{n0} \tag{29}$$

On $\psi_s=0\, on\, s=s_0$

$$A_{n+1}(n+1)(n+2)Q_{n+1}(s_0)\sqrt{s_0{}^2-1}+\sum_{m=1}^{\infty}C_m\left((s^2-1)\frac{d}{ds}R^{(3)}_{1m}(\mathrm{i}\alpha,s)+sR^{(3)}_{1m}(\mathrm{i}\alpha,s)\right)_{s=s_0}d_n^{1m}(i\alpha)==s_0\sqrt{s_0{}^2-1}\,\delta_{n0} \tag{30}$$

Multiplying (29) with (n+1) (n+2) $Q_{n+1}(s_0)$, multiplying (30) with $Q'_{n+1}(s_0)$, and subtracting the second result from the first we get

$$\sum_{m=1}^{\infty}C_m\left((n+1)(n+2)Q_{n+1}(s_0)R^{(3)}{}_{1m}(i\alpha,s_0)-Q'_{n+1}(s_o)\left((s^2-1)\frac{d}{ds}R^{(3)}_{1m}(\mathrm{i}\alpha,s)+sR^{(3)}_{1m}(\mathrm{i}\alpha,s)\right)_{s=s_0}\right)d_n^{1m}(i\alpha)$$

$$=\left(\sqrt{(s_0{}^2-1)}\,Q_1(s_0)-s_0\sqrt{(s_0{}^2-1)}Q'_1(s_0)\right)\delta_{n0}$$

$$=\frac{1}{\sqrt{(s_0{}^2-1)}}\delta_{n0} \tag{31}$$

The system of equations determining { C_m } is now given by

$$\sum_{n=0}^{\infty'}\Delta_{nm}C_m=\delta_n \tag{32}$$

where

$$\Delta_{nm}=d_n^{1m}(\mathrm{i}\alpha)\left((n+1)(n+2)Q_{n+1}(s_0)R^{(3)}_{1m}(\mathrm{i}\alpha,s_0)-Q'_{n+1}(s_o)\left((s^2-1)\frac{d}{ds}R^{(3)}_{1m}(\mathrm{i}\alpha,s)+sR^{(3)}_{1m}(\mathrm{i}\alpha,s)\right)_{s=s_0}\right) \tag{33}$$

$$\delta_n=\frac{1}{\sqrt{(s_0{}^2-1)}}\delta_{n0} \tag{34}$$

where δ_{n0} is Kronecker delta i.e. $\delta_{n0}=1 if\ n=0$

$\delta_{n0} = 0 \, if \; n \neq 0$

This is an infinite nonhomogeneous system of linear equations in the unknowns $\{C_n\}$. It is known from the theory of spheroidal wave functions that the coefficients d_n^{1m} have to be defined as zero when n+m+1 is a positive odd integer.

In the system (32), we see that the right hand side vector { δ_n } has only one nonvanishing component δ_n corresponding to n=0. Hence the subsystem of (32) given by

$$\sum_{m=1}^{\infty} \Delta_{2n+1,2m} C_{2m} = 0 \tag{35}$$

is a homogeneous system and this leads to C_m=0 for even values of m. From (29) we conclude that A_n's are all zero for even values of n. Thus for the determination of C_n's where n is odd, we get a non homogeneous linear subsystem of (32). The analytical determination of C_n's is not possible and we have to necessarily resort to numerical evaluation after suitably truncating the system for specific values of the material parameters. Herein we have truncated the system to a 5by 5 system in view of the negligible nature of the coefficients C_n.

Pressure distribution:

The equation (2) is seen to represent the following equations:

$$\frac{\sqrt{(s^2-1)}}{c\sqrt{s^2-t^2}} \frac{\partial p}{\partial s} = \mu \frac{-1}{c^2\sqrt{s^2-t^2}\sqrt{(s^2-1)}} \frac{\partial}{\partial t}(E^2\psi) - \frac{\mu}{k} \frac{-1}{c^2\sqrt{s^2-t^2}\sqrt{(s^2-1)}} \frac{\partial \psi}{\partial t} \tag{36}$$

$$\frac{\sqrt{(1-t^2)}}{c\sqrt{s^2-t^2}} \frac{\partial p}{\partial t} = \mu \frac{-1}{c^2\sqrt{s^2-t^2}\sqrt{(s^2-1)}} \frac{\partial}{\partial s}(E^2\psi) - \frac{\mu}{k} \frac{1}{c^2\sqrt{s^2-t^2}\sqrt{(1-t^2)}} \frac{\partial \psi}{\partial s} \tag{37}$$

These are equivalent to

$$\sqrt{(s^2-1)} \frac{\partial p}{\partial s} = \frac{\mu}{c\sqrt{(s^2-1)}} \frac{\partial}{\partial t}(E^2\psi) - \frac{\mu}{k} \frac{1}{c\sqrt{s^2-1}} \frac{\partial \psi}{\partial t} \tag{38}$$

$$\sqrt{(1-t^2)} \frac{\partial p}{\partial t} = \frac{-\mu}{c\sqrt{(1-t^2)}} \frac{\partial}{\partial s}(E^2\psi) - \frac{\mu}{k} \frac{1}{c\sqrt{(1-t^2)}} \frac{\partial \psi}{\partial s} \tag{39}$$

Using the expression of ψ from (21) and integrating (38) and (39) we get

$$p=\frac{\mu\alpha^2 U}{c}\left(-2\mathrm{st}+\sum_{n=1}^{\infty}A_{n+1}(n+1)(n+2)Q_{n+1}(s)P_{n+1}(t)\right)+$$
an arbitrary constant (40)

where the arbitrary constant can be absorbed into the hydrostatic pressure.

Drag on the spheroid

Drag on the spheroid is given by

$$2\pi c^2\sqrt{(s_0{}^2-1)}\left(\int_{-1}^{1}\left(t\sqrt{(s^2-1)}\,t_{\xi\xi}-s\sqrt{(1-t^2)}\,t_{\xi\eta}\right)dt\right)_{S=S_0} \quad (41)$$

Here the required components of stress $t_{\xi\xi}$, $t_{\xi\eta}$ are given by

$$t_{\xi\xi}=-p+2\mu e_{\xi\xi}\;,\; t_{\xi\eta}=2\mu e_{\xi\xi} \quad (42)$$

where

$$e_{\xi\xi}=\frac{1}{c^3(s^2-1)}\left(\psi_{st}+\frac{t\psi_s}{(s^2-\mathrm{t}^2)}-\frac{s(2s^2-1-\mathrm{t}^2)\psi_t}{(s^2-\mathrm{t}^2)(s^2-1)}\right)$$

Using the conditions on the boundary $s=s_0,\psi=0,\psi_s=0,\psi_t=0,\psi_{st}=0,\psi_{tt}=0.$ we have

$$(e_{\xi\xi})_{on\,s=s_0}=0,\ (\mathrm{t}_{\xi\xi})_{on\,s=s_0}=-p(s_0,t)$$

$$e_{\xi\eta}=\frac{\left((s_0{}^2-1)\psi_{ss}-(1-\mathrm{t}^2)\psi_{tt}\right)}{h_1h_2h_3}-\frac{s\sqrt{(s_0{}^2-1)}\,\psi_s}{c^3(s^2-\mathrm{t}^2)^2\sqrt{(1-\mathrm{t}^2)}}-\frac{t\sqrt{(1-\mathrm{t}^2)}\,\psi_t}{c^3(s^2-\mathrm{t}^2)^2\sqrt{(\mathrm{s}^2-1)}} \quad (43)$$

$$t_{\xi\eta\ on\,s=s_0}=\frac{2\mu(s^2-1)}{h_1h_2h_3}\left(\psi_{ss}\right)_{on\,s=s_0}$$

To calculate $\psi_{ss}\ on\,s=s_0$, we note that

$$E^2\psi=\frac{1}{\mathrm{c}^2(s^2-t^2)}\left((s_0{}^2-1)\frac{\partial^2\psi}{\partial s^2}-(1-\mathrm{t}^2)\frac{\partial^2\psi}{\partial t^2}\right)$$

$$= \frac{\alpha^2}{c^2} Uc\sqrt{(s_0{}^2-1)(1-t^2)} \sum_{n=1}^{\infty} C_m R_{1m}^{(3)}(\mathrm{i}\alpha, s_0) S_{1m}^{(1)}(\mathrm{i}\alpha, s_0) \tag{44}$$

On $s = s_0$

$$\frac{1}{\mathrm{c}^2(s_0{}^2-t^2)}(s_0{}^2-1)\left(\psi_{ss}\right)_{s=s_0} = \frac{\alpha^2}{\mathrm{c}^2} Uc\sqrt{(s_0{}^2-1)(1-t^2)} \sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0) S_{1m}^{(1)}(\mathrm{i}\alpha, \mathrm{t})$$

Hence

$$\left(\psi_{ss}\right)_{s=s_0} = \frac{Uc\alpha^2}{\sqrt{(s_0{}^2-1)}}(s_0{}^2-t^2)\sqrt{(1-t^2)} \sum_{n=1}^{\infty} C_m R_{1m}^{(3)}(i\alpha, s_0) S_{1m}^{(1)}(i\alpha, \mathrm{t}) \tag{45}$$

and

$$\left(t_{\xi\eta}\right)_{s=s_0} = \frac{(s_0{}^2-1)U\,\mathrm{c}\,\alpha^2(s_0{}^2-t^2)\sqrt{(1-t^2)}}{\mathrm{c}^3(s_0{}^2-t^2)\sqrt{(s_0{}^2-1)(1-t^2)}} \sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0) S_{1n}^{(1)}(i\alpha, \mathrm{t}) \tag{46}$$

Drag on the spheroid =

$$2\pi c^2\sqrt{(s_0{}^2-1)} \int_{-1}^{1} \left(-t\sqrt{(s_0{}^2-1)}\, P(s_0,t) - \frac{s_0\sqrt{(1-t^2)}\,Uc\alpha^2\sqrt{(s_0{}^2-1)}}{c^3} \sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0) S_{1n}^{(1)}(i\alpha, \mathrm{t}) \right) dt$$

$$= 2\pi c^2(s_0{}^2-1) \int_{-1}^{1} \left(-t\, p(s_0,t) - \frac{s_0 U\alpha^2}{c^2} \sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0) S_{1n}^{(1)}(i\alpha, \mathrm{t})\sqrt{(1-t^2)} \right) dt \tag{47}$$

Consider

$$\int_{-1}^{1} t\, P(s_0,t)dt = \frac{\mu U\alpha^2}{c} \left(-2s_0 \int_{-1}^{1} t^2\, dt + \sum_{n=1}^{\infty} A_{n+1}(n+1)(n+2) Q_{n+1}(s_0) \int_{-1}^{1} t\, P_{n+1}(t) dt \right)$$

$$= \frac{\mu U\alpha^2}{c} \left(-\frac{4}{3} s_0 + \frac{4}{3} A_1 . Q_1(s_0) \right) \tag{48}$$

Consider

$$\int_{-1}^{1}\sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0)\sqrt{(1-t^2)}\,\mathrm{S}^{(1)}{}_{1n}(i\alpha,\mathrm{t})dt$$

Let us simplify

$$\int_{-1}^{1}\sqrt{(1-t^2)}\,\mathrm{S}^{(1)}{}_{1m}(i\alpha,\mathrm{t})dt = \int_{-1}^{1}\sum_{n=0,1}^{\infty}{}' d_r^{\,1m}\, p_1^{(1)}(t)\, p_{n+1}^{(1)}(t)dt \;= d_0^{\,1m}\,\frac{4}{3} \tag{49}$$

This leads to

$$\int_{-1}^{1}\sum_{n=1}^{\infty} C_m R_{1n}^{(3)}(i\alpha, s_0)\sqrt{(1-t^2)}\,S_{1n}^{(1)}(i\alpha,\mathrm{t})dt = \sum_{n=1}^{\infty} C_m d_0^{\,1m}\,\frac{4}{3} R_{1n}^{(3)}(i\alpha, s_0) \tag{50}$$

Now drag is given by

$$Drag = 2\pi c^2\sqrt{(s_0{}^2-1)}\left(\int_{-1}^{1}\sqrt{(s_0{}^2-1)}\left(2s_0t^2-\sum_{n=1}^{\infty}A_{n+1}(n+1)(n+2)Q_{n+1}(s_0)\mathrm{t}\mathrm{P}_{n+1}(t)\right)dt-2s\sum_{n=1}^{\infty}C_mR^{(3)}{}_{1m}(i\alpha,s_0)\int_{-1}^{1}\sqrt{(1-\mathrm{t}^2)}\,\mathrm{S}^{(1)}{}_{1m}(i\alpha,\mathrm{t})\mathrm{dt}\right) \tag{51}$$

$$\text{Drag=}\; 2\pi c^2\sqrt{(s_0{}^2-1)}\left(2s_0\frac{2}{3}\sqrt{(s_0{}^2-1)}-A_1.2.Q_1(s_0)\sqrt{(s_0{}^2-1)}\frac{2}{3}-2s\sum_{n=1}^{\infty}C_mR_{1n}^{(3)}(i\alpha,s_0)\frac{4}{3}d_0^{\,1m}(i\alpha)\right) \tag{52}$$

The equation (29) with n=0 gives

$$A_1\sqrt{(s_0{}^2-1)}Q_1^1(s_0)+\sum_{m=1}^{\infty}C_mR^{(3)}{}_{1m}(i\alpha,s_0)d_0^{\,1m}(i\alpha)=\frac{1}{2}\sqrt{(s_0{}^2-1)} \tag{53}$$

Using this, drag simplifies to

$$2\pi c^2\sqrt{(s_0{}^2-1)}\,\frac{4}{3}A_1\sqrt{(s_0{}^2-1)}\left(s_0Q'_{n+1}(s_0)-Q_1(s_0)\right)$$

$$= \frac{8\pi c^2}{3}(s_0{}^2-1)A_1\frac{1}{(s_0{}^2-1)}$$

$$= \frac{8\pi c^2}{3}A_1 \tag{54}$$

Drag on the body= $2\pi c^2\sqrt{(s_0{}^2-1)}\left[t\sqrt{(s_0{}^2-1)}t_{\xi\xi}-s\sqrt{(1-t^2)}t_{\xi\eta}\right]$

In dimensional form

$$D=\frac{8\pi c^2\mu Uc}{3k}A_1$$

$$D=\frac{8\pi c^3\mu U}{3k}A_1 \qquad (55)$$

Numerical work and discussion:
To understand the variation in drag with respect to permeability parameter α and size of the spheroid s_0 we resort to numerical work. Since C_n's are zero when n is even, we suppress the even suffixed C_n's in system (32). We truncate the system to 5 by 5 system and resort to numerical determination of C_1, C_3 , C_5 , C_7, and C_9. Then using these values and a consequence of the equation (29) with n=0, we determine A_1 numerically. Thus the formula in (54) allows us to estimate the drag on the spheroid numerically. For numerical evaluation we have taken

$$\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 \text{ and}$$

$$s_0 = 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 2.0$$

Fig-2 shows the variation of drag for different values of α. From the Fig-2 we note that as the permeability parameter α is increasing, for a fixed $value\ of\ s_0$ the drag is seen to be increasing

Fig-3 shows the variation of drag for different values of the size $s_0$of the spheroid. As the size of the spheroid increases the drag is seen to be increasing.

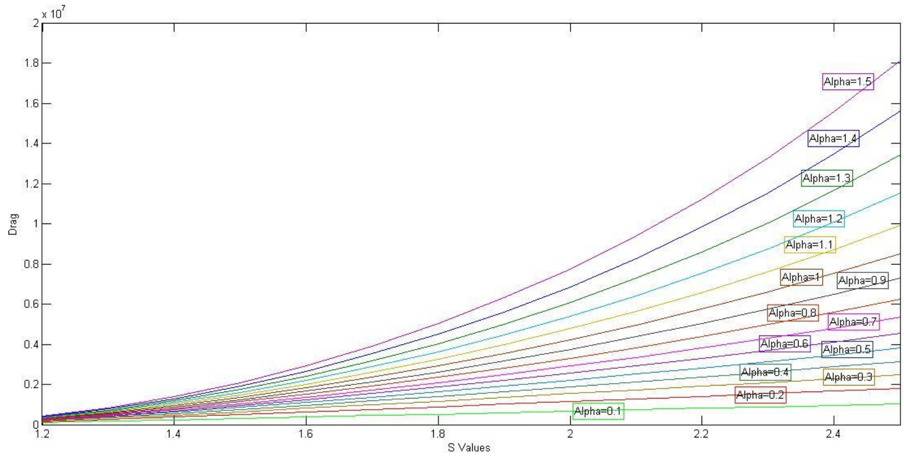

Fig-2: VARIATION OF DRAG WITH RESPECT TO ALPHA

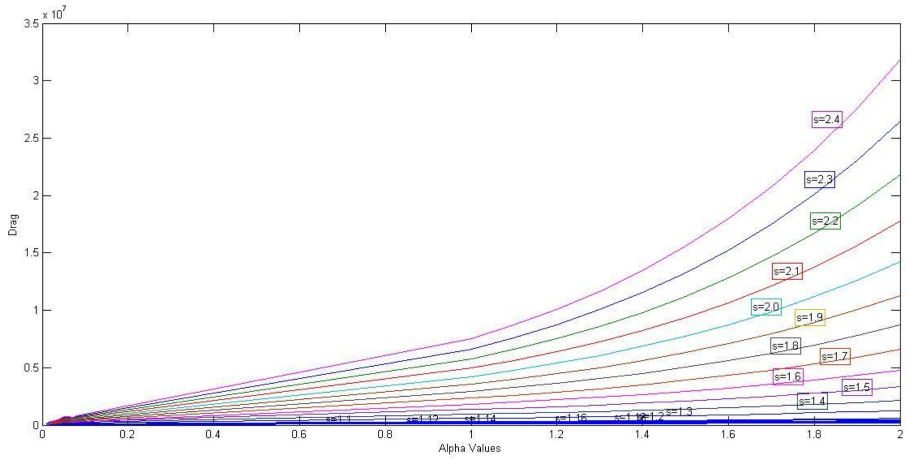

Fig-3:VARIATION OF DRAG WITH RESPECT TO S_0

References

1. Stokes, G. G. "On the effect of fluids on the motion of pendulums, Trans. Camb. Phil," Soc, Vol 9, 1851.
2. Donald A.Nield and Adrian Bejan."Convection in porous media Springar science,"inc 2006.
3. Payne, L.E. and Pell, W.H. "The Stokes flow problems for a class of axially symmetric bodies," J. Fluid Mech, Vol 7 ,pp 529-549, 1960.
4. Lakshmana Rao, S.K. and Iyengar, T.K.V."The slow stationary flow of incompressible micropolar fluid past a spheroid," Int. J. Engg. Sci, Vol 19, pp 189-220, 1981.
5. Abramowitz, M. and Stegun, I.A "Handbook of Mathematical functions with formulas, graphs and mathematical tables," Dover publications, INC, NewYork, 1965.