Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 3 (2015), pp. 1363-1377

http://www.ripublication.com

Application of Fuzzy Multi Objective Linear Programming In The Efficient Treatment of Communicable Diseases

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Abstract

Fuzzy Multi objective optimization is the process of optimizing systematically and simultaneously a collection of objective functions with fuzzy variable. A wide variety of problems in medicine, engineering, industry etc., involve the simultaneous optimization of several objectives in the presence of uncertainty. In this paper, Fuzzy Multi Objective Linear Programming Model is developed based on Fuzzy Multi Objective Transportation Model to minimize the overall treatment cost and curing time by distributing the various treatments to the disease population in order to minimize the human productivity loss. This model will be handy to the health department in controlling the communicable diseases with minimum cost and time.

Key Words: Multi objective optimization, Fuzzy Linear Programming Problem, Fuzzy Transportation Problem, Fuzzy Number, Communicable diseases.

Introduction

Optimization is the act of achieving the best possible result under given circumstances. In design, construction, maintenance etc, professionals have to take decisions. The goal of all such decisions is either to minimize effort or to maximize benefit. The effort or the benefit can be usually expressed as a function of certain design variables. Hence, optimization is the process of finding the conditions given the maximum or the minimum value of the function. In the last three decades many optimization techniques have been invented and successfully applied to optimizations problems in Computer Sciences, Information Technology, Engineering, Chemistry, Biology, Biochemistry, Medicine, Economics etc.

Different modeling techniques are developed to meet the requirements of different types of optimization problems. Major categories of modeling approaches are classical optimization techniques, linear programming, non-linear programming, geometric programming, dynamic programming, integer programming etc. Transportation problem refers to a special class of linear programming problem. Since ancient day, the transportation problem is placing an important role in optimization problems to keep the balance in economic world. Transportation problem was developed in earlier days with single objective and with assumption that supply, demand and cost parameters are exactly known, where as in real life situations, the transportation problem with multi-objective and the parameters are not defined precisely. The necessity to optimize more than one objective or goal while satisfying the physical limitations led to the development of multi-objective programming methods. Fuzzy multi-objective programming satisfies all the requirements of real life situation to optimize the effort and benefit.

In recent years there has been a dramatic increase in the application to optimize techniques to the study the medical problems and the delivery of health care. To indicate the wide spread scope of the subject, some special typical applications in medical discipline are as follows: In 2008, Caetano et al **[9]** discussed optimal medication in HIV seropositive patient treatment using fuzzy cost function, Crina Grosan et al **[3]** developed Multi-Criteria programming model for medical diagnosis and treatment. In 2009, Denton et al **[4]** developed the optimization model to optimize the start time of strain therapy for patients with diabetes, Lee et al **[7]** have discussed about modeling and optimizing the public health infrastructure for emergency response. In 2010, Zhao Jingwei **[16]** discussed Fuzzy Multi-Objective Routing Inventory Problem in recycling infectious medical waste. In 2012, Mason et al **[10]** developed the revised optimization model to optimizing strain treatment decisions for diabetes patients in the presence of uncertain future adherence. In 2013, Persi Pamela et al **[11]** proposed a fuzzy optimization technique for the prediction of coronary Heart Disease using Decision Tree. Recently, in 2014, Lee et al **[8]** developed optimization modeling for emergency department workflow.

This paper proposes Fuzzy Multi Objective Linear Programming Model to minimize the overall treatment cost and curing time of a disease population who have to be cured by the various treatments. The main aim of this paper is to develop the multi objective optimization model based on Fuzzy Multi Objective Linear Programming technique to minimize the human productivity loss by distributing the various treatments to the different disease population so as to minimize the overall treatment cost and to minimize the overall curing time.

Preliminaries:

In this section, some basic definitions, generalized triangular fuzzy number, generalized trapezoidal fuzzy numbers and defuzzification, are presented.

Definition 2.1

Let R be the set of all real numbers. We assume a fuzzy number $\tilde{A}$ that can be expressed for all $x \in R$ in the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}_L}(x) & a \le x \le b \\ w & b \le x \le c \\ \mu_{\tilde{A}_R}(x) & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

Where $0 \le w \le 1$ is a constant, a, b, c, d are real numbers, such that $a < b \le c < d$, $\mu_{\tilde{A}_L}(x) : [a,b] \to [0,w]$, $\mu_{\tilde{A}_R}(x) : [c,d] \to [0,w]$ are two strictly monotonic and continuous functions from R to the close interval $[0,w]$.

Since $\mu_{\tilde{A}_L}(x)$ is continuous and strictly increasing, the inverse function of $\mu_{\tilde{A}_L}(x)$ exists. Similarly $\mu_{\tilde{A}_R}(x)$ is continuous and strictly decreasing, the inverse function of $\mu_{\tilde{A}_R}(x)$ also exist. The inverse functions of $\mu_{\tilde{A}_L}(x)$ and $\mu_{\tilde{A}_R}(x)$ can be denoted by $\mu_{\tilde{A}_L^{-1}}(x)$ and $\mu_{\tilde{A}_R^{-1}}(x)$, respectively. $\mu_{\tilde{A}_L^{-1}}(x)$ and $\mu_{\tilde{A}_R^{-1}}(x)$ are continuous on [0,w] that means both $\int_0^w \mu_{\tilde{A}_L^{-1}}(x)$ and $\int_0^w \mu_{\tilde{A}_R^{-1}}(x)$ exist.

Definition 2.2

A fuzzy number $\tilde{A}$ = (a, b, c, d; w) is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\left(\frac{x-a}{b-a}\right) & a \le x \le b \\ w & b \le x \le c \\ w\left(\frac{x-d}{c-d}\right) & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.3

A fuzzy number $\tilde{A}$ = (a, b, c; w) is said to be generalized triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a \\ w\left(\frac{x-a}{b-a}\right) & a \le x \le b \\ w\left(\frac{c-x}{c-b}\right) & b \le x \le c \\ 0 & x > c \end{cases}$$

Defuzzification

The process of converting the fuzzy output to a crisp value is said to be defuzzification. A number of defuzzification techniques are known, including centre-of-area, centre of gravity, and mean of maximums. A common and useful defuzzification technique is center of gravity. This technique was developed by Sugeno in 1985. This is the most commonly used technique and is very accurate. In 2011 and 2012, Phani Bushan Rao and others **[12, 14, 15]** have proposed a centroid formula for defuzzfication using circumcenter of centroids, orthocenter of centroids, and centroid of centroids. In 2014, Hari Ganesh & Jayakumar **[6]** have proposed a centroid by using radius of gyration of centroid for ranking of fuzzy numbers. Herewith, a new centroid is proposed for defuzzification based on centroid of centroid which is presented as follows:

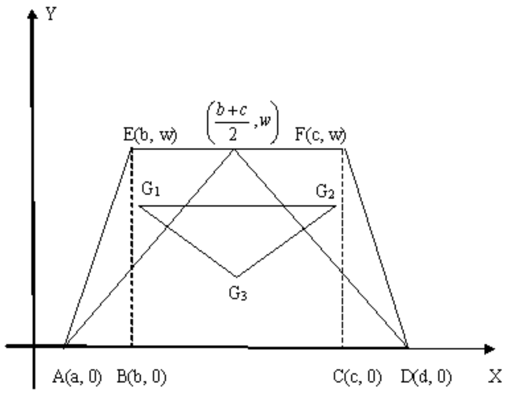

Figure 1: Centroid of Centroids

We define the centroid $G(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ as

$$G(\bar{x}_0, \bar{y}_0) = \left(\frac{4a + 5b + 5c + 4d}{18}, \frac{5w}{9}\right) \tag{1}$$

Its Ranking Function is defined as

$$R(\tilde{A}) = \frac{4a + 5b + 5c + 4d}{18} \tag{2}$$

As a special case, for triangular fuzzy number $\tilde{A} = (a, b, d; w)$, i.e., $c = b$ the centroid of Centroids is given by

$$G(\bar{x}_0, \bar{y}_0) = \left(\frac{2a + 5b + 2d}{9}, \frac{5w}{9}\right) \tag{3}$$

Its Ranking Function is defined as

$$R(\tilde{A}) = \frac{2a + 5b + 2d}{9} \tag{4}$$

Mathematical Formulation of Fuzzy Multi-Objective Transportation Model

In this section, mathematical formulation of fuzzy multi-objective transportation model with fuzzy cost and fuzzy time is presented. Generally the fuzzy transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various sources, to different destinations in such a way that the total fuzzy transportation cost is a minimum. Let there be m sources, i^{th} source possessing fuzzy supply units of a certain product, n destinations (n may or may not be equal to m) with destination j requiring fuzzy demand units. Cost of shipping of an item from each of m sources to each of the n destinations are known either directly or indirectly in terms of mileage, shipping hours, etc. If the objective of a transportation problem is to minimize fuzzy cost, and fuzzy time, then this type of fuzzy problem is treated as a fuzzy multi-objective transportation problem.

Mathematically, the fuzzy multi-objective transportation problem can be stated as:

Minimize $\tilde{z}_k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\tilde{p}_{ij}^{k}\right)x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} \cong \tilde{a}_i \qquad i = 1,2,.....,m$

$\sum_{i=1}^{m} x_{ij} \cong \tilde{b}_j \qquad j = 1,2,.......,n$

Where $\tilde{z}_k = \{\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_k\}$

If the objective function $\tilde{z}_1$ denotes the fuzzy cost function,

$$\tilde{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\, x_{ij}$$

If the objective function $\tilde{z}_2$ denotes the fuzzy cost function,

$$\tilde{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}\, x_{ij}$$

Then it is a bi-objective fuzzy transportation problem, which is represented by using weights of objectives to consider the priorities of the objective as follows:

$$\tilde{z}_1 = w_1\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\, x_{ij} + w_2\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}\, x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} \cong \tilde{a}_i \qquad i = 1,2,.....,m$

$$\sum_{i=1}^{m} x_{ij} \cong \tilde{b}_j \qquad j = 1,2,\ldots\ldots, n$$

$$x_{ij} \geq 0 \quad i = 1,2,\ldots\ldots, m; \; j = 1,2,\ldots\ldots, n$$

and $w_1+w_2 = 1$
where

$\tilde{c}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$: Fuzzy cost from i^{th} source to j^{th} destination

$\tilde{t}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$: Fuzzy time from i^{th} source to j^{th} destination

$\tilde{a}_i = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$: Fuzzy supply from i^{th} source to j^{th} destination

$\tilde{b}_j = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$: Fuzzy demand from i^{th} source to j^{th} destination

All denotes $\tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i, \tilde{b}_j$ a non-negative triangular fuzzy numbers.

$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$: Total fuzzy cost for shipping from i^{th} source to j^{th} destination.

$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{t}_{ij}\, x_{ij}$: Total fuzzy time for shipping from i^{th} source to j^{th} destination.

The same fuzzy transportation problem with two objectives may be represented in the following form of m × n fuzzy matrix (Table 1) where each cell having a fuzzy cost, and fuzzy time.

Table 1: Fuzzy Multi Objective Transportation Model with Fuzzy Cost and Fuzzy Time

Source / Destination	1	2	…..	j	…..	n	**Supply**
1	$\tilde{c}_{11}; \tilde{t}_{11}$	$\tilde{c}_{12}; \tilde{t}_{12}$	…..	$\tilde{c}_{1j}; \tilde{t}_{1j}$	…..	$\tilde{c}_{1n}; \tilde{t}_{1n}$	$\tilde{a}_1$
2	$\tilde{c}_{21}; \tilde{t}_{21}$	$\tilde{c}_{22}; \tilde{t}_{22}$	…..	$\tilde{c}_{2j}; \tilde{t}_{2j}$	…..	$\tilde{c}_{2n}; \tilde{t}_{2n}$	$\tilde{a}_2$
⋮			⋮		⋮		⋮
i	$\tilde{c}_{i1}; \tilde{t}_{i1}$	$\tilde{c}_{i2}; \tilde{t}_{i2}$	….	$\tilde{c}_{ij}; \tilde{t}_{ij}$	…..	$\tilde{c}_{in}; \tilde{t}_{in}$	$\tilde{a}_i$
⋮			⋮		⋮		⋮
m	$\tilde{c}_{m1}; \tilde{t}_{m1}$	$\tilde{c}_{m2}; \tilde{t}_{m2}$	…..	$\tilde{c}_{mj}; \tilde{t}_{mj}$	…..	$\tilde{c}_{mn}; \tilde{t}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_j$		$\tilde{b}_n$	

The Total Optimum Solution For Fuzzy Multi-Objective Transportation Problem

In this section, the method to solve fuzzy multi-objective transportation problem is presented as follows:

Step 1: Test whether the given fuzzy multi-objective transportation problem is a balanced transportation problem or not. If it is balanced (i.e., sum of supply units equal to the sum of demand units) then go to step 3. If it is unbalanced (i.e., sum of supply units is not equal to the sum of demand units) then go to step 2.

Step 2: Introduce dummy rows and / or columns with zero fuzzy costs, and time so as to form balanced transportation problem.

Step 3: Consider the fuzzy linear programming model as presented in section 4.

Step 4: Convert the fuzzy multi-objective transportation problem in to the following crisp linear programming problem by using defuzzification method presented in section 3.

$$\tilde{z}_1 = w_1 \sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{c}_{ij})\, x_{ij} + w_2 \sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{t}_{ij})\, x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} \cong R(\tilde{a}_i) \qquad i = 1,2,\ldots\ldots,m$

$$\sum_{i=1}^{m} x_{ij} \cong R(\tilde{b}_j) \qquad j = 1,2,\ldots\ldots.,n$$

$$x_{ij} \geq 0 \quad i = 1,2,\ldots\ldots,m; \; j = 1,2,\ldots\ldots,n$$

and $w_1+w_2 = 1$

Step 5: Convert the fuzzy multi – objective transportation problem into the following single objective crisp transportation problem using the assigned values of w_1 and w_2:

$$\tilde{z}_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} Q_{ij}\; x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} \cong R(\tilde{a}_i) \qquad i = 1,2,\ldots\ldots,m$

$$\sum_{i=1}^{m} x_{ij} \cong R(\tilde{b}_j) \qquad j = 1,2,\ldots\ldots.,n$$

$$x_{ij} \geq 0 \quad i = 1,2,\ldots\ldots,m; \; j = 1,2,\ldots\ldots,n$$

and $w_1+w_2 = 1$

where Q_{ij} is constant

Step 6: To find the optimal solution {xij}, solve the crisp linear programming problem obtained in step 5 by using any one of the suitable conventional method like simplex, Big-M, Two Phase, etc.

Step 7: Find the optimal total fuzzy transportation cost, and total fuzzy transportation time by substituting the optimal solution obtained in step 6 in the objective function of step 3.

Proposed Fuzzy Model For Efficient Treatment of Communicable Diseases

In this section, a Fuzzy Multi Objective Linear Programming Model is proposed based on multi objective fuzzy transportation model for computing minimum treatment cost and curing time of a disease population affected by various communicable diseases in order to minimize the human productivity loss.

This model is concerned with finding the overall minimum treatment cost and curing time of a disease population affected by various communicable diseases which are to be cured by various treatments in a region.

The data of the model include

1. The size of patients affected by each disease to be taken the treatment and the total availability of various treatments in a particular region.
2. The unit treatment cost and curing time (i.e. treatment cost and curing time per patient) of the disease.

The objective is to determine how the various treatments may be distributed to the different disease population so as to minimize the overall treatment cost and to minimize the curing time. Therefore, the decision variables are:

x_{ij} = the affordability of the j^{th} treatment to the i^{th} disease, where i = 1, 2, ……., m and j = 1, 2, ……., n.

That is a set of m × n variables.

In order to minimize the treatment cost and period, the following problem must be solved

The Objective Function

Consider the size of patients to be taken the treatment i who have been affected by disease j. For any i and any j, the unit treatment cost is c_{ij}, unit curing time t_{ij}, affordability of the treatment to the disease x_{ij}. Since we assume that the cost and time functions are linear, the total treatment cost and total curing time is given by $c_{ij}x_{ij}$ and $t_{ij}x_{ij}$ respectively. Summing over all i and all j now yields the overall treatment cost and curing time for all disease – treatment combinations. That is, our objective functions are

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$$

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}x_{ij}$$

Then it is a two objective transportation using the weights of the objectives which consider the priorities of the objective.

$$\tilde{Z} = w_1\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} + w_2\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}x_{ij}$$

The constraints

Consider treatment i, total affordability of this treatment for the various given diseases in the region is the sum $x_{i1} + x_{i2} + \ldots\ldots\ldots + x_{in}$. Since the availability of this treatment for various diseases in the region is a_i, the affordability of this treatment for the various given diseases cannot exceed a_i.

(i.e.) $\sum_{j=1}^{n} x_{ij} \leq a_i$ for i = 1,2,……,m

Consider disease j. the total affordability of various given treatments for this disease in the region is the sum $x_{1j} + x_{2j} + \ldots\ldots + x_{mj}$. Since the total size of patients affected by this disease to be taken the treatment is b_j, the total affordability of various treatments should not be less than b_j.

(i.e.) $\sum_{i=1}^{m} x_{ij} \geq b_j$ for j = 1,2,……,n

where $x_{ij} \geq 0$ for all i and j

The above implies that the total availability of various given treatments for various given diseases $\sum_{i=1}^{m} a_i$ is greater than or equal to the total number of patients affected by the various given diseases $\sum_{j=1}^{n} b_j$. When the total availability of various given treatments is equal to the total number of patients affected by the various given diseases (i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$) then the model is said to be balanced. In a balanced model, each of the constraints is an equation:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i = 1,2,\ldots\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{for } j = 1,2,\ldots\ldots,n$$

A model in which total availability of various given treatments and total number of patients affected by the various given diseases are unequal is called unbalanced. It is always possible to balance an unbalanced problem.

The fuzzy problem, in which the treatment cost c_{ij}, curing time t_{ij}, total availability of treatment a_i and total number of patients to be taken the treatment b_j quantities are fuzzy quantities, can be formulated as follows:

Minimize Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i$ for i = 1,2,……,m

and $\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j$ for j = 1,2,……,n

where $x_{ij} \geq 0$ for all i and j

This fuzzy problem is explicitly represented by the following table.

Table 2: Fuzzy Model for Optimization of Cost and Time of Treatment of Diseases

Treatments / Diseases	D_1	D_2	…..	D_j	…..	D_n	**Supply (availability of treatment T_j)**
T_1	$\tilde{c}_{11}; \tilde{t}_{11}$	$\tilde{c}_{12}; \tilde{t}_{12}$	…..	$\tilde{c}_{1j}; \tilde{t}_{1j}$	…..	$\tilde{c}_{1n}; \tilde{t}_{1n}$	$\tilde{a}_1$
T_2	$\tilde{c}_{21}; \tilde{t}_{21}$	$\tilde{c}_{22}; \tilde{t}_{22}$	…..	$\tilde{c}_{2j}; \tilde{t}_{2j}$	…..	$\tilde{c}_{2n}; \tilde{t}_{2n}$	$\tilde{a}_2$
⋮			⋮		⋮		⋮
T_i	$\tilde{c}_{i1}; \tilde{t}_{i1}$	$\tilde{c}_{i2}; \tilde{t}_{i2}$	….	$\tilde{c}_{ij}; \tilde{t}_{ij}$	…..	$\tilde{c}_{in}; \tilde{t}_{in}$	$\tilde{a}_i$
⋮			⋮		⋮		⋮
T_m	$\tilde{c}_{m1}; \tilde{t}_{m1}$	$\tilde{c}_{m2}; \tilde{t}_{m2}$	…..	$\tilde{c}_{mj}; \tilde{t}_{mj}$	…..	$\tilde{c}_{mn}; \tilde{t}_{mn}$	$\tilde{a}_m$
Demand (no. of patients affected by the disease D_i to be taken the treatment)	$\tilde{b}_1$	$\tilde{b}_2$	…..	$\tilde{b}_j$	…..	$\tilde{b}_n$	

Example

Communicable diseases are diseases that are as a result of the causative organism spreading from one person to another. They are among the major causes of illnesses in many countries. These diseases affect people of all ages but more so children due to their exposure to environmental conditions that support the spread. Communicable diseases are preventable base on interventions placed on various levels of transmission of the disease. Health Departments have an important role to play in the control of these diseases by applying effective and efficient management, prevention and control measures.

In Thanjavur Region, the availability of various treatments like Allopathy (T_1), Ayurvedic (T_2), Homeopathy (T_3) and Unani (T_4) for all type of diseases are

(50000,52000,55000), (31000,34000,37000), (10500,12500,14500), and (5500,7500,9500) respectively. Moreover, the size of patients affected by the communicable diseases in winter season like Dengue (D_1), Malaria (D_2) and Tuberculosis (D_3) are (21500,22500,25500), (14250,17250,19500), and (10250,12450,15500) respectively. Treatment cost and curing time for all above said treatment - disease combination per patient are as follows:

Table 3: Treatment Cost & Time per Patient

Treatment	**Disease**	**Treatment Cost per Patient (in Rupees)**	**Curing Time per Patient (in days)**
Allopathy	Dengue	(2700,3500,3600)	(31,35,37)
	Malaria	(2100,2500,2800)	(20,22,25)
	Tuberculosis	(8300,8500,9100)	(250,270,300)
Ayurvedic	Dengue	(1700,2000,2300)	(19,20,24)
	Malaria	(2900,3200,3400)	(29,30,34)
	Tuberculosis	(4600,4800,5100)	(90,120,130)
Homeopathy	Dengue	(4000,4200,4500)	(70,90,110)
	Malaria	(4400,4800,5000)	(55,75,95)
	Tuberculosis	(3700,4100,4300)	(325,345,355)
Unani	Dengue	(3500,3800,4000)	(100,120,130)
	Malaria	(3800,4100,4400)	(60,90,120)
	Tuberculosis	(5300,5500,5700)	(400,420,450)

The data collected from the Department of Medical and Rural Health Services at Thanjavur District.

Let us consider a optimization problem with rows representing treatments Allopathy (T_1), Ayurvedic (T_2), Homeopathy (T_3) and Unani (T_4) and column representing communicable diseases Dengue (D_1), Malaria (D_2) and Tuberculosis (D_3) which are affected in the winter season at Thanjavur Region.

Table 4: Unbalanced Table with Fuzzy Treatment Cost and Fuzzy Curing Time

Treatments / diseases	Dengue (D_1)	Malaria (D_2)	Tuberculosis (D_3)	**Supply (availability of treatment T_j)**
Allopathy (T_1)	(2700,3500,3600) (31,35,37)	(2100,2500,2800) (20,22,25)	(8300,8500,9100) (250,270,300)	**(50000,52000, 55000)**
Ayurvedic (T_2)	(1700,2000,2300) (19,20,24)	(2900,3200,3400) (29,30,34)	(4600,4800,5100) (90,120,130)	**(31000,34000, 37000)**
Homeopathy (T_3)	(4000,4200,4500) (70,90,110)	(4400,4800,5000) (55,75,95)	(3700,4100,4300) (325,345,355)	**(10500,12500, 14500)**
Unani (T_4)	(3500,3800,4000) (100,120,130)	(3800,4100,4400) (60,90,120)	(5300,5500,5700) (400,420,450)	**(5500,7500, 9500)**
Demand (no. of patients affected by the disease D_i to be taken the treatment)	**(21500,22500, 25500)**	**(14250,17250, 19500)**	**(10250,12450, 15500)**	

Using the ranking function in equation (4), the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$, $R(\tilde{a}_i)$ and $R(\tilde{b}_j)$ for all i and j are calculated and given in the following table.

Table 5: Unbalanced Table with Treatment Cost and Curing Time

Treatments / diseases	Dengue (D_1)	Malaria (D_2)	Tuberculosis (D_3)	**Supply (availability of treatment T_j)**
Allopathy (T_1)	3344 35	2478 22	8589 272	**52222**
Ayurvedic (T_2)	2000 21	3178 31	4822 116	**34000**
Homeopathy (T_3)	4222 90	4756 75	4056 343	**12500**
Unani (T_4)	3778 118	4100 90	5500 422	**7500**
Demand (no. of patients affected by the disease D_i to be taken the treatment)	**22944**	**17083**	**12639**	

The above problem is unbalanced. For make it as a balanced one, the dummy column is introduced as follows:

Table 6: Balanced Table with Treatment Cost and Curing Time

Treatments / diseases	Dengue (D_1)	Malaria (D_2)	Tuberculosis (D_3)	(D_4)	**Supply (availability of treatment T_j)**
Allopathy (T_1)	3344 35	2478 22	8589 272	0 0	**52222**
Ayurvedic (T_2)	2000 21	3178 31	4822 116	0 0	**34000**
Homeopathy (T_3)	4222 90	4756 75	4056 343	0 0	**12500**
Unani (T_4)	3778 118	4100 90	5500 422	0 0	**7500**
Demand (no. of patients affected by the disease D_i to be taken the treatment)	**22944**	**17083**	**12639**	**53556**	**106222**

The above crisp multi-objective transportation problem is converted into the following crisp linear programming problem by using step 6.

Minimize

$(1358.6)x_{11} + (1004.4)x_{12} + (3598.8)x_{13} + (0)x_{14} +$
$(812.6)x_{21} + (1289.8)x_{22} + (1998.4)x_{23} + (0)x_{24} +$
$(1742.8)x_{31} + (1947.4)x_{32} + (1828.2)x_{33} + (0)x_{34} +$
$(1582)x_{41} + (1694)x_{42} + (2453.2)x_{43} + (0)x_{44}.$

Subject to:

$x_{11} + x_{21} + x_{31} + x_{41} = 22944$ $\quad$ $x_{12} + x_{22} + x_{32} + x_{42} = 17083$
$x_{13} + x_{23} + x_{33} + x_{43} = 12639$ $\quad$ $x_{14} + x_{24} + x_{34} + x_{44} = 53556$
$x_{11} + x_{12} + x_{13} + x_{14} = 52222$ $\quad$ $x_{21} + x_{22} + x_{23} + x_{24} = 34000$
$x_{31} + x_{32} + x_{33} + x_{34} = 12500$ $\quad$ $x_{41} + x_{42} + x_{43} + x_{44} = 7500$

Using TORA software, the crisp linear programming problem is solved to find the optimum solution which is as follows:

x_{12}=22944, x_{21}=17083, x_{33}=12500, x_{34}=139, x_{41}=35139, x_{42}=11056, x_{44}=7361.

The overall minimum fuzzy treatment cost and fuzzy curing time are obtained by using step 7 as follows:

Overall Minimum Fuzzy Treatment Cost = ₹ (288472800, 321633800, 346486500)

Overall Minimum Fuzzy Curing Time = (9023217, 10370648, 11315882) days

After defuzzification, by using the ranking function in eqn. (4), the overall minimum treatment cost and curing time respectively are ₹ 319,78,7511.10 and 10,28,1270.89 days.

Conclusion

In multi-objective optimization problems, the problem of minimizing the total cost and time of transportation is studied as multi-objective transportation problem since long time and is well known in which time – cost minimization problem has been studied by many authors. In 1977, Bhatia **[1]** and others have given a procedure for time minimization in transportation problem. In 1981, Prakash **[13]** introduced transportation problem with objectives to minimize total cost and duration of transportation. Recently, in 2010, Chakraborty Ananya **[2]** has proposed a method for the minimization of transportation cost as well as time of transportation when the demand, supply and transportation cost are fuzzy. In 2014, Hari Ganesh & Jayakumar **[5]** proposed a fuzzy linear programming model for minimizing agricultural production cost. Hence, in this paper, Fuzzy Multi Objective Linear Programming Model has been developed in order to distribute the various treatments to the different disease population so as to minimize the overall treatment cost and to minimize the curing time by employing the supply, demand, cost and time parameters as triangular fuzzy numbers. As minimizing the overall treatment cost and curing time, the human productivity loss may be minimized. This work will be an innovative application of Fuzzy multi objective linear programming technique in healthcare.

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