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On Homomorphic Images of Soft Subgroups

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Abstract

In this paper, we construct a soft group and soft subgroups by assigning a group structure to the universe set X and investigate some interesting results on homomorphic images of soft subgroups.

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Key Words: Soft set, Soft Group, Soft Subgroup, Soft normal subgroup, Soft cyclic subgroup, Homomorphism.

Introduction

The soft set theory was introduced by *Molodtsov* [5] in 1999 as a general mathematical tool to deal with problems involving uncertainty or vagueness. *Molodtsov* recognized the importance of the role of parameters in modeling various problems involving uncertainty and the soft set models worked very well in dealing with such problems. He has shown several applications of this theory in many fields like economics, engineering, medical sciences, etc. Among many theories like Fuzzy set theory, Rough set theory etc., the soft set theory became very significant because of its wide range of applicability. The soft set theory became a very good source of research for many mathematicians and computer scientists of recent years. The development in the fields of soft set theory and its application has been taking place in a rapid pace. The theory of soft topology, soft group theory, soft algebras are some branches of soft set theory.

The notion of Soft Groups was first introduce by *Aktas* and *Cagman* [1] in 2007. Later, some authors [6] have studied various properties of soft groups. The Soft Group theory has many applications in the theory of computers science.

In this present work, we assign a Group structure to the universe set X and introduce some soft group theoretical concepts like soft subgroup, soft normal subgroup, soft cyclic subgroup etc., in a different approach. We also investigate some interesting results on the homomorphic images of soft subgroups.

Preliminaries

In this section, we present some basic definitions which are needed in further study of this paper. Let U be an initial universe set and E_U (or simply E) be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let $P(U)$ be the collection of all subsets of U.

Definition 1.1

A pair (F, A) is called a *soft set* over U, if $A \subset E$ and $F : A \to P(U)$. We write F_A for (F, A).

Definition 1.2

Let F_A and G_B be soft sets over a common universe set U and $A, B \subset E$. Then we say that

a) F_A is a *soft subset* of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if (i) $A \subset B$ and (ii) $F(e) \subset G(e) \ \forall\, e \in A$.

b) F_A equals G_B, denoted by $F_A \tilde{=} G_B$, if $F_A \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_A$.

Definition 1.3

A soft set F_A over U is called a *null soft set*, denoted by Φ, if for $e \in A$, $F(e) = \phi$.

Definition 1.4

A soft set F_A over U is called an *absolute soft set*, denoted by $\tilde{A}$, if for $e \in A$, $F(e) = U$.

Definition 1.5

The *union* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcup B,$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & if \quad e \in A - B \\ G(e) & if \quad e \in B - A \\ F(e) \bigcup G(e) & if \quad e \in A \bigcap B \end{cases} \qquad \text{We write } F_A \bigcup G_B \tilde{=} H_C.$$

Definition 1.6
The *intersection* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C = A \bigcap B,$ and for all $e \in C$, $H(e) = F(e) \bigcap G(e)$. We write $F_A \bigcap G_B \tilde{=} H_C$.

Definition 1.7
For a soft set F_A over U, the relative *complement* of F_A is denoted by F_A^c and is defined by $F_A^c \tilde{=} F_A^1$, where $F^1 : A \to P(U)$ is a mapping given by $F^1(e) = U - F(e)$ for all $e \in A$.

Definition 1.8
If F_A and G_B are two soft sets over U, then F_A *AND* G_B is denoted by $F_A \wedge G_B$ and it is defined as a soft set $(H, A \times B)$ where

$$H(\alpha,\beta) = F(\alpha) \bigcap G(\beta), \forall (\alpha,\beta) \in A \times B.$$

Definition 1.9
If F_A and G_B are two soft sets over U, then F_A *JOIN* G_B is denoted by $F_A \vee G_B$ and it is defined as $(H, A \times B)$ where

$$H(\alpha,\beta) = F(\alpha) \bigcup G(\beta), \forall (\alpha,\beta) \in A \times B.$$

Definition 1.10
Suppose a binary operation denoted by $\circ$, is defined for all subsets of U. Let F_A and G_B be two soft sets over U. Then the operation $\circ$ for the soft sets is defined in the following way: $F_A \circ G_B = (H, A \times B)$ Where
$H(\alpha,\beta) = F(\alpha) \circ G(\beta)$, $\alpha \in A$ and $\beta \in B$.

A New Approach : Soft Groups

In this section, we introduce the notions of soft subgroups, soft normal subgroups and then we define soft group. These definitions are different from the concepts introduced by *Aktas* and *Cagman* in their research article [1].

In what follows G and $P(G)$ stand for a group and the collection of all subsets of G. Let E be the set of parameters. We denote the identity element in G by the symbol $\boldsymbol{e_G}$.

Definition 2.1
We say that a soft set F_E is a *soft subgroup* over G if $F(e)$ is a subgroup of G for every $e \in E$.

Example 2.2
Let $G = \ 1,-1,i,-i\ $ and $H = \ 1,-1\ $. Then G forms an abelian group with respect to multiplication and H is a subgroup of G if $F : E \to P(G)$ is defined by $F(e) = H$ for every $e \in E$ then F_E is a soft subgroup over G. Here E any set of parameters.

Definition 2.3
We define the *identity soft subgroup* over G to be a soft set $I : E \to P(G)$, such that $I(e) = \ \boldsymbol{e_G}\ $ for every $e \in E$.

Definition 2.4
We say that a soft set F_E is a *soft normal subgroup* over G if $F(e)$ is a normal subgroup of G for every $e \in E$.

Definition 2.5
Let $S(G)$ be the collection of all soft subgroups over G. That is $S(G)$ the collection of all soft sets F_E, where $F : E \to P(G)$ is defined by $F(e)$ is a subgroup of G for every $e \in E$. We say that $S(G)$ is a *soft group* over G.

Definition 2.6
We define an *absolute soft subgroup* to be a soft set $F : E \to P(G)$ such that $F(e) = G$ for every $e \in E$. We denote by $\tilde{G}$.

Remark 2.7
We call the soft sets I_E and $\tilde{G}$, improper soft subgroups over G.

Proposition 2.8
The intersection of any two soft subgroups over G is also a soft subgroup over G.

Proposition 2.9
The intersection of any two soft normal subgroups over G is also a soft normal subgroup over G.

Proposition 2.10
The union of two soft subgroups is a soft subgroup over G if one is contained in the other.

Remark 2.11
Sometimes the union of two soft subgroups can be a soft subgroup without the condition in the above proposition – 2.10. Hence the condition is sufficient but not necessary. It can be observed from the following example.

Example 2.12

Let G be a group and e_0, a fixed parameter in E. Let $F_1 : E \to P(G)$ and $F_2 : E \to P(G)$ be two soft subgroup over G defined by

$$F_1 = \begin{cases} \{\boldsymbol{e_G}\} & if \quad e = e_0 \\ H & if \quad e \neq e_0 \end{cases}$$

$$F_2 = \begin{cases} H & if \quad e = e_0 \\ \{\boldsymbol{e_G}\} & if \quad e \neq e_0 \end{cases}$$

Where H is a proper subgroup of G. Then $F_1 \cup F_2 = F_3$ and $F_3 : E \to P(G)$ is given by $F_3(e) = H$ for every $e \in E$.

Proposition 2.13

The union of two soft normal subgroups is a soft normal subgroup over G if one is contained in the other.

Proposition 2.14

Every soft subgroup over an abelian group is soft normal.

Definition 2.15

We say that a soft set F_E is a *soft cyclic subgroup* over G if $F(e)$ is a cyclic subgroup of G for every $e \in E$. If $a(e) \in G$ is a generator of $F(e)$ then we say that the soft set $F_{a(e)} : E \to P(G)$ defined by $F_{a(e)}(e) = a(e)$ is a *soft generator* of F_E.

Proposition 2.16

Every soft subgroup over an abelian group is soft cyclic.

Homomorphic Images of Soft Subgroups

In this section, we investigate few interesting results on the homomorphic images of soft subgroups. Throughout this section, G and G' denote two groups. Let g and g' be the identity elements in G and G' respectively. Let $P\ G$ and $P\ G'$ be the power sets of G and G' respectively.

Definition 3.1

A mapping $\varphi : G \to G'$ is said to be a *homomorphism* of G into G' if $\varphi(xy) = \varphi(x)\ \varphi(y)$ for all x and y in G.

Definition 3.2

A homomorphism $\varphi : G \to G'$ is said to be an *isomorphism* of G into G' if φ is one-one.

Definition 3.3
A homomorphism $\varphi : G \rightarrow G'$ is said to be an *automorphism* of G onto itself if φ is a bijection.

Definition 3.4
Let $F : E \rightarrow P(G)$ be a soft set and $\varphi : G \rightarrow G'$, a mapping of G into G'. Then we define a soft set $\varphi_F : E \rightarrow P(G')$ as follows.

$$\varphi_F(e) = \varphi(F(e)) = \{\varphi(x) : x \in F(e)\}$$

If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' then we call the soft set $\varphi_F : E \rightarrow P(G')$ to be the *homomorphic image* of the soft set F_E.

Proposition 3.5
If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' and $F : E \rightarrow P(G)$ is a soft subgroup over G then $\varphi_F : E \rightarrow P(G')$ is soft subgroup over G'.

Proposition 3.6
If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' and $F : E \rightarrow P(G)$ is a soft normal subgroup over G then $\varphi_F : E \rightarrow P(G')$ is soft normal subgroup over G'.

Proposition 3.7
If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' and $F : E \rightarrow P(G)$ is a soft cyclic subgroup over G then $\varphi_F : E \rightarrow P(G')$ is soft cyclic subgroup over G'.

Definition 3.8
If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' then the *kernel* of φ is denoted by K_φ and it is defined by $K_\varphi = \{x \in G : \varphi(x) = g'\}$

Definition 3.9
We define a *soft kernel* K_E to be the soft set $K : E \rightarrow P(G)$ defined by $K(e) = K_\varphi$ for every $e \in E$.

Proposition 3.10
If $\varphi : G \rightarrow G'$ is a homomorphism of G into G' and if K_E is a soft kernel then φ_K is the identity soft subgroup over G'.

Proposition 3.11

If $I : E \to P \;\; G$ and $I' : E \to P \;\; G'$ are identity soft subgroups over G and G' respectively then $\varphi_I \cong I'_E$.

Proposition 3.12

An automorphic image of a soft subgroup over G is again a soft subgroup over G.

Proposition 3.13

If $\varphi : G \to G'$ is a homomorphism of G into G' then $\varphi_K \cong I'_E$, where $K : E \to P \;\; G$ is a soft kernel over G.

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