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Quantum Algorithm for Isomorphic Graphs Problem by Numbering Method

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Abstract

A quantum algorithm for the isomorphic graphs problem by a numbering method and its example are reported. It is decided whether two graphs are isomorphic or not. When the number of graph vertexes is n, a computational complexity of a classical computation is $n!$. The computational complexity becomes about n^2 by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the numbering method. Therefore, a polynomial time process becomes possible.

AMS subject classification: Primary 81-08; Secondary 68R10, 68W40.

Keywords: Quantum algorithm, isomorphic graphs problem, numbering method, computational complexity, polynomial time.

1. Introduction

A quantum computer can move quickly to resolve a problem by doing a parallel calculation that uses quantum entangled states. Haroche and Wineland [1] made the very first steps towards building the quantum computer. Quantum algorithms that have been started by Deutsch-Jozsa's algorithm for the rapid solution [2−4] are expanded the application range by Shor's algorithm for the factorization [3−5], Grover's algorithms for the database search [3, 6, 7], and so on. A quantum algorithm for the 3-SAT problem by a numbering method has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The isomorphic graphs problem [9] is examined by the numbering method this time. Therefore, its result is reported.

2. Isomorphic Graphs Problem

Two graphs which contain the same number of graph vertexes connected in the same way are said to be isomorphic. It is searched whether two graphs are isomorphic or not [9].

3. Quantum Algorithm

It is assumed that vertexes and edges of graphs X and Y are $x(i)$, $x(i, j)$ and $y(i)$, $y(i, j) = y(j, i)$ [$0 \le i \le j \le n-1$. i, j and n are integers.], respectively, where each of $x(i, j)$ and $y(i, j)$ is a number of edge i-j, especially, each of $x(i, i)$ and $y(i, i)$ is a number of loop.].

(1) The number of the repeated permutation of n vertexes is n^n.

(2) The number of the permutation of n vertexes is $n!$.

When n vertexes are $y(a_0)$, $y(a_1)$, ⋯ , $y(a_{n-2})$ and $y(a_{n-1})$, $a_0 n^{n-1} + a_1 n^{n-2} + \cdots + a_{n-2} n^1 + a_{n-1} n^0 = U$ is the numbering datum from 0 to n^n-1 [The 0-th datum is 0, 0, ⋯ , 0 and 0. The (n^n-1)-th datum is $(n-1)$, $(n-1)$, ⋯ , $(n-1)$ and $(n-1)$.] in (1). In (2), it is assumed that the first datum is 0, 1, ⋯ , $n-1$, and the $n!$-th datum is $(n-1)$, $(n-2)$, ⋯ , 0, the V-th datum is obtained from $v_1(n-1)! + v_2(n-2)! + \cdots + v_{n-1}1!$. Each of t_i [$1 \le i \le n$. i is an integer.] is 1 piece of permutation from 0 to $n-1$. When v_i is 0 from $i = 1$ to $i = n-2$ sequentially, t_i is the smallest number in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and v_{i+1}, v_{i+2}, ⋯ , v_{n-2} and v_{n-1} are 0, t_i is the v_i-th small number in remained numbers, and $t_{i+1} > t_{i+2} > \cdots > t_{n-1} > t_n$ is selected in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and there are $v_{i+1} \neq 0$ or $v_{i+2} \neq 0$ or ⋯ or $v_{n-2} \neq 0$ or $v_{n-1} \neq 0$, t_i is the (v_i+1)-th small number in remained numbers. When v_{n-1} is 1, $t_{n-1} < t_n$ is selected in remained numbers. Therefore, $t_1 n^{n-1} + t_2 n^{n-2} + \cdots + t_{n-1} n^1 + t_n n^0$ is $U(V)$. This method is named the numbering method for this problem. g is the minimum integer that follows $n!/1 \leq 4^g = 2^{2g}$, because a number of combinations of an answer is at least 1. $U(V=1)$, $U(V=(n!/4)-1)$, $U(V=(n!/16)-1)$, $\cdots$, $U(V=(n!/4^{g-1})-1)$ and $U(V=n!/4^g)$ are calculated. Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_0>$, $|a_1>$, $\cdots$, $|a_{n-1}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{n-2}>$, $|d_0>$, $|d_1>$ $|e>$ and $|f>$ are prepared. When P is the minimum integer that is $log_2 n$ or more, each of $|a_h>$ that h is an integer from 0 to $n-1$ is consisted of P quantum bits [= qubits]. States of $|a_0>$, $|a_1>$, $\cdots$, $|a_{n-1}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{n-2}>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$ are a_0, a_1, $\cdots$, a_{n-1}, b, c_0, c_1, $\cdots$, c_{n-2}, d_0, d_1, e and f, respectively.

1. Step 1: Each qubit of $|a_0>$, $|a_1>$, $\cdots$, $|a_{n-1}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{n-2}>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$ is set $|0>$.
2. Step 2: The Hadamard gate $\boxed{H}$ [3, 4] acts on each qubit of $|a_0>$, $|a_1>$, $\cdots$, $|a_{n-2}>$ and $|a_{n-1}>$. It changes them for entangled states. The total states are $(2^P)^n$.
3. Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|1>$ in $a_h < n$, or it changes $|b>$ for $|0>$ in the others of a_h. As a target state for $|b>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3, 6, 7] act on $|b>$. When Q is the minimum even integer that is $(2^P/n)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is Q, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{n-1}>$. Therefore, each state of $|a_h>$ is 0, 1, $\cdots$, $n-2$ or $n-1$, and the total states become n^n.
4. Step 4: It is assumed that a quantum gate (B) changes $|c_0>$, $|c_1>$, $\cdots$, $|c_{n-3}>$ and $|c_{n-2}>$ for $|c_0 + 1>$, $|c_1 + 1>$, $\cdots$, $|c_{n-3} + 1>$ and $|c_{n-2} + 1>$ in $a_h = 0, 1, \cdots , n-3$ and $n-2$, respectively. This action is repeated from $|a_0>$ to $|a_{n-1}>$. As the target state for $|c_0>$ is 1, (PI) and (IM) act on $|c_0>$. When R_0 is the minimum even integer that is $(n/(n-1))^{(n-1)/2}$ or more, the total number that (PI) and (IM) act on $|c_0>$ is R_0. Next, (OB) observes $|c_0>$. Therefore, only the graphs that contain 1 piece of 0 remain. The number of data is $n(n-1)^{n-1}$. As the target state for $|c_1>$ is 1, (PI) and (IM) act on $|c_1>$. When R_1 is the minimum even integer that is $((n-1)/(n-2))^{(n-2)/2}$ or more, the total number that (PI) and (IM) act on $|c_1>$ is R_1. Next, (OB) observes $|c_1>$. Therefore, only the graphs that contain 1 piece of 1 remain. The number of data is $n(n-1)(n-2)^{n-2}$. Similarly, these actions are repeated sequentially from $|c_2>$ to $|c_{n-2}>$. Only the graphs that contain 1 piece

of number from 0 to $n-1$, respectively, remain. The number of data is $n!$ [$=W_0$].

5. Step 5: It is assumed that a quantum gate (C_0) changes $|d_0>$ for $|d_0+1>$ at $x(0, 0) = y(a_0, a_0)$, or it changes $|d_0>$ for $|d_0+0>$ in $x(0, 0) \neq y(a_0, a_0)$, and it changes $|d_1>$ for $|d_1 + a_0 n^{n-1}>$. Similarly, (C_i) [$1 \le i \le n-1$. i is the integer.] changes $|d_0>$ for $|d_0+1>$ at $x(i, i) = y(a_i, a_i)$, or it changes $|d_0>$ for $|d_0+0>$ in $x(i, i) \neq y(a_i, a_i)$, and it changes $|d_1>$ for $|d_1 + a_i n^{n-1-i}>$. These actions are repeated sequentially from 1 to $n-1$ at i. Therefore, $|d_0>$ becomes $|0>, |1>, \cdots,$ $|n-1>$ or $|n>$, and $|d_1>$ becomes $|a_0 n^{n-1} + a_1 n^{n-2} + \cdots + a_{n-1} n^0 = U>$.
6. Step 6: It is assumed that a quantum gate ($D(0, 1)$) changes $|d_0>$ for $|d_0+1>$ at $x(0, 1) = y(a_0, a_1)$, or it changes $|d_0>$ for $|d_0+0>$ in $x(0, 1) \neq y(a_0, a_1)$. Similarly, ($D(i, j)$) [$0 \le i < j \le n-1$. i and j are integers.] changes $|d_0>$ for $|d_0+1>$ at $x(i, j) = y(a_i, a_j)$, or it changes $|d_0>$ for $|d_0+0>$ in $x(i, j) \neq y(a_i, a_j)$. These actions are repeated sequentially from 0 and 2 to $n-2$ and $n-1$ at i and j, respectively. Therefore, $|d_0>$ becomes $|0>, |1>, \cdots, |n + (n(n-1)/2) - 1>$ or $|n + n(n-1)/2 = n(n+1)/2>$.
7. Step 7: It is assumed that a quantum gate (E) changes $|e>$ for $|0>$ at $d_0 = n(n+1)/2$, or it changes $|e>$ for $|d_1>$in the others of d_0.
8. Step 8: It is assumed that a quantum gate (F_1) changes $|f>$ for $|1>$ in $U(V=1) \le e \le U(V=(n!/4)-1)$ or $e=0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of the data that is included in $U(V=1) \le e \le U(V=(n!/4)-1)$ or $e=0$ is $W_1 \approx n!/4$. When T_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|f>$ is $T_1 \approx 2$. Next, (OB) observes $|f>$, and the data of W_1 remain. Similarly, (F_i) [$2 \le i \le g-1$. i is the integer.] changes $|f>$ for $|1>$ in $U(V=1) \le e \le U(V=(n!/4^i)-1)$ or $e=0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of the data that is included in $U(V=1) \le e \le U(V=(n!/4^i)-1)$ or $e=0$ is $W_i \approx n!/4^i$. When T_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|f>$ is $T_i \approx 2$. Next, (OB) observes $|f>$, and the data of W_i remain. These actions are repeated sequentially from 2 to $g-1$ at i. (F_g) changes $|f>$ for $|1>$ at $e=0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of the data that is included at $e=0$ is $W_g \approx n!/4^g$. When T_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI)

and (IM) act on $|f>$ is $T_g \approx 2$. Next, (OB) observes $|a_0>$, $|a_1>$, $\cdots$,$|a_{n-1}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{n-2}>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$, and one of the data of W_g remains. Therefore, one example of combinations that are $b_0 = n(n + 1)/2$ is obtained.

4. Numerical Computation

It is assumed that there are $n = 5$, $x(0, 1) = 1$, $x(0, 2) = 1$, $x(0, 4) = 1$, $x(1, 2) = 2$, $x(2, 3) = 1$, $x(2, 4) = 1$, $x(4, 4) = 1$, $y(0, 3) = y(3, 0) = 1$, $y(0, 4) = y(4, 0) = 2$, $y(1, 1) = 1$, $y(1, 3) = y(3, 1) = 1$, $y(1, 4) = y(4, 1) = 1$, $y(2, 4) = y(4, 2) = 1$, $y(3, 4) = y(4, 3) = 1$, $g = 4$ [$5!/1 = 120 \leq 4^4 = 256$], $U(V{=}1) = 194$, $U(V{=}(5!/4) - 1 = 29) = 738$ [for example, $V = 29 = 1 \cdot 4! + 0 \cdot 3! + 2 \cdot 2! + 1 \cdot 1!$, $U = 738 = 1 \cdot 5^4 + 0 \cdot 5^3 + 4 \cdot 5^2 + 2 \cdot 5^1 + 3 \cdot 5^0$], $U(V = (5!/16) - 1 \approx 7) = 294$ and $U(V = (5!/64) - 1 \approx 1) = 194$.

First of all, $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$, $|a_4>$, $|b>$, $|c_0>$, $|c_1>$, $|c_3>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$ are prepared. When P is the minimum integer that is $log_2\, n = log_2\, 5 \approx 2.322 \leq 3 = P$, each of $|a_h>$ that h is the integer from 0 to 4 is consisted of $P = 3$ qubits. States of $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$, $|a_4>$, $|b>$, $|c_0>$, $|c_1>$, $|c_2>$, $|c_3>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$ are a_0, a_1, a_2, a_3, a_4, b, c_0, c_1, c_2, c_3, d_0, d_1, e and f, respectively.

1. Step 1: Each qubit of $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$, $|a_4>$, $|b>$, $|c_0>$, $|c_1>$, $|c_2>$, $|c_3>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$ is set $|0>$.
2. Step 2: $\boxed{H}$ acts on each qubit of $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$ and $|a_4>$. It changes them for entangled states. The total states are $(2^P)^n = (2^3)^5$.
3. Step 3: (A) changes $|b>$ for $|1>$ in $a_h < 5$, or it changes $|b>$ for $|0>$ in the others of a_h. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When Q is the minimum even integer that is $(2^P/n)^{1/2} = (2^3/5)^{1/2} \approx 1.265 \leq 2 = Q$, the total number that (PI) and (IM) act on $|b>$ is $Q \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_4>$. Therefore, each state of $|a_h>$ is 0, 1, 2, 3 or 4, and the total states become $n^n = 5^5$.
4. Step 4: (B) changes $|c_0>$, $|c_1>$, $|c_2>$ and $|c_3>$ for $|c_0 + 1>$, $|c_1 + 1>$, $|c_2 + 1>$ and $|c_3 + 1>$ in $a_h = 0, 1, 2$ and 3, respectively. This action is repeated from $|a_0>$ to $|a_4>$. As the target state for $|c_0>$ is 1, (PI) and (IM) act on $|c_0>$. When R_0 is the minimum even integer that is $(5/4)^{4/2} \approx 1.563 \leq 2 = R_0$, the total number that (PI) and (IM) act on $|c_0>$ is R_0. Next, (OB) observes $|c_0>$. Therefore, only the graphs that contain 1 piece of 0 remain. The number of data is $5 \cdot 4^4$. As the target state for $|c_1>$ is 1, (PI) and (IM) act on $|c_1>$. When R_1 is the minimum even integer that is $(4/3)^{3/2} \approx 1.540 \leq 2 = R_1$, the total number that (PI) and

(IM) act on $|c_1>$ is R_1. Next, (OB) observes $|c_1>$. Therefore, only the graphs that contain 1 piece of 1 remain. The number of data is $5 \cdot 4 \cdot 3^3$. Similarly, these actions are repeated sequentially from $|c_2>$ to $|c_3>$. Only the graphs that contain 1 piece of number from 0 to 4, respectively, remain. The number of data is 5! [$=W_0$].

5. Step 5: (C_0) changes $|d_0>$ for $|d_0 + 1>$ at $x(0, 0) = y(a_0, a_0)$, or it changes $|d_0>$ for $|d_0 + 0>$ in $x(0, 0) \neq y(a_0, a_0)$, and it changes $|d_1>$ for $|d_1 + a_0 5^4>$. Similarly, (C_i) [$1 \leq i \leq 4$. i is the integer.] changes $|d_0>$ for $|d_0 + 1>$ at $x(i, i) = y(a_i, a_i)$, or it changes $|d_0>$ for $|d_0 + 0>$ in $x(i, i) \neq y(a_i, a_i)$, and it changes $|d_1>$ for $|d_1 + a_i 5^{4-i}>$. These actions are repeated sequentially from 1 to 4 at i. Therefore, $|d_0>$ becomes $|0>$, $|1>$, $|2>$, $|3>$, $|4>$ or $|5>$, and $|d_1>$ becomes $|a_0 5^4 + a_1 5^3 + a_2 5^2 + a_3 5^1 + a_4 5^0 = U>$.
6. Step 6: ($D(0, 1)$) changes $|d_0>$ for $|d_0 + 1>$ at $x(0, 1) = y(a_0, a_1)$, or it changes $|d_0>$ for $|d_0 + 0>$ in $x(0, 1) \neq y(a_0, a_1)$. Similarly, ($D(i, j)$) [$0 \leq i < j \leq 4$. i and j are integers.] changes $|d_0>$ for $|d_0 + 1>$ at $x(i, j) = y(a_i, a_j)$, or it changes $|d_0>$ for $|d_0 + 0>$ in $x(i, j) \neq y(a_i, a_j)$. These actions are repeated sequentially from 0 and 2 to 3 and 4 at i and j, respectively. Therefore, $|d_0>$ becomes $|0>$, $|1>$, $\cdots$, $|14>$ or $|15>$.
7. Step 7: (E) changes $|e>$ for $|0>$ at $d_0 = 15$, or it changes $|e>$ for $|d_1>$ in the others of d_0.
8. Step 8: (F_1) changes $|f>$ for $|1>$ in $U(V = 1) = 194 \leq e \leq U(V = 29) = 738$ or $e = 0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of the data that is included in $194 \leq e \leq 738$ or $e = 0$ is $W_1 \approx 5!/4$. When T_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (5!/(5!/4))^{1/2} = 2 \leq 2 = T_1$, the total number that (PI) and (IM) act on $|f>$ is $T_1 \approx 2$. Next, (OB) observes $|f>$, and the data of W_1 remain. Similarly, (F_i) [$2 \leq i \leq 3$. i is the integer.] changes $|f>$ for $|1>$ in $194 \leq e \leq U(V=(5!/4^i) - 1)$ or $e = 0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of the data that is included in $194 \leq e \leq U(V=(5!/4^i) - 1)$ or $e = 0$ is $W_i \approx 5!/4^i$. When T_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx ((5!/4^{i-1})/(5!/4^i))^{1/2} = 2 \leq 2 = T_i$, the total number that (PI) and (IM) act on $|f>$ is $T_i \approx 2$. Next, (OB) observes $|f>$, and the data of W_i remain. These actions are repeated sequentially from 2 to 3 at i. (F_4) changes $|f>$ for $|1>$ at $e = 0$, or it changes $|f>$ for $|0>$ in the others of e. As the target state for $|f>$ is 1, (PI) and (IM) act on $|f>$. The number of

the data that is included at $e = 0$ is $W_4 \approx 5!/4^4$. When T_4 is the minimum even integer that is $(W_3/W_4)^{1/2} \approx ((5!/4^3)/(5!/4^4))^{1/2} = 2 \leq 2 = T_4$, the total number that (*PI*) and (*IM*) act on $|f>$ is $T_4 \approx 2$. Next, (*OB*) observes $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$, $|a_4>$, $|b>$, $|c_0>$, $|c_1>$, $|c_2>$, $|c_3>$, $|d_0>$, $|d_1>$, $|e>$ and $|f>$, and one of the data of W_4 remains. Therefore, a_0, a_1, a_2, a_3, a_4, b, c_0, c_1, c_2, c_3, d_0, d_1, e and f are 3, 0, 4, 2, 1, 1, 1, 1, 1, 1, 15, 1986, 0 and 1, respectively. As a result, $x(0)$, $x(1)$, $x(2)$, $x(3)$ and $x(4)$ of graph X are $y(3)$, $y(0)$, $y(4)$, $y(2)$ and $y(1)$ of graph Y, respectively.

5. Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is Pn at $\boxed{H}$, n at (A), $Qn \approx 2n$ at (*PI*) and (*IM*), n at (*OB*), n at (B), $\sum_{i=0\to n-2} R_i \approx 2(n-1)$ at (*PI*) and (*IM*), and $(n-1)$ at (*OB*), n at (C_i) [$0 \leq i \leq n-1$. i is the integer.], $n(n-1)$ at ($D(i, j)$) [$1 \leq i < j \leq n-1$. i and j are integers.], 2 at (E), g at (F_i) [$1 \leq i \leq g$. i is the integer.], $\sum_{i=1\to g} T_i \approx 2g$ at (*PI*) and (*IM*), and g at (*OB*). Therefore, S becomes $n^2 + (P + 8)n - 1 + 4g$. In the example of the numerical computation, S is 95. The computational complexity of the classical computation [= Z] is $n! = 5! = 120$. After all, S/Z becomes about 4/5. When n is large enough, S becomes about n^2. And then, S/Z is about $n^2/n!$. For example, as for $n = 50$, S/Z is about $1/10^{61}$. Therefore, the polynomial time process becomes possible.

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