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Elzaki Transform And Power Series Expansion On A Bulge Heaviside Step Function

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Abstract

The aim of this paper is to introduce the Heaviside step function on a bulge function and formulate the Elzaki transform of the Heaviside step functionof a bulge function by applying the Power series expansion.

Keywords: Elzaki transform, Bulge function, Heaviside step function

1 Introduction

Some problems in applied mathematics and physics can be solved by using ELzaki transform. In mathematics, Elzaki transform can be appliedto find a solution of the linear ordinary andpartial differential equations with constantcoefficient and variable coefficients. In addition, Elzakitransforms ofderivatives have been studied in many ways to solve the ODEs. Ig. Cho andHj. Kim [12] showed that Laplace transform of derivative can be expressedby an infinite series or Heaviside function. T. Lee and H. Kim [13] foundthe representation of energy equation by Laplace transform.
In the resent years, . P. Haarsa and S. Pothat [6] introduce the Heaviside step function on a bulge function and formulate the Laplace transform of the Heaviside step function of a bulge function by applying the Power series expansion
In this paper,we introduce the Heaviside step function on a bulge function and formulateElzaki transform of the Heaviside step function of a bulge function byapplying the Power series expansion.

We introduce the study by handing out the Elzaki transform, Heaviside stepfunction and the Power series expansion which can be used our study.This method is illustrated by giving lemma already described in [6].

Definition 1.

Elzaki Transform [2]. Given a function $f(t)$ defined for all $t \geq 0$, Elzaki transform of f is the function T defined as follow:

$$E[f(t), v] = T(v) = v\int_0^t f(t)e^{-\frac{t}{v}}dt,\ v \in (k_1, k_2) \qquad (1)$$

for all values of$s$ for which the improper integral converges.

The non-homogeneous differential equation with constant coefficientsof the form,

$$a_n\frac{d^n y}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}\frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_1\frac{d\ \ y}{dx} + a_0 = f(x) \qquad (2)$$

is called the higher order non-homogeneous linear differential equations. In thispaper, we study the non-homogeneous second order differential equation with,a bulge function in the form,

$$y'' + w^2 y = f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}}, 0 < t < \delta \\ t, t > \delta \end{cases}$$

Elzaki transform of the first and second derivatives are expressed respectively by:

$$E\{y'\} = \frac{T(v)}{v} - vf(0)\text{and}E\{y''\} = \frac{T(v)}{v^2} - f(0) - vf'(0)$$

The Power series expansion of$e^{-\frac{(t-l)^2}{2}}$is derived by:

$$e^{-\frac{(t-l)^2}{2}} = e^{-\frac{l^2}{2}} + e^{-\frac{l^2}{2}}lt + e^{-\frac{l^2}{2}}\left(-\frac{1}{2} + \frac{l^2}{2}\right)t^2 + e^{-\frac{l^2}{2}}\left(-\frac{l}{2} + \frac{l^3}{6}\right)t^3 \qquad (3)$$

Lemma 1.

The Elzaki transform of the bulge function $e^{-\frac{(t-l)^2}{2}}$is expressed by,

$$E\left\{e^{-\frac{(t-l)^2}{2}}\right\} = e^{-\frac{l^2}{2}}[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] \qquad (4)$$

Heaviside step function of a bulge function of a piecewise continuous function is given by:

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}}, 0 < t < \delta \\ t, t > \delta \end{cases}$$

Is expressedby

$$f(t) = e^{-\frac{(t-l)^2}{2}} + tu(t-\delta) - e^{-\frac{(t-l)^2}{2}} u(t-\delta) \quad (5)$$

wherea, δ are constants.

Lemma 2.

Elzaki transform of$e^{-\frac{(t-l)^2}{2}} u(t-\delta)$is expressed by:

$$E\left\{e^{-\frac{(t-l)^2}{2}} u(t-\delta)\right\} = Ae^{\frac{-\delta}{V}} v^2 + Ale^{\frac{-\delta}{V}}[v^3 + \delta v^2] + ABe^{\frac{-\delta}{V}}[2v^4 + 2\delta v^3 + \delta^2 v^2] + ACe^{\frac{-\delta}{V}}[6v^5 + 6\delta v^4 + 3\delta^2 v^3 + \delta^3 v^2] \quad (6)$$

Where

$$A = e^{-\frac{l^2}{2}}, B = \left(-\frac{1}{2} + \frac{l^2}{2}\right), C = \left(-\frac{1}{2} + \frac{l^3}{6}\right).$$

Proof

From equation (3) and the Heaviside step function, we have

$$e^{-\frac{(t-l)^2}{2}} u(t-\delta) = e^{-\frac{l^2}{2}} u(t-\delta) + e^{-\frac{l^2}{2}} ltu(t-\delta) + e^{-\frac{l^2}{2}} \left(-\frac{1}{2} + \frac{l^2}{2}\right) t^2 u(t-\delta)$$

$$+ e^{-\frac{l^2}{2}} \left(-\frac{1}{2} + \frac{l^3}{6}\right) t^3 u(t-\delta) \quad (7)$$

Therefore, by takingElzaki transform to equation (7), we obtain:

$$E\left\{e^{-\frac{(t-l)^2}{2}} u(t-\delta)\right\} = \Gamma_1 E\{u(t-\delta)\} + \Gamma_1 l E\{tu(t-\delta)\} + \Gamma_1 \Gamma_2 E\{t^2 u(t-\delta)\} + \Gamma_1 \Gamma_3 E\{t^3 u(t-\delta)\}$$

$$= Ae^{\frac{-\delta}{V}} v^2 + Ale^{\frac{-\delta}{V}}[v^3 + \delta v^2] + ABe^{\frac{-\delta}{V}}[2v^4 + 2\delta v^3 + \delta^2 v^2]$$

$$+ ACe^{\frac{-\delta}{V}}[6v^5 + 6\delta v^4 + 3\delta^2 v^3 + \delta^3 v^2] = k (8)$$

Where

$$A = e^{-\frac{l^2}{2}}, B = \left(-\frac{1}{2} + \frac{l^2}{2}\right), C = \left(-\frac{1}{2} + \frac{l^3}{6}\right)$$

2 Main Result

Lemma 3.

Elzaki transform of Heaviside step function of a bulgefunction of a piecewise continuous function

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}}, 0 < t < \delta \\ t, t > \delta \end{cases}$$

can be expressed by

$$A[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] + e^{\frac{-\delta}{v}}[v^3 + \delta v^2] + k \qquad (9)$$

whereα, δ are constants.

Proof.

By taking the Elzaki transform to equation (5) and lemma2, we have

$$E\{f(t)\} = E\left\{e^{-\frac{(t-l)^2}{2}}\right\} + E\left\{\left[t - e^{-\frac{(t-l)^2}{2}}\right]u(t-\delta)\right\}$$

$$= E\left\{e^{-\frac{(t-l)^2}{2}}\right\} + E\{tu(t-\delta)\} - E\left\{e^{-\frac{(t-l)^2}{2}}u(t-\delta)\right\} = \Gamma_1[v^2 + lv^3 + (-1 + l^2)v^4 + (-3l + l^3)v^5] + e^{\frac{-\delta}{v}}[v^3 + \delta v^2] - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \qquad (10)$$

Where

$\alpha_1 = \Gamma_1 e^{\frac{-\delta}{v}}v^2, \alpha_2 = \Gamma_1 l e^{\frac{-\delta}{v}}[v^3 + \delta v^2], \alpha_3 = \Gamma_1\Gamma_2 e^{\frac{-\delta}{v}}[2v^4 + 2\delta v^3 + \delta^2 v^2],$

$\alpha_4 = \Gamma_1\Gamma_3 e^{\frac{-\delta}{v}}[6v^5 + 6\delta v^4 + 3\delta^2 v^3 + \delta^3 v^2]$

Lemma 4.

The solution of the non-homogeneous differential equation withconstant coefficients,

$$f(t) = \begin{cases} e^{-\frac{(t-l)^2}{2}}, 0 < t < \delta \\ t, t > \delta \end{cases}$$

Where$y(0) = w_0, y'(0) = w_1$.is expressed by:

$$y(t) = w_0 \cos wt + \frac{w_1}{w}\sin wt + \Gamma_1\left[1 + lt - \frac{t^2}{2} + \frac{t^2 l^2}{2} - \frac{lt^3}{3} + \frac{l^3 t^3}{6}\right] + k_1 + k_2 (11)$$

Where$w$, $w_0$and w_1 are constant. $k_1 = E^{-1}\left\{e^{\frac{-\delta}{v}}[v^3 + \delta v^2]\right\}$,$k_2 = E^{-1}\{K\}$

Proof.

By taking the Elzaki transform to the non-homogeneous differentialequation with constant coefficients and the Heaviside step function and lemma3. we obtain:

$$\frac{E\{y\ \ (t)\}}{v^2} - w_0 - vw_1 + w^2 E\{y(t)\} = E\{Heavisidestepfuctionoff(t)\},$$

Or

$$E\{y(t)\} = \frac{w_0 v^2}{1 + w^2 v^2} + \frac{w_1 v^3}{1 + w^2 v^3} + A\left[1 + lt - \frac{t^2}{2} + \frac{t^2 l^2}{2} - \frac{lt^3}{3} + \frac{l^3 t^3}{6}\right] + e^{\frac{-\delta}{V}}[v^3 + \delta v^2] + K \tag{12}$$

The inverse Elzaki transform can be used to equation (12) to obtain thesolution of the non-homogeneous differential equation with the Heaviside stepfunction as:

$$y(t) = w_0 \cos wt + \frac{w_1}{w} \sin wt + A\left[1 + lt - \frac{t^2}{2} + \frac{t^2 l^2}{2} - \frac{lt^3}{3} + \frac{l^3 t^3}{6}\right] + k_1 + k_2 \tag{13}$$

Where

$$k_1 = E^{-1}\left\{e^{\frac{-\delta}{V}}[v^3 + \delta v^2]\right\}, k_2 = E^{-1}\{K\}$$

3 Conclusion

In this paper, we introduce the Heaviside step function on a bulge functionand formulate Elzaki transform of the Heaviside step function of a bulgefunction. And also we discovered the method to solve theNon-homogeneous second order differential equation with a bulge function involved the Heaviside step function. Elzaki transform, the inverse Elzakitransform and the Power series expansion are used in this method.

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