

## Quantum Algorithm For Minimum Multiprocessor Scheduling With Speed Factors Problem By Numbering Method

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### Abstract

A quantum algorithm for the minimum multiprocessor scheduling with speed factors problem by a numbering method and its example are reported. When  $n$  tasks are parted by  $m$  processors with speed factors  $s_i$  [ $1 \leq i \leq m$ .  $i$  is an integer.], and a sum of length of each task in the  $i$ -th processor is  $t_i$ , it is decided whether  $t_i/s_i$  is a finish time  $M$  or less or not. A computational complexity of a classical computation is  $m^n$ . The computational complexity becomes about  $3(\log_2 m)n$  by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the numbering method. Therefore, a polynomial time process becomes possible.

**AMS subject classification:** Primary 81-08; Secondary 68R05, 68W40.

**Keywords:** Quantum algorithm, minimum multiprocessor scheduling with speed factors problem, numbering method, computational complexity, polynomial time.

### Introduction

Haroche and Wineland [1] developed methods for the very first steps towards building a quantum computer. The algorithms of the quantum computer by Deutsch and Jozsa [2–4], Shor [3–5], Grover [3, 6, 7] and so on are known. A quantum algorithm for the 3-SAT problem by a numbering method has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The minimum multiprocessor scheduling with speed factors problem [9] is examined by the numbering method this time. Therefore, its result is reported.

### Minimum Multiprocessor Scheduling With Speed Factors Problem

When  $n$  tasks are parted by  $m$  processors with speed factors  $s_i$  [ $1 \leq i \leq m$ .  $i$  is an integer.], and a sum of length of each task in the  $i$ -th processor is  $t_i$ , it is decided whether  $t_i/s_i$  is a finish time  $M$  or less or not [9].

### Quantum Algorithm

It is assumed that  $n$  tasks are parted by  $m$  processors [ $1 \leq m \leq n$ .  $m$  is an integer.] with speed factors  $s_i$  [ $1 \leq i \leq m$ .  $i$  is the integer.], each task is  $x((\text{task number}), (\text{processor number of the task})) = x(f, j_f)$  [ $1 \leq f \leq n$ .  $1 \leq j_f \leq m$ .  $f$  and  $j_f$  are integers.] that are lengths, and a sum of length of each task in the  $i$ -th processor is  $t_i$ , it is decided whether  $t_i/s_i$  is the finish time  $M$  or less or not. When a combination  $(x(1, j_1), x(2, j_2), \dots, x(n-1, j_{n-1}), x(n, j_n))$  is labeled  $(j_1-1, j_2-1, \dots, j_{n-1}-1, j_n-1)$ ,  $V(N)$  is the  $N$ -th  $(j_1-1, j_2-1, \dots, j_{n-1}-1, j_n-1)$  [ $0 \leq N \leq m^n - 1$ .  $N$  is an integer.  $V(0)$  is  $(0, 0, \dots, 0, 0)$ .  $V(m^n - 1)$  is  $(m-1, m-1, \dots, m-1, m-1)$ .]. This method is named the numbering method for this problem.  $g$  is the minimum integer that follows  $m^n/1 \leq 4^g = 2^{2g}$ , because a number of combinations of an answer is at least 1.

First of all, quantum registers  $|a_1\rangle, |a_2\rangle, \dots, |a_n\rangle, |b_1\rangle, |b_2\rangle, |c_1\rangle, |c_2\rangle, \dots, |c_m\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  are prepared. When  $P$  is the minimum integer that is  $\log_2 m$  or more, each of  $|a_f\rangle$  that  $f$  is an integer from 1 to  $n$  is consisted of  $P$  quantum bits [= qubits]. States of  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  are  $a_f, b_1, b_2, c_i, d, e_1$  and  $e_2$ , respectively.

Step 1: Each qubit of  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  is set  $|0\rangle$ .

Step 2: The Hadamard gate  $\boxed{H}$  [3, 4] acts on each qubit of  $|a_f\rangle$ . It changes them for entangled states. The total states are  $(2^P)^n$ .

Step 3: It is assumed that a quantum gate ( $A$ ) changes  $|b_1\rangle$  for  $|1\rangle$  in  $a_f < m$ , or it changes  $|b_1\rangle$  for  $|0\rangle$  in the others of  $a_f$ , it changes  $|b_2\rangle$  for  $|a_1, a_2, \dots, a_{f-1}, a_f\rangle$  at  $|a_f\rangle$ , and it changes  $|c_i\rangle$  for  $|c_i + x(f, a_f + 1)\rangle$  at  $a_f = i - 1$ . As a target state for  $|b_1\rangle$  is 1, quantum phase inversion gates ( $PI$ ) and quantum inversion about mean gates ( $IM$ ) [3, 6, 7] act on  $|b_1\rangle$ . When  $Q$  is the minimum even integer that is  $(2^P/m)^{1/2}$  or more, the total number that ( $PI$ ) and ( $IM$ ) act on  $|b_1\rangle$  is  $Q$  because they are a couple. Next, an observation gate ( $OB$ ) observes  $|b_1\rangle$ . These actions are repeated sequentially from  $|a_1\rangle$  to  $|a_n\rangle$ . Therefore, each state of  $|a_f\rangle$  is 0, 1,  $\dots, m-2$  or  $m-1$ , and the total states become  $m^n$  [=  $W_0$ ].

Step 4: It is assumed that a quantum gate ( $B$ ) changes  $|d\rangle$  for  $|d+1\rangle$  in  $c_i/s_i$  ( $= t_i/s_i$ )  $\leq M$ , or it doesn't change  $|d\rangle$  in the others of  $c_i$ . These actions are repeated sequentially from  $|c_1\rangle$  to  $|c_m\rangle$ .

Step 5: It is assumed that a quantum gate ( $C$ ) changes  $|e_1\rangle$  for  $|V(m^n - 1)\rangle$  at  $d = m$ , or it changes  $|e_1\rangle$  for  $|b_2\rangle$  in the others of  $d$ .

Step 6: It is assumed that a quantum gate ( $D_1$ ) changes  $|e_2\rangle$  for  $|1\rangle$  in  $V(0) = (0, 0, \dots, 0, 0) \leq e_1 \leq V((m^n/4) - 2)$  or  $e_1 = V(m^n - 1)$ , or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1, ( $PI$ ) and ( $IM$ ) act on  $|e_2\rangle$ . The number of the data that is included in  $V(0) \leq e_1 \leq V((m^n/4) - 2)$  or  $e_1 = V(m^n - 1)$  is  $W_1 \approx m^n/4$ . When  $R_1$  is the minimum even integer that is  $(W_0/W_1)^{1/2} \approx (m^n/(m^n/4))^{1/2}$  or more, the total number that ( $PI$ ) and ( $IM$ ) act on  $|e_2\rangle$  is  $R_1 \approx 2$ . Next, ( $OB$ ) observes  $|e_2\rangle$ , and

the data of  $W_1$  remain. Similarly,  $(D_h)$  [ $2 \leq h \leq g - 1$ .  $h$  is an integer.] changes  $|e_2\rangle$  for  $|1\rangle$  in  $V(0) \leq e_1 \leq V((m^n/4^h) - 2)$  or  $e_1 = V(m^n - 1)$ , or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$ . The number of the data that is included in  $V(0) \leq e_1 \leq V((m^n/4^h) - 2)$  or  $e_1 = V(m^n - 1)$  is  $W_h \approx m^n/4^h$ . When  $R_h$  is the minimum even integer that is  $(W_{h-1}/W_h)^{1/2} \approx ((m^n/4^{h-1})/(m^n/4^h))^{1/2}$  or more, the total number that  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$  is  $R_h \approx 2$ . Next,  $(OB)$  observes  $|e_2\rangle$ , and the data of  $W_h$  remain. These actions are repeated sequentially from 2 to  $g - 1$  at  $h$ .  $(D_g)$  changes  $|e_2\rangle$  for  $|1\rangle$  at  $e_1 = V(m^n - 1)$ , or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$ . The number of the data that is included at  $e_1 = 0$  is  $W_g \approx 1 \approx m^n/4^g$ . When  $R_g$  is the minimum even integer that is  $(W_{g-1}/W_g)^{1/2} \approx ((m^n/4^{g-1})/(m^n/4^g))^{1/2}$  or more, the total number that  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$  is  $R_g \approx 2$ . Next,  $(OB)$  observes  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$ , and one of the data of  $W_g$  remains. Therefore, one example of combinations that are  $c_i/s_i \leq M$  is obtained.

### Numerical Computation

It is assumed that there are  $n = 6$ ,  $x(f, j_f)$  [ $1 \leq f \leq 6$ .  $1 \leq j_f \leq 3$ .  $f$  and  $j_f$  are integers.  $x(1, j_1) = 1$ ,  $x(2, j_2) = 2$ ,  $x(3, j_3) = 3$ ,  $x(4, j_4) = 4$ ,  $x(5, j_5) = 5$ ,  $x(6, j_6) = 6$ .],  $m = 3$ ,  $s_1 = 1$ ,  $s_2 = 2$ ,  $s_3 = 3$ ,  $M = 4$ ,  $g = 5$ ,  $1 \leq i \leq 3$  [ $i$  is the integer.],  $V(728) = (2, 2, 2, 2, 2, 2)$ ,  $V(180) = (0, 2, 0, 2, 0, 0)$ ,  $V(44) = (0, 0, 1, 1, 2, 2)$ ,  $V(9) = (0, 0, 0, 1, 0, 0)$  and  $V(1) = (0, 0, 0, 0, 0, 1)$ .

First of all,  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  are prepared. When  $P$  is the minimum integer that is  $\log_2 3 \approx 1.6 \leq 2 = P$ , each of  $|a_f\rangle$  that  $f$  is the integer from 0 to 5 is consisted of 2 qubits. States of  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  are  $a_f, b_1, b_2, c_i, d, e_1$ , and  $e_2$ , respectively.

Step 1: Each qubit of  $|a_f\rangle, |b_1\rangle, |b_2\rangle, |c_i\rangle, |d\rangle, |e_1\rangle$  and  $|e_2\rangle$  is set  $|0\rangle$ .

Step 2:  $\boxed{H}$  acts on each qubit of  $|a_f\rangle$ . It changes them for entangled states. The total states are  $(2^2)^6$ .

Step 3:  $(A)$  changes  $|b_1\rangle$  for  $|1\rangle$  in  $a_f < 3$ , or it changes  $|b_1\rangle$  for  $|0\rangle$  in the others of  $a_f$ , it changes  $|b_2\rangle$  for  $|a_1, a_2, \dots, a_{f-1}, a_f\rangle$  at  $|a_f\rangle$ , and it changes  $|c_i\rangle$  for  $|c_i + x(f, a_f + 1)\rangle$  at  $a_f = i - 1$ . As the target state for  $|b_1\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|b_1\rangle$ . When  $Q$  is the minimum even integer that is  $(2^2/3)^{1/2} \approx 1.2 \leq 2 = Q$ , the total number that  $(PI)$  and  $(IM)$  act on  $|b_1\rangle$  is  $Q \approx 2$ . Next,  $(OB)$  observes  $|b_1\rangle$ . These actions are repeated sequentially from  $|a_1\rangle$  to  $|a_6\rangle$ . Therefore, each state of  $|a_f\rangle$  is 0, 1 or 2, and the total states become  $3^6 [= W_0]$ .

Step 4:  $(B)$  changes  $|d\rangle$  for  $|d + 1\rangle$  in  $c_i/s_i (= t_i/s_i) \leq 4$ , or it doesn't change  $|d\rangle$  in the others of  $c_i$ . These actions are repeated sequentially from  $|c_1\rangle$  to  $|c_3\rangle$ .

Step 5:  $(C)$  changes  $|e_1\rangle$  for  $|V(728)\rangle$  at  $d = 3$ , or it changes  $|e_1\rangle$  for  $|b_2\rangle$  in the others of  $d$ .

Step 6:  $(D_1)$  changes  $|e_2\rangle$  for  $|1\rangle$  in  $V(0) = (0, 0, 0, 0, 0, 0) \leq e_1 \leq V((3^6/4) - 2) \approx V(180) = (0, 2, 0, 2, 0, 0)$  or  $e_1 = V(728)$ , or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$ . The number of the data that is included in  $V(0) \leq e_1 \leq V(180)$  or  $e_1 = V(728)$  is  $W_1 \approx 3^6/4$ . When  $R_1$  is the minimum even integer that is  $(W_0/W_1)^{1/2} \approx (3^6/(3^6/4))^{1/2} \approx 2 \leq 2 = R_1$ , the total

number that  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$  is  $R_1 \approx 2$ . Next,  $(OB)$  observes  $|e_2\rangle$ , and the data of  $W_1$  remain. Similarly,  $(D_h)$  [ $2 \leq h \leq 4$ .  $h$  is the integer.] changes  $|e_2\rangle$  for  $|1\rangle$  in  $V(0) \leq e_1 \leq V((3^6/4^h) - 2)$  or  $e_1 = V(728)$  [ $h = 2 : V((3^6/16) - 2) \approx V(44)$ .  $h = 3 : V((3^6/64) - 2) \approx V(9)$ .  $h = 4 : V((3^6/256) - 2) \approx V(1)$ .], or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$ . The number of the data that is included in  $V(0) \leq e_1 \leq V((3^6/4^h) - 2)$  or  $e_1 = V(728)$  is  $W_h \approx 3^6/4^h$ . When  $R_h$  is the minimum even integer that is  $(W_{h-1}/W_h)^{1/2} \approx ((3^6/4^{h-1})/(3^6/4^h))^{1/2} \approx 2 \leq 2 = R_h$ , the total number that  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$  is  $R_h \approx 2$ . Next,  $(OB)$  observes  $|e_2\rangle$ , and the data of  $W_h$  remain. These actions are repeated sequentially from 2 to 4 at  $h$ .  $(D_5)$  changes  $|e_2\rangle$  for  $|1\rangle$  at  $e_1 = V(728)$ , or it changes  $|e_2\rangle$  for  $|0\rangle$  in the others of  $e_1$ . As the target state for  $|e_2\rangle$  is 1,  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$ . The number of the data that is included at  $e_1 = V(728)$  is  $W_5 \approx 1 \approx 3^6/4^5$ . When  $R_5$  is the minimum even integer that is  $(W_4/W_5)^{1/2} \approx ((3^6/4^4)/(3^6/4^5))^{1/2} \approx 2 \leq 2 = R_5$ , the total number that  $(PI)$  and  $(IM)$  act on  $|e_2\rangle$  is  $R_5 \approx 2$ . Next,  $(OB)$  observes  $|a_f\rangle$ ,  $|b_1\rangle$ ,  $|b_2\rangle$ ,  $|c_i\rangle$ ,  $|d\rangle$ ,  $|e_1\rangle$  and  $|e_2\rangle$ , and one of the data of  $W_5$  remains. For example, when  $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, c_1, c_2, c_3, d, e_1$  and  $e_2$  are 1, 1, 2, 0, 1, 2, 1, (1, 1, 2, 0, 1, 2), 4, 8, 9, 3, (2, 2, 2, 2, 2, 2) and 1, respectively, it is obtained that tasks of the 1st, 2nd and 3rd processors are (4), (1, 2, 5) and (3, 6), respectively.

## Discussion and Summary

The computational complexity of this quantum algorithm [=  $S$ ] becomes the following. In the order of the actions by the gates, the number of them is  $Pn$  at  $\square$ ,  $n$  at  $(A)$ ,  $Qn \approx 2n$  at  $(PI)$  and  $(IM)$ ,  $n$  at  $(OB)$ ,  $m$  at  $(B)$ , 2 at  $(C)$ ,  $g$  at  $(D_h)$  [ $1 \leq h \leq g$ .  $h$  is the integer.],  $\sum_{h=1 \rightarrow g} R_h \approx 2g$  at  $(PI)$  and  $(IM)$ , and  $g$  at  $(OB)$ . Therefore,  $S$  becomes  $(P + 4)n + m + 2 + 4g$ . In the example of the numerical computation,  $S$  is 61. The computational complexity of the classical computation [=  $Z$ ] is  $m^n = 3^6 = 729$ . After all,  $S/Z$  becomes about 1/12. When  $n$  is large enough,  $S$  becomes about  $3(\log_2 m)n$ , where  $P$  is about  $\log_2 m$ , and  $g$  is about  $(n/2)\log_2 m$ . And then,  $S/Z$  is about  $3(\log_2 m)n/m^n \approx n/m^n$ . For example, as for  $n = 100$  and  $m = 5$ ,  $S/Z$  is about  $100/5^{100} \approx 1/10^{68}$ .

Therefore, the polynomial time process becomes possible.

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