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Heuristic Algorithm for finding Sensitivity Analysis of a More for Less Solution to Transportation Problems

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Abstract

In this paper, a new algorithm is proposed for finding the costsensitivity analysiswhich determines the interval of perturbation to keep the current more-for-less (MFL) optimal solution to the transportation problem remaining optimal. The existence of a more-for-less situation in distribution problems provides useful information to a manager in deciding on the plant in which capacities are to be increased, and to assess the markets in which it is worthwhile to pursue efforts to increase demand.

Keywords: Transportation problem, MFL optimal solution,cost sensitivity analysis

1. Introduction

The transportation problem (TP) considers minimum-cost planning problems for shipping a product from origins to destinations. The MFL paradox in a TP occurs when it is possible to transport more total goods for less (or equal) total cost while transporting the same quantity or more from each source and to each destination, keeping all transporting costs non-negative. In literature, many researchers [1-5] have developed various algorithms for finding MFL solution to the Transportation problem. Pandian and Anuradha [6] have introduced path method for finding a MFL optimal solution to a TP. Sensitivity analysis(SA) in TP is used to acquire information about how decisions are affected as the input data are varied. For instance, when the cost of an activity or the available amount of resources is changed, we often need information about how the total cost of the current decision is altered, in order to obtain a new optimal decision for the new situation. In this case, SA can be applied.In literature many researchers [7-13] have examined the performance of the SA in different types of transportation problems.

This paper is organized as follows: section 2 and 3 projects the basics of transportation problem and MFL optimal solution. In section 4, we determine the theorems about the sensitivity interval of allocated and unallocated cells to the MFL solution of the TP and we describe our proposed algorithm. A numerical example is solved in section 5. Finally the paper is concluded in section 6.

2. Preliminaries

Consider the following transportation problem

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1, 2, \ldots, m;$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1, 2, \ldots, n;$$

$$x_{ij} \geq 0, \; i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

where m is the number of supply (S) points; n is the number of demand (D) points; x_{ij} is the number of units shipped from supply point i to demand point j; a_i is the supply at supply point i and b_j is the demand at demand point j.

3. MFL Optimal Solution

According to Arsham [8],if Modi index is negative (non-negative) at each cell (i, j) , then one can (cannot) increase the ith plant capacity/the demand of the jthmarket , that is, the current level is their maximum level.

4. Cost Sensitivity Analysis

Costsensitivity analysis in the objective function coefficient of unallocated variables of the TP will not alter the existing MFL solution, as the current cost itself is too high and there has been no allocation along this route. Cost SA in the objective function coefficient of allocated variables of the TP is likely to change the transportation schedule. Unlike in the earlier case, this will disturb the allocations to more than one cell. In this case, the TP is re-studied with the current MFL solution such that the MFL optimal solution is invariant. In TP, the interval of costs coefficients are a function of *k*MODI indices where $k = (m + n)$ is the number of unallocated cells in the

MFL solution. Using the optimal conditions, we obtain the intervals of the cost in the TP.

Now, we need the following theorems which is used in the proposed method

Theorem 4.1: Let $(i,j)^{th}$ position be an unallocated cell corresponding to an MFL solution of the TP with $\delta_{ij} = d_{ij} - u_i - v_j (\geq 0)$. If $d_{ij} + \Delta d_{ij}$ is the perturbed cost of d_{ij}, then the interval of $\Delta d_{ij} = [-\delta_{ij}, \infty)$.

Proof: Now, since $(i,j)^{th}$ positionis aunallocated cell and the perturbed cost $d_{ij} + \Delta d_{ij}$ is not affected the current MFL solution to the problem, $d_{ij} + \Delta d_{ij} - u_i - v_j \geq 0$. This implies, $\Delta d_{ij} \geq -\delta_{ij}$. Therefore, the interval of Δd_{ij}= $[-\delta_{ij}, \infty)$.Hence the theorem.

Theorem 4.2: Let $(i,j)^{th}$ position be an allocated cell corresponding to an MFL solution of the TP with $\delta_{ij} = d_{ij} - u_i - v_j (= 0)$. If $d_{ij} + \Delta d_{ij}$ is the perturbed value of d_{ij} and U_i is theleast value of δ_{ij} for all unallocatedcells in the i^{th} source and V_j is the least value of δ_{ij} for all unallocatedcells in the j^{th} destination, then the interval of $\Delta d_{ij} = (-\infty, M_{ij}]$ where $M_{ij} =$ maximum$\{U_i, V_j\}$.

Proof: Now, since $d_{ij} + \Delta d_{ij}$ is the perturbed value of d_{ij} and the current MFL solution remains optimal, $\delta_{ij} = d_{ij} - u_i - v_j \geq 0$, for all unallocated cells in ithsource and jth destination are positive.

Now, attaching Δd_{ij} to first u_i then v_j , we have the following:

$d_{is} - (u_i + \Delta d_{ij}) - v_s \geq 0$, (i,s) is unallocated cells, for all s and

$d_{rj} - u_r - (v_j + \Delta d_{ij}) \geq 0$, (r,j) is unallocated cells, for all r .

Thus, we can conclude on the basis of the above implications that $\Delta d_{ij} \leq U_i$ and $\Delta d_{ij} \leq V_j$.

We choose, $M_{ij} =$ maximum $\{U_i, V_j\}$ for getting some better interval. Therefore, the intervalof $\Delta d_{ij} = (-\infty, M_{ij}]$.Hence the theorem.

A heuristic algorithm for determiningthe costSA of aMFL optimal solution to the TP is proposed below:

Step 1: Findthe MFL optimal solution to the given problem using the path method [6].
Step 2: Find the value $k = (m+n) -$ number of non zero allocated cells in the MFL optimalsolution. We construct the MODI indices table in parameters, in which we take $u_i = \theta_i$, $i = 1,2,...,k$ (if $m < n$); otherwise $v_i = \theta_i$ $i = 1,2,...,k$ such that

$d_{ij}-(u_i+v_j)=0$ for all allocated cells (i, j) and $d_{ij}-(u_i+v_j)\geq 0$ for all unallocated cells (i, j) for the MFL optimal solution to the given TP.

Step 3: Calculate the relations among MODI indices parameters $\theta_i,\ i=1,2,...,k$ and their intervalsusing the parametric MODI indices table and the optimality conditions $d_{ij}-(u_i+v_j)\geq 0$ for all unallocatedcells (i, j).

Step 4: (a) If (i, j) is an unallocated cell and $d_{ij}+\Delta d_{ij}$ is the $(i, j)^{\text{th}}$ cost of the perturbed problem,we find the interval of Δd_{ij} using the optimality condition $(d_{ij}+\Delta d_{ij})-(u_i+v_j)\geq 0$.

(b)If (i, j) is an allocated cell and $d_{ij}+\Delta d_{ij}$ is the $(i, j)^{\text{th}}$ cost of the perturbed problem, wefind the intervals of Δd_{ij} using the optimality conditions

(i) For any fixed i and for any $s=1,2,j-1,j+1,...,m$;
$(d_{is}+\Delta d_{is})-(u_i+(v_j+\Delta d_{is}))\geq 0$

(ii) For any fixed j and for any $t=1,2,i-1,i+1,...,n$;
$(d_{tj}+\Delta d_{tj})-((u_i+\Delta d_{tj})+v_j)\geq 0$

Step 5: Compute the intervals of Δd_{ij} of all unallocated cells using the Theorem 4.1 and then, calculate the intervals of Δd_{ij} of all allocated cells using the Theorem 4.2 to the given problem.

5. Numerical Example:

Consider the following transportation problem in Table 1.

Table 1. Transportation problem

	D_1	D_2	D_3	D_4	D_5	S
O_1	6	17	8	15	16	9
O_2	18	11	24	15	18	24
O_3	10	7	13	6	7	10
O_4	14	6	20	11	12	19
D	6	15	16	10	15	

Now, we obtain the following MFL optimal solution to the TP by the path method [6] in Table 2.

Table 2. MFL Optimal solution

	D_1	D_2	D_3	D_4	D_5	S
O_1	6	17	8 (16)	15	16	16
O_2	18 (6)	11 (8)	24	15(10)	18	24
O_3	10	7	13	6	7 (15)	15
O_4	14	6 (19)	20	11	12	19
D	6	27	16	10	15	

Using Step 2, the MODI index matrix corresponding to the above solution is given below in Table 3.

Table 3. MODI Index matrix

	$v_1 = 18 - \theta_2$	$v_2 = 11 - \theta_2$	$v_3 = 8 - \theta_1$	$v_4 = 15 - \theta_2$	$v_5 = 7 - \theta_3$	S
$u_1 = \theta_1$	6	17	8 (16)	15	16	16
$u_2 = \theta_2$	18 (6)	11 (8)	24	15(10)	18	24
$u_3 = \theta_3$	10	7	13	6	7 (15)	15
$u_4 = -5 + \theta_2$	14	6 (19)	20	11	12	19
D	6	27	16	10	15	

Using Step 3, we obtain the following relations for unallocated cells as $\theta_1 - \theta_2 \le -12$; $\theta_1 - \theta_2 \le 6$; $\theta_1 - \theta_2 \le 0$; $\theta_1 - \theta_3 \le 9$; $\theta_2 - \theta_1 \le 16$; $\theta_2 - \theta_3 \le 11$; $\theta_3 - \theta_2 \le -8$; $\theta_3 - \theta_2 \le -4$; $\theta_3 - \theta_1 \le 5$; $\theta_3 - \theta_2 \le 9$; $\theta_2 - \theta_1 \le 17$ and $\theta_2 - \theta_3 \le 10$. Now, the corresponding limits for the above relation is $-16 \le \theta_1 - \theta_2 \le -12$; $-5 \le \theta_1 - \theta_3 \le 9$; $8 \le \theta_2 - \theta_3 \le 10$ (1)

Using Step 4(a), we determine the interval of an unallocated cell. Now, we consider the unallocated cell (1,1) has the optimality condition $(6 + \Delta d_{11}) - (\theta_1 + 18 - \theta_2) \ge 0.$ Using equation (1), we get $\Delta d_{11} \ge 12 + \theta_1 - \theta_2 \ge 12 - 16 = -4.$ Hence by using Step 5, the sensitivity range of an unallocated cell (1,1) is $[-4, \infty)$. Proceeding in this same manner, we can find the sensitivity range of other unallocated cells.

Using Step 4(b), we determine the interval of anallocated cell. Now, we consider the allocated cell (1,3), and by using the optimality condition (i), we have

$24 - [\theta_2 + (8 - \theta_1 + \Delta d_{13})] \ge 0$, using (1) we get $\Delta d_{13} \le 16 - \theta_2 + \theta_1 \le 16 - 12 = 4$

$13 - [\theta_3 + (8 - \theta_1 + \Delta d_{13})] \ge 0$, using (1) we get $\Delta d_{13} \le 5 - \theta_3 + \theta_1 \le 5 + 9 = 14$

$20 - [-5 + \theta_2 + (8 - \theta_1 + \Delta d_{13})] \ge 0$, using (1) we get $\Delta d_{13} \le 17 - \theta_2 + \theta_1 \le 17 - 12 = 5$

Similarly, by using the optimality condition (ii), we have

$6-[(\theta_1+\Delta d_{13})+18-\theta_2]\ge 0$, using (1) we get $\Delta d_{13}\le -12-\theta_1+\theta_2\le -12+16=4$

$17-[(\theta_1+\Delta d_{13})+11-\theta_2]\ge 0$, using (1) we get $\Delta d_{13}\le 6-\theta_1+\theta_2\le 6+16=22$

$15-[(\theta_1+\Delta d_{13})+15-\theta_2]\ge 0$, using (1) we get $\Delta d_{13}\le -\theta_1+\theta_2\le 16$

$16-[(\theta_1+\Delta d_{13})+7-\theta_3]\ge 0$, using (1) we get $\Delta d_{13}\le 9-\theta_1+\theta_3\le 9-9=0$.

Hence by using Step 5, the sensitivity range of an allocatedcell (1,3) is $(-\infty,4]$. Proceeding in this same manner, we can find the sensitivity range of other allocated cells.

Now, the sensitivity ranges of all $\Delta d_{ij}\,'s$ to the given transportation problem are shown in the following table 4.

Table 4. Sensitivity ranges of all $\Delta d_{ij}\,'s$

$[-4,\infty)$	$[-22,\infty)$	$(-\infty,4]$	$[-16,\infty)$	$[-14,\infty)$
$(-\infty,3]$	$(-\infty,6]$	$[-4,\infty)$	$(-\infty,3]$	$[-1,\infty)$
$[0,\infty)$	$[-6,\infty)$	$[-14,\infty)$	$[-1,\infty)$	$(-\infty,2]$
$[1,\infty)$	$(-\infty,6]$	$[-5,\infty)$	$[-1,\infty)$	$[-2,\infty)$

6. Conclusion

Anew algorithm for computing the cost sensitivity ranges of transportation problems is considered, when the MFL optimal solution is invariant.Costsensitivity analysis of a transportation problem can obtain an exact perturbation range, which keeps the variables with positive MFL optimal solutions still positive and zero variables still unchanged. This method can help the decision makers in the logistics related issues of real life problems.

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