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Special Pythagorean Triangles and 5-Digit Dhuruva Numbers

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Abstract

Pythagorean triangles, each with a leg represented by a 5-digit Dhuruva number are obtained. A few interesting results are given.

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Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-22].

In [23-25], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [26], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. In [27], Pythagorean triangles in connections with 5 digit dhuruva numbers, namely, 53955,59994 are given. However we have four more 5 digit durva numbers, Thus the objective of this paper is to find out the special Pythagorean triangles in connection with 5 digit Dhuruva numbers, namely, $N_1 = 75933, N_2 = 61974, N_3 = 82962, N_4 = 63954$.

Also, a few special Pythagorean triangles in connection with 75933, 61974, 82962, 63954 are obtained.

Basic Definitons:

Definition 2.1:

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2:

Most cited solution of the Pythagorean equation is $y = m^2 - n^2, x = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

Method of Analysis:

Consider the dhuruva number $N_1 = 75933$.

To start with, it is noted that the leg x cannot be represented by N_1 as x is even and N_1 is odd. Also z cannot be written as sum of two squares, since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P = p_1^{r_1} p_2^{r_2} .... p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 (\bmod 4)$ then r_i is even. A prime $p \equiv 1 (\bmod 4)$ can be written as $p = a^2 + b^2$

Now, consider $y = N_1 => m^2 - n^2 = 75933$

which is a binary quadratic Diophantine equation. Solving the above equation for m, n we get 12 integer solutions and thus, we have 12 Pythagorean triangles, each having the leg y to be represented by the 5 digit Dhuruva number $N_1 = 75933$ as shown in the table below:

m	n	x	y	z	A	P
12657	12654	320323356	75933	320323365	12161556695574	640722654
4223	4214	35591444	75933	35591525	1351282558626	71258902
3457	3446	23825644	75933	23825765	904576312926	47727342
2927	2914	17058556	75933	17058725	647653666374	34193214
1167	1134	2646756	75933	2647845	100488061674	5370534

993	954	1894644	75933	1896165	71933001426	3866742
673	614	826444	75933	829925	31377186126	1732302
433	334	289244	75933	299045	10981582326	664222
383	266	203756	75933	217445	7735902174	497134
337	194	130756	75933	151205	4964347674	357894
303	126	76356	75933	107685	2898970074	259974
37967	37966	2882910244	75933	2882910245	109454011778826	5765896422

Note that there are 8 primitive and 4 non-primitive triangles. Also the expression

$$\frac{4A}{P} - x + z$$

represents a 5-digit Dhuruva number for each of the above Pythagorean triangles, where A and P represent the area and perimeter of the Pythagorean triangle.

In a similar manner, it is seen that there are 24 Pythagorean triangles wherein, each of the following expressions $\frac{2A}{P}, \frac{1}{2}(y+x-z)$ represents 75933 as shown in the table below.

m	n	x	y	z	A	P
75934	1	151868	5765972355	5765972357	437833344804570	11532096580
25314	3	151884	640798587	640798605	48663526293954	1281749076
8446	9	152028	71334835	71334997	5422446147690	142821860
6914	11	152108	47803275	47803517	3635630276850	95758900
5854	13	152204	34269147	34269485	2607950624994	68690836
2334	33	154044	5446467	5448645	419497781274	11049156
1986	39	154908	3942675	3945717	305375949450	8043300
1346	59	158828	1808235	1815197	143599174290	3782260
866	99	171468	740155	759757	63456448770	1671380
766	117	179244	573067	600445	51359410674	1352756
674	143	192764	433827	474725	41813113914	1101316
606	177	214524	335907	398565	36030056634	948996
606	429	519948	183195	551277	47625936930	1254420
674	531	715788	172315	736237	61670504610	1624340
766	649	994268	165555	1007957	82303019370	2167780
866	767	1328444	161667	1338245	107382778074	2828356
1346	1287	3464604	155347	3468085	269107918794	7088036
1986	1947	7733484	153387	7735005	593107955154	15621876
2334	2301	10741068	152955	10742157	821450027970	21636180
5854	5841	68386428	152035	68386597	5198565290490	136925060
6914	6903	95454684	151987	95454805	7253935528554	191061476
8446	8437	142517804	151947	142517885	10827576382194	285187636
25314	25311	1281445308	151875	1281445317	97309753076250	2563042500
75934	75933	11531792844	151867	11531792845	875649391919874	23063737556

Note that there are 16 primitive and 8 non-primitive triangles.

Also, it is observed that there are 12 Pythagorean triangles wherein each of the expressions $y-\frac{2A}{P},\frac{1}{2}(z+y-x)$ is represented by 75933 as shown in the table below:

m	n	x	y	Z	A	P
25311	25308	1281141576	151857	1281141585	97275158153316	2562435018
8437	8428	142214072	151785	142214153	10792981459260	284580010
6903	6892	95150952	151745	95151073	7219340605620	190453770
5841	5828	68082696	151697	68082865	5163970367556	136317258
2301	2268	10437336	150777	10438425	786855105036	21026538
1947	1908	7429752	150345	7431273	558513032220	15011370
1287	1228	3160872	148385	3164353	234512995860	6473610
767	668	1024712	142065	1034513	72787855140	2201290
649	532	690536	138177	704225	47708096436	1532938
531	388	412056	131417	432505	27075581676	975978
429	252	216216	120537	247545	13031013996	584298
75933	75932	11531489112	151865	11531489113	875614796996940	23063130090

Note that there are 8 primitive and 4 non-primitive triangles.

For simplicity, we exibit the connections between special phythagorean triangles and the five digit dhuruva numbers N2,N3 and N4 in the following table respectively.

Dhurva Numbers	Expressions	Number of Phythagorean triangles	Remark
$N_2=61974$	$\{(\frac{4A}{P}-x+z),x\}$ $\{\frac{2A}{P},\frac{1}{2}(y+x-z)\}$ $\{y-\frac{2A}{P},\frac{1}{2}(z+y-x)\}$	6 24 12	6-Non primitive (6-primitive, 18-Non primitive) (4-primitive, 8-Non primitive)
$N_3=82962$	$\{(\frac{4A}{P}-x+z),x\}$ $\{\frac{2A}{P},\frac{1}{2}(y+x-z)\}$ $\{y-\frac{2A}{P},\frac{1}{2}(z+y-x)\}$	6 24 12	6-Non primitive (9-primitive, 15-Non primitive e) (4-primitive, 8-Non primitive)
$N_4=63954$	$\{(\frac{4A}{P}-x+z),x\}$	12	12-Non primitive

	$\{\frac{2A}{P}, \frac{1}{2}(y+x-z)\}$ $\{y-\frac{2A}{P}, \frac{1}{2}(z+y-x)\}$	48 24	(16-primitive, 32-Non primitive) (10-primitive, 14-Non primitive)

Conclusion:

In this paper, we have presented the relations between special Pythagorean triangles and five digit dhuruva numbers. To conclude, one may search for the relations between Pythagorean triangles and higher order dhuruva numbers.

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