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New Definition Of The Imaginary Unit And Proof Of The Riemann Hypothesis

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Abstract

In this paper, we present a new definition of the imaginary unit and the proof of the Riemann hypothesis as well. We prove that the divergent series 1+1+1+1+... is equivalent to the divergent series 1+2+3+4+..., and we know that one can assign a finite value -1/12 to the series 1+2+3+4+..., and also can assign a finite value -1/2 to the series 1+1+1+1+... by using the analytic continuation of the Riemann zeta function. Since the two series are both equivalent, then the finite values of the two series are both equivalent as well, then ($\frac{-1}{12} = \frac{-1}{2}$), which gives ($\frac{1}{2} = 3$). By applying this wonderful result on the definition of the imaginary unit we obtain : $(-1)^{\frac{1}{2}} = (-1)^3$, then (i = $\sqrt{-1}$ = -1), which is considered a new definition of the imaginary unit, which means that the imaginary unit could be equal to a real number, and hence the complex numbers could be equal to the real numbers. After that we prove that the fraction 1/2 equals any number, so the real part of the complex zeros of the Riemann zeta function could be equal to 1/2, then the complex zeros of the Riemann zeta function can be written in the form of 1/2+it, which proves the Riemann hypothesis.

Keywords: Riemann zeta function; Riemann hypothesis; Zeta function regularization; Imaginary unit

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1. Introduction

The Riemann hypothesis, proposed by Bernhard Riemann (1859), is a conjecture that the non-trivial zeros of the Riemann zeta function all have real part 1/2.Thus the non-trivial zeros should lie on the critical line consisting of the complex numbers 1/2+ it, where t is a real number and i is the imaginary unit.[1]

2. The Riemann zeta function

The Riemann zeta function is defined as:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\cdots \qquad (2.1)$$

3. Zeta function regularization

In zeta function regularization, the series $\sum_{n=1}^{\infty} n$ is replaced by the series $\sum_{n=1}^{\infty}(n)^{-s}$.The latter series is an example of a Dirichlet series. When the real part of s is greater than 1, the Dirichlet series converges, and its sum is the Riemann zeta function (s). On the other hand, the Dirichlet series diverges when the real part of s is less than or equal to 1, so, in particular, the series 1 + 2 + 3 + 4 +... that results from setting s = -1 does not converge. The benefit of introducing the Riemann zeta function is that it can be defined for other values of s by analytic continuation. One can then defined the zeta-regularized sum of 1 + 2 + 3 + 4 +... to be $\zeta(-1)$, which equals -1/12, and similarly defined the zeta-regularized sum of 1 + 1 + 1 + 1 + ... to be $\zeta(0)$, which equals -1/2. [2][3]

4. The relationship between the series 1+2+3+4+... , and the series 1+1+1+1+...

We know that each positive integer n has 2^{n-1} distinct compositions[4] (e.g.(2=2,1+1), (3=3,2+1,1+2,1+1+1), (4=4,3+1,1+3,2+2,2+1+1,1+2+1,1+1+2,1+1+1+1)),… ,etc. Now the series 1+2+3+4+… can be written in many equivalent forms such as: 1+2+(2+1)+(2+2)+(3+2)+… or 1+(1+1)+3+4+… or 1+2+3+(1+1+1+1)+... or 1+1+1+1+1+1+1+1+1+1+...,...,etc. We showed in the previous section that one can assign the finite value -1/2 to the series 1+1+1+1+..., and the finite value of -1/12 to the series 1+2+3+4+... by using the analytic continuation of the Riemann zeta function, and we showed that the series 1+2+3+4+... can be written in many equivalent forms and the only form that have a finite value is the form: "1+1+1+1+..." So, the series 1+1+1+1+… is considered the only equivalent form to the series 1+2+3+4+… that has a finite value by using the analytic continuation of the Riemann zeta function, and the other equivalent forms of the series 1+2+3+4+… do not have finite values. The reason will be explained as follows: We agree that the Riemann zeta function converges for all complex numbers s with real part greater than1 :

$$\zeta(s) = \sum_{n=1}^{\infty}(n)^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \ldots$$

Now let's assigns the values of s that make the above series diverges:

Let's put s=0 in the above series we obtain: ζ(0) = 1 + 1 + 1 + 1 + … (which is considered an equivalent form of the series 1+2+3+4+...).

Let's put s= -1 in the above series we obtain: ζ (-1) = 1 + 2 + 3 + 4 + …

Let's put s= -2 in the above series we obtain: ζ (-2) = 1 + 4 + 9 + … (which is not an equivalent form of the series 1+2+3+4+... because the second term of the series 1+2+3+4+... should be (2 or 1+1).

Let's put s= -3 in the above series we obtain: ζ (-3) = 1 + 8 + 27 + …(which is not an equivalent form of the series 1+2+3+4+... because the second term of the series 1+2+3+4+... should be (2 or 1+1).And so on for all negative integers.

Let's put s= 1 in the above series we obtain: ζ (1) = 1 + 1/2 + 1/3 + …(which is not an equivalent form of the series 1+2+3+4+... because the second term of the series 1+2+3+4+... should be (2 or 1+1). From the above explanation, one can deduce that by setting s = 0 in the Riemann zeta function can produce the only equivalent form to the divergent series1+2+3+4+..., which is the series 1+1+1+1+..., and the other values of s (i.e. except s=-1) that make the Riemann zeta function diverge cannot produce the equivalent forms of the divergent series 1+2+3+4+..., so, the other equivalent forms of the series 1+2+3+4+... such as: 1+2+(2+1)+(2+2)+... or 1+(1+1)+3+4+... or1+2+3+(1+1+1+1)+... or ...,etc. could not have finite values.

5. New definition of the imaginary unit

We showed in the previous section that the only form that equivalent to the divergent series1+2+3+4+... and have a finite value by using the analytic continuation of the Riemann zeta function is the series 1+1+1+1+..., since the two series are both equivalent, then the finite values of the two series are both equivalent as well,

Then,

$$\frac{-1}{12} = \frac{-1}{2} \quad (5.1)$$

which gives:

$$\frac{1}{12} = \frac{1}{2} \quad (5.2)$$

By multiplication of both sides of the equation (5.2) by 6 we get:

$$\frac{1}{2} = 3 \quad (5.3)$$

We know that the imaginary unit equals the square root of -1.[5]

By applying the value obtained in the equation (5.3) on the definition of the imaginary unit we get:

$$i = \sqrt{-1} = (-1)^{\frac{1}{2}} = (-1)^{3} = -1 \quad (5.4)$$

Then,

$$(-1)^{\frac{1}{2}} = -1 \quad (5.5)$$

From the equation (5.5), we can deduce that the complex numbers could be equal to the real numbers.

By squaring both sides of the equation (5.5) we get:

$$-1 = 1 \quad (5.6)$$

6. Proof of the fraction 1/2 equals any number

Let's rewrite the equation (5.5) in the following form:

$$(-1)^{\frac{1}{2}} = (-1)^{n} \quad (6.1)$$

where n represents any number (i.e. real or complex number).

As we showed in equation (5.6) that 1= -1, so the equation (6.1) can be written as :

$$(1)^{\frac{1}{2}} = (1)^{n} \quad (6.2)$$

let X be equal the right hand side of the equation (6.2),

Then,

$$(1)^{n} = X \quad (6.3)$$

By taking the natural logarithm for both sides of the equation (6.3) gives:

$$\ln(1)^{n} = \ln X \quad (6.4)$$

Which gives:

$$n \ln(1) = \ln(X) \quad (6.5)$$

Then,

$$\ln(X) = 0 \quad (6.6)$$

Which gives:

$$X = 1 \tag{6.7}$$

Therefore, X=1 for any value of the exponent n.
So, the equation (6.3) can be written as:

$$(1)^n = X = 1 \tag{6.8}$$

And, the equation (6.2) can be written as:

$$(1)^{\frac{1}{2}} = (1)^n = 1 \tag{6.9}$$

Which gives,

$$\frac{1}{2} = n \tag{6.10}$$

Since n represents any number, and any value of n will verify the equation (6.9), then 1/2 could be equal any number (i.e. real or complex number).

7. Proof of the Riemann hypothesis

We showed in the previous section that 1/2 could be equal any number.

Let (x+it) represents any complex zeros of the Riemann zeta function, where x is the real part of the complex zero, t is the imaginary part of the complex zero, and i is the imaginary unit. Since we proved that 1/2 could be equal any number, then we can get:

$$\frac{1}{2} = x + it \tag{7.1}$$

By equating the real parts for both sides of the equation (7.1), we obtain:

$$x = \frac{1}{2} \tag{7.2}$$

From the equation (7.2), the real part of the complex zeros of the Riemann zeta function equals 1/2.

Therefore the complex zeros of the Riemann zeta function can be written as:

$$\frac{1}{2} + it \tag{7.3}$$

So, the real part of every non-trivial zero of the Riemann zeta function is 1/2, which proves the Riemann hypothesis.

8. Conclusion

In this paper , we presented a new definition of the imaginary unit and the proof of the Riemann hypothesis as well. We proved that the divergent series 1+1+1+1+... is equivalent to the divergent series 1+2+3+4+..., and we know that one can assign a finite value -1/12 to the series 1+2+3+4+..., and also can assign a finite value -1/2 to the series 1+1+1+1+... by using the analytic continuation of the Riemann zeta function. Since the two series are both equivalent , then the finite values of the two series are both equivalent as well, then ($\frac{-1}{12} = \frac{-1}{2}$), which gives ($\frac{1}{2} = 3$). By applying this wonderful result on the definition of the imaginary unit we obtain: $(-1)^{\frac{1}{2}} = (-1)^3$, then (i =$\sqrt{-1}$ = -1), which is considered a new definition of the imaginary unit, which means that the imaginary unit could be equal to a real number, and hence the complex numbers could be equal to the real numbers which can be used in many aspects of sciences such as quantum mechanics , cosmology and electrical engineering. After that we proved that the fraction 1/2 equals any number, so the real part of the complex zeros of the Riemann zeta function could be equal to 1/2, then the complex zeros of the Riemann zeta function could be written in the form of 1/2+it, which proves the Riemann hypothesis.

References

[1] John Derbyshire, Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003, ISBN 978-0-309-08549-6

[2] Stopple, Jeffrey (2003), A Primer of Analytic Number Theory: From Pythagoras to Riemann, p. 202, ISBN 0-521-81309-3

[3] Knopp, Konrad (1990) [1922]. Theory and Application of Infinite Series. Dover. pp. 490492. ISBN 0-486-66165-2.

[4] Heubach, Silvia; Mansour, Toufik (2004). "Compositions of n with parts in a set". Congressus Numerantium 168: 3351.

[5] Nahin, Paul J. (1998). An Imaginary Tale: The Story of $\sqrt{-1}$. Chichester: Princeton University Press. ISBN 0-691-02795-1.