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Finite Difference Solution of Free Convective Heat Transfer of Non-Newtonian Power Law Fluids from a Vertical Plate

Itishree Swain
Department of Mathematics, Krupajal Engineering School,
Bhubaneswar-751002, Odisha, India.
E-mail: itishreeswain1970@gmail.com

Hadibandhu Pattanayak
Department of Mathematics,
Institute of Mathematics and Applications,
Andharua, Bhubaneswar-751003, Odisha, India.
E-mail: h.pattnayak@gmail.com

Mrutyunjay Das
Department of Mathematics,
School of Applied Sciences, KIIT University,
Bhubaneswar-751024, Odisha, India.
E-mail: mdasfma@kiit.ac.in

Trailokyanath Singh
Department of Mathematics,
C. V. Raman College of Engineering,
Bhubaneswar-752054, Odisha, India.
E-mail: trailokyanaths108@gmail.com

Abstract

A laminar free convective flow of a power law fluid over a vertical plate is investigated. The equations governing the fluid flow are solved numerically using an implicit finite difference scheme which is shown to be unconditionally stable. The effects of the flow parameters on the velocity field and temperature profile are reflected through graphs.

AMS subject classification:
Keywords: Power law fluid, magnetic field, Reynold number, Prandtl number.

1. Introduction

Although natural convection has been of interest for a number of years most of the studies published, so far have been concerned with Newtonian rather than non-Newtonian fluids. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Because of the application of non-Newtonian fluids in industries processing molten plastics, polymers etc. The problem of steady flow and heat transfer in power law fluids by free convection along a vertical plate has been investigated by many researchers. The fundamental case of laminar natural convection heat transfer from an isothermal vertical plate to a power law fluid was analyzed by Acrivos [1]. Hence investigation concerning combined thermal and species diffusion driven flows were carried out by several Researchers such as Sommer [2], Wilcox [3], Emery et al. [4] and Som and Chen [5]. Huang, et al. [6] obtained the local similarity solutions of free convection heat transfer from a vertical plate to non-Newtonian power law fluids. All these workers obtained the solution of the problem when the vertical plate has uniform temperature using asymptotic methods, integral methods or numerical methods. Padhy and Pattanayak [7] have studied the mass transfer and free convective effects of a power law fluids past an impulsively started vertical plate. Hassain and Wilson [8] discussed the natural convection flow in a fluid saturated porous medium enclosed by non-isothermal walls with heat generation. Similarly Nasser [9] studied the problem of unsteady free convection with heat and mass transfer from an isothermal vertical flat plate to a non-Newtonian power law fluid immersed in a saturated porous medium and also Olajuwon [10] investigated the flow and convection heat transfer in a pseudo plastic power law fluid past a vertical plate with heat generation. Chamkha et al. [11] presented in tabular and graphical form to show the effects of material parameters of the problem on the solution.

Recently, Prasad et al. [12] studied finite difference analysis of radioactive free convection flow past an impulsively started vertical plate with variable heat and mass flux. Again Prasad et al. [13] investigated the unsteady free convection heat and mass transfer in a Walters-B viscoelastic flow past a semi finite vertical palte.Thus the aim of the present study is to investigate laminar free convective flow of a power law fluid over a vertical plate in presence of magnetic filed. The interaction of magnetic field is proved to be counter productive in enhancing velocity distribution. The effect of different flow parameters on velocity and temperature are reflected through the figures.

2. Formulation of the Problem

There are different types of non-Newtonian fluids exist but the simplest and most common type is the power-law fluid for which the theological equation of the state between stress components and strain rate components defined by Vujanovic [14] is

$$\tau_{i,j} = P\delta_{i,j} + K \left| \sum_{m=1}^{3} \sum_{I=1}^{3} e_{Im} e_{Im} \right|^{(n-1)/2} e_{i,j} \tag{1}$$

where P is the pressure, $\delta_{(i,j)}$ is the Kronecker delta and K and n are the consistency and flow behavior indices of the fluid. When $n > 1$ the fluid is described as dilatants, $n < 1$ as pseudo-plastic and when $n = 1$ it is known as the Newtonian fluid. Consider unsteady, laminar, hydromagnetic, boundary-layer, two-dimensional flow of a non-Newtonian power-law fluid from a vertical plate. $x^{'}-$ axis is taken along the infinite vertical plate in the upward direction and $y^{'}$ axis is taken normal to it. The fluid is permeated by a uniform transverse magnetic field B_0 applied parallel to $y^{'}$-axis. Initially, at time $t^{'} \leq 0$, the fluid and plate are at rest and at a uniform temperature $T^{'}_{\infty}$. When $t' > 0$ the plate is maintained at constant temperature $T^{'}_{w}$. Since the plate is of infinite extent in $x^{'}$ direction and is electrically non-conducting all physical quantities, except pressure, are functions of $y^{'}$ and $t^{'}$ only. The fluid flow is induced due to the constant heating of the plate.

The power law model for non Newtonian fluids the convective laminar and unsteady conservations equations can be written as

$$\frac{\partial u^{'}}{\partial x^{'}} + \frac{\partial v^{'}}{\partial y^{'}} = 0, \tag{2}$$

$$\frac{\partial u^{'}}{\partial t^{'}} + u^{'}\frac{\partial u^{'}}{\partial x^{'}} + v^{'}\frac{\partial u^{'}}{\partial y^{'}} = g\beta_T(T^{'} - T^{'}_{\infty}) + \frac{k}{\rho}\frac{\partial}{\partial y^{'}}\left|\frac{\partial u^{'}}{\partial y^{'}}\right|^{n-1}\frac{\partial u^{'}}{\partial y^{'}} - \frac{\sigma B_0^2 u^{'}}{\rho}, \tag{3}$$

$$\frac{\partial T^{'}}{\partial t^{'}} + u^{'}\frac{\partial T^{'}}{\partial x^{'}} + v^{'}\frac{\partial T^{'}}{\partial y^{'}} = \alpha\left(\frac{\partial^2 T^{'}}{\partial y^{'2}}\right) \tag{4}$$

where g is acceleration due to gravity, α represents the thermal diffusivity, β_T is coefficient of thermal expansion of fluid, k is fluid consistency index for power law fluid, ρ is fluid density, n is power law index, $u^{'}, v'$ are stream wise and transverse velocity respectively.

Similarly $x^{'}$ and $y^{'}$ are steam wise and transverse co-ordinate. $T^{'}$ is temperature of the fluid and $t^{'}$ is time.

The initial boundary conditions are

$$\begin{aligned}
&t^{'} = 0, u^{'} = v^{'} = 0, T^{'} = T^{'}_{\infty} \quad \forall\, x^{'} \text{ and } y^{'} \\
&t^{'} > 0, x^{'} = 0, u^{'} = 0, T^{'} = T^{'}_{\infty} \\
&t^{'} > 0, u^{'} = v^{'} = 0, T^{'} = T^{'}_{\omega} \text{ at } y^{'} = 0, x^{'} > 0 \\
&t^{'} > 0, u^{'} = 0, T^{'} = T^{'}_{\infty} \text{ at } y^{'} \to \infty, x^{'} > 0.
\end{aligned} \tag{5}$$

The dimensionless variables are defined as follows:

$$x = \frac{x^{'}}{l}, y = \frac{y^{'}}{l}, u = \frac{u^{'}}{U_0}, v = \frac{v^{'}}{U_0}, T = \frac{T^{'} - T^{'}_{\infty}}{T^{'}_{w} - T^{'}_{\infty}}, t = \frac{U_0 t^{'}}{l} \tag{6}$$

where $U_0 = \left[\frac{\rho l^n}{k}\right]^{\frac{1}{n-2}}$ and l is the suitable length scale.

Substituting the above non-dimensional variables into equations (2)-(4) yields the following dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + G_r T - Mu, \tag{8}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr Re}\frac{\partial^2 T}{\partial y^2}, \tag{9}$$

where $Re = \frac{U_0 l}{v}$ is the Reynold's number, $Gr = \frac{g\beta_T(T' - T'_\infty)l}{U_0^2}$ is the Grashof number, $Pr = \frac{\rho \nu C_p}{k}$ is the Prandtl number, $M = \frac{\sigma B_0^2 l}{\rho U_0}$ is the Hartmann number, C_p is the specific heat at constant pressure, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity and μ is the constant viscosity of the fluid in boundary layer region.

The initial and boundary conditions will be reduced to

$$\begin{aligned}
&t = 0, u = v = 0, T = 0 \quad \forall\, x \text{ and } y\\
&t > 0, u = v = 0 \text{ at } x = 0\\
&t > 0, u = v = 0, T = 1 \text{ at } y = 0, x > 0\\
&t > 0, u = 0, T = 0 \text{ at } y \to \infty, x > 0.
\end{aligned} \tag{10}$$

Of special significance of this type of flow and heat transfer situation are the skin-friction co-efficient C_f and the Nusselt number Nu. These physical quantities are defined in non-dimensional form, respectively, as follows:

$$C_f Re^{\frac{1}{2}} = \left|\frac{\partial u}{\partial y}\right|^n_{y=0}. \tag{11}$$

$$Nu Re^{-\frac{1}{2}} = -\left[\frac{\partial T}{\partial y}\right]_{y=0}. \tag{12}$$

3. Solution of the problem

The equations (7)–(10) are solved by implicit finite difference method. For discretisation in space and time a uniform mesh of space step Δx and Δy along x and y direction respectively and time Δt are employed so that the grid points are $(x_i, y_j, t_k;) = (i\Delta x, j\Delta y, k\Delta t)$ $i = 0, 1, 2, \ldots, N$, $j = 0, 1, 2, \ldots, M$ and $k = 0, 1, 2, \ldots, K$.

The discretised form of equations (7), (8) and (9) obtained respectively as,

$$\frac{u_{i+1,j}^{(k+1)} - u_{i-1,j}^{(k+1)}}{\Delta x} + \frac{v_{i,j}^{(k+1)} - v_{i,j-1}^{(k+1)}}{\Delta y} = 0, \tag{13}$$

$$\begin{aligned}
&\frac{u_{i,j}^{(k+1)} - u_{i,j}^{(k)}}{\Delta t} + u_{i,j}^{(k)}\left(\frac{u_{i,j}^{(k)} - u_{i-1,j}^{(k)}}{\Delta x}\right) + v_{ij}^{(k)}\left(\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right) \\
&= GrT_{i,j}^{(k+1)} - Mu_{i,j}^{(k+1)} + \frac{1}{\Delta y}\left[\left|\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right)\right] \\
&- \frac{1}{\Delta y}\left[\left|\frac{u_{i,j}^{(k)} - u_{i,j-1}^{(k)}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j}^{(k)} - u_{i,j-1}^{(k)}}{\Delta y}\right)\right].
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\left[\frac{T_{i,j}^{(k+1)} - T_{i,j}^{(k)}}{\Delta t} + u_{i,j}^{(k)}\left(\frac{T_{i,j}^{(k)} - T_{i-1,j}^{(k)}}{\Delta x}\right) + v_{i,j}^{(k)}\left(\frac{T_{i,j+1}^{(k)} - T_{i,j}^{(k)}}{\Delta y}\right)\right] \\
&= \frac{1}{PrRe}\frac{T_{i,j+1}^{(k)} - 2T_{i,j}^{(k)} + T_{i,j-1}^{(k)}}{(\Delta y)^2}.
\end{aligned} \tag{15}$$

$$i = 1, 2, 3, \ldots, N,$$
$$j = 1, 2, 3, \ldots, M,$$
$$k = 0, 1, 2, \ldots, K.$$

The above discretised equation (13) to (15) are solved iteratively using the following algorithm.

Step-I Initialize $u_{i,j}^{(0)}, v_{i,j}^{(0)}, T_{i,j}^{(0)}$, for all (i, j)

Step-II

For $k = 0, 1, 2, 3, \ldots, K$ max
For $j = 1, 2, 3, \ldots, M$
For $i = 1, 2, 3, \ldots, N$

$$u_{i,j}^{(k+1)} = \frac{1}{1+M\Delta t}u_{i,j}^{(k)} + \frac{\Delta t}{1+M\Delta t}GrT_{i,j}^{(k+1)} + \frac{\Delta t}{(1+M\Delta t)\Delta y}\left[\left|\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right)\right] - \frac{\Delta t}{(1+M\Delta t)\Delta y}\left[\left|\frac{u_{i,j}^{(k)} - u_{i,j-1}^{(k)}}{\Delta y}\right|^{n-1}\left(\frac{u_{i,j}^{(k)} - u_{i,j-1}^{(k)}}{\Delta y}\right)\right] - \frac{\Delta t}{1+M\Delta t}\left[u_{i,j}^{(k)}\left(\frac{u_{i,j}^{(k)} - u_{i-1,j}^{(k)}}{\Delta x}\right) + v_{i,j}^{(k)}\left(\frac{u_{i,j+1}^{(k)} - u_{i,j}^{(k)}}{\Delta y}\right)\right]. \quad (16)$$

Step-III

$$v_{i,j}^{(k+1)} = \Delta y\left[\frac{u_{i-1,j}^{(k+1)} - u_{i,j}^{(k+1)}}{\Delta x}\right] + v_{i,j-1}^{(k+1)}. \quad (17)$$

Step-IV

$$T_{i,j}^{(k+1)} = T_{i,j}^{k} + \frac{\Delta t}{P_r R_e}\left[\frac{T_{i,j+1}^{k} - 2T_{i,j}^{k} + T_{i,j-1}^{k}}{(\Delta y)^2} - P_r R_e\left\{u_{i,j}^{k}\left(\frac{T_{i,j}^{k} - T_{i-1}^{k}}{\Delta x}\right) + v_{i,j}^{k}\left(\frac{T_{i,j+1}^{k} - T_{i,j}^{k}}{\Delta y}\right)\right\}\right]. \quad (18)$$

The steps (II) to (IV) are repeated until the relative errors of two consecutive values of $u_{i,j}$, $v_{i,j}$ and $T_{i,j}$ are less than a given tolerance.

4. Results and Discussion

We have solved the governing equations (7) to (9) with the boundary conditions (10) using finite difference method. The velocity and temperature of the fluid for different Prandtl numbers are shown in Fig. 1. Prandtl number increases the viscous diffusivity of the fluid at the surface which enhanced the velocity of the fluid near the surface as depicted by Fig. 1. This is inversely proportional to thermal diffusivity which is the reason behind the increase of thermal boundary layer thickness.

The velocity is highly influenced by the other parameters such as magnetic parameter, Grashof number and power-law index which are shown through Fig. 2-4. Transverse magnetic field creates a Lorentz drag force which resist the flow velocity of the fluid. As a result the momentum boundary layer thickness decreases with increase of the magnetic parameter as shown in Fig. 2. Viscous force acting on the fluid is inversely proportional

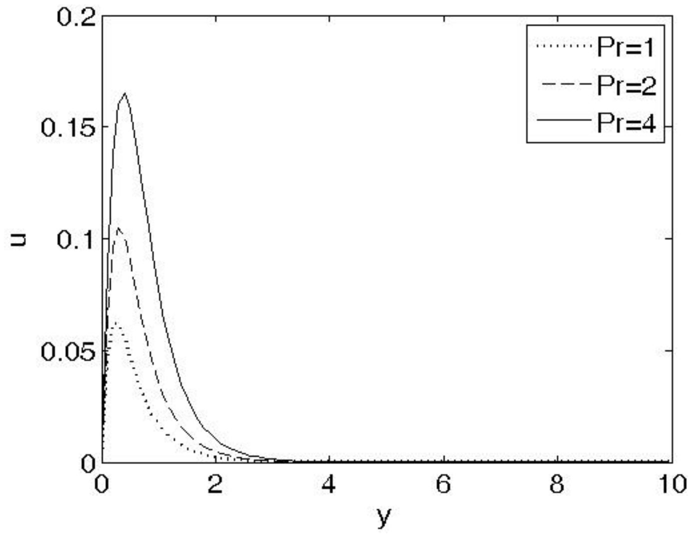

Figure-1(a)
Variation of Pr on the velocity profile of the fluid when $M = 4$, $n = 1$ and $Gr = 5$.

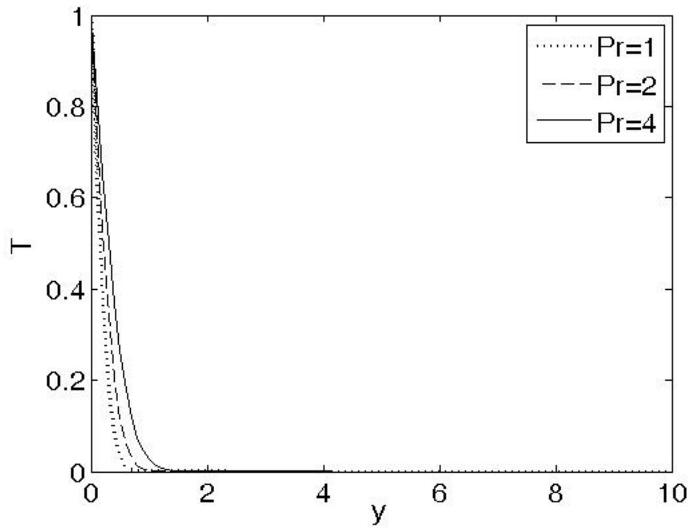

Figure-1(b)
Variation of Pr on the temperature profile of the fluid when $M = 4$, $n = 1$ and $Gr = 5$.

to the Grashof number and so the velocity of the fluid increases with increase of the Grashof number. It is interesting to note that the velocity of the fluid is lower in the case of Newtonian fluid ($n = 1$) while for $n \neq 1$, the velocity of the fluid increases i.e., for both the pseudo-plastic and dilatant fluid, the velocity is more than the Newtonian fluid as shown in Fig. 4. For the physical interest in view, we found the influence of magnetic

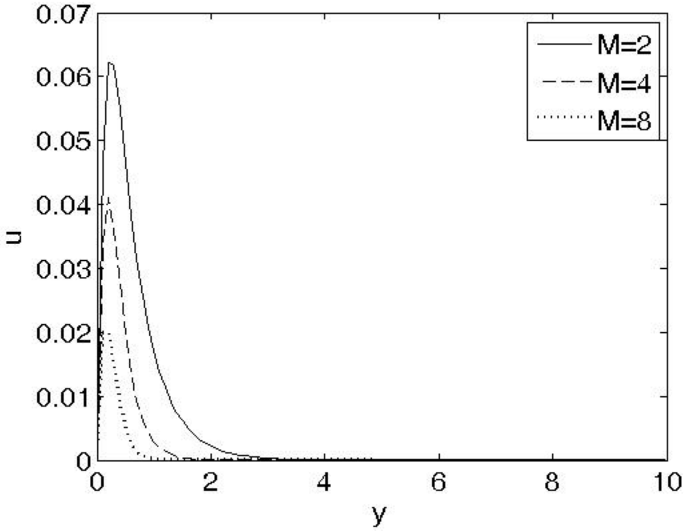

Figure-2
Variation of M on the velocity profile of the fluid when $Pr = 3$, $n = 1$ and $Gr = 5$.

Table 1: The Effect of All the Parameters on Skin Friction and Nusselt Number

parameters>	M	n	P_r	R_e	G_r	C_f	N_u
	2	1	1	1	5	0.5536	0.3857
	4					0.1746	0.3857
	8					0.0468	0.3857
	2	1.5	1	1	5	0.4618	0.3857
		2				0.3924	0.3857
	2	1	2	1	5	0.6771	0.2775
			4			0.7748	0.1979
	2	1	1	3	5	0.3266	0.6122
				6		0.2016	0.7529
	2	1	1	1	10	1.1070	0.3857
					15	1.6593	0.3861

parameter, power-law index, Prandtl number, Reynolds number and Grashof number on the skin friction and local Nusselt number which is shown by table 1. It is interesting to note that magnetic parameter decreases the velocity of the fluid which helps to reduce the skin friction at the surface. While the local heat transfer rate is not influenced by the magnetic parameter. The skin friction at the surface decreases when the fluid changes from dilatant to pseudo-plastic.

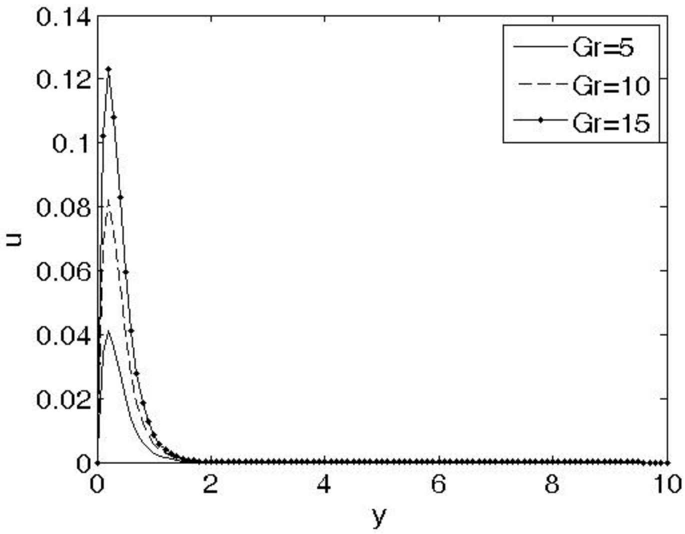

Figure-3
Variation of Gr on the velocity profile of the fluid when $M = 4$, $n = 1$ and $Pr = 3$.

5. Conclusion

In the present work we have considered the free convective heat transfer of Non-newtonian power law fluid, in presence of magnetic field. The system of nonlinear equations obtained by discretisation is solved numerically by finite difference method. The solution obtained well agrees with the Newtonian case. It gives improved result taking into consideration the behavior of magnetic field. One of the major observation in the present investigation is that the velocity decreases with an increase in magnetic field. This method well suits for other non-newtonian fluid flow problems.

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