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A Study on the *k*-Convexity of Sequences

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Abstract

For any integer $k\geq 2$, a real sequence $(a_n)_1^\infty$ is said to be k-convex if

$$a_{n+1} \leq a_n + \frac{a_{n+k}-a_n}{k} \text{ and } a_{n+k-1} \leq a_n + \frac{(k-1)(a_{n+k}-a_n)}{k}$$

for all positive integer n. In this paper, we present some properties on the k-convexity of real sequences.

Keywords: *k*-convexity, sequences

1. Introductionand Preliminaries

Let $(a_n)_1^\infty$ be a real sequence. The sequence $(a_n)_1^\infty$ is said to be convex if

$$0 \leq a_{n+2} - 2a_{n+1} + a_n$$

for all positive integer n.In [1, 4, 6, 7, 8], some properties of convex sequenceswere presented.For any$r\geq 0$, the sequence $(a_n)_1^\infty$ is said to be r-convex if

$$0 \leq a_{n+2} - (1+r)a_{n+1} + ra_n$$

for all positive integer n.In [3], for any $r\geq 0$, some properties of r-convex sequenceswere presented.For any real numbers$p$ and q, the sequence $(a_n)_1^\infty$ is said to be (p, q)-convex if

$$0 \leq a_{n+2} + (p+q)a_{n+1} + pqa_n$$

for all positive integer n.In [2], for any real numbers$p$ and q, some properties of (p,q)-convex sequenceswere presented.

For any integer $k\geq 2$, the sequence $(a_n)_1^\infty$ is said to be k-convex if

$$a_{n+1} \leq a_n + \frac{a_{n+k}-a_n}{k} \text{ and } a_{n+k-1} \leq a_n + \frac{(k-1)(a_{n+k}-a_n)}{k}$$

for all positive integer n.For any integer $k \geq 2$, if the sequence $(a_n)_1^\infty$ is k-convex then we have the inequality

$$a_{n+1} - a_n \leq a_{n+k} - a_{n+k-1}$$

for all positive integer n.In [5], for any integer $k \geq 2$, some properties of k-convex sequenceswere presented.In this paper, we present some properties on the k-convexity of real sequences.

Theorem 1.1. [5] Let k be an integer such that $k \geq 2$, and let c, d be real numbers such that c > 0. Assume that $(a_n)_1^\infty$ and $(b_n)_1^\infty$ are k-convex sequences. Then

(i) $(ca_n + d)_1^\infty$ is a k-convex sequence,

(ii) $(a_n + dn)_1^\infty$ is a k-convex sequence,

(iii) if $(a_n)_1^\infty$ is positive then $a_n^2 \; {}_1^\infty$ is a k-convex sequence, and

(iv) $(a_n + b_n)_1^\infty$ is a k-convex sequence,

(v) if $\left(\frac{a_n}{n}\right)_1^\infty$ is a non-increasing sequence then $\left(\frac{a_n}{n}\right)_1^\infty$ is a k-convex sequence.

2. Main Results

Theorem 2.1. Let k be an integer such that $k \geq 2$, and let $(a_n)_1^\infty$ be a k-convex sequence. Then, for any positive integer m, if $(n^{m-1}a_n)_1^\infty$ is non-decreasing then $(n^m a_n)_1^\infty$ is a k-convex sequence.

Proof. First, we will show that if $(a_n)_1^\infty$ is non-decreasing then $(na_n)_1^\infty$ is a k-convex sequence. Assume that $(a_n)_1^\infty$ is non-decreasing.

By the k-convexity of $(a_n)_1^\infty$, we have

$$a_{n+1} \leq a_n + \frac{a_{n+k} - a_n}{k} \text{ and } a_{n+k-1} \leq a_n + \frac{(k-1)(a_{n+k} - a_n)}{k}$$

for all positive integer n.

By the assumption of $(a_n)_1^\infty$, we have

$$\begin{aligned}(n+1)a_{n+1} &\leq (n+1)a_n + \frac{(n+1)(a_{n+k} - a_n)}{k} \\ &= na_n + \frac{(n+k)a_{n+k} - na_n + (k-1)(a_n - a_{n+k})}{k} \\ &\leq na_n + \frac{(n+k)a_{n+k} - na_n}{k}\end{aligned}$$

and

$$(n+k-1)a_{n+k-1} \leq (n+k-1)a_n + \frac{(n+k-1)(k-1)(a_{n+k} - a_n)}{k}$$

$$= na_n + \frac{(k-1)((n+k)a_{n+k} - na_n + a_n - a_{n+k})}{k}$$

$$\le na_n + \frac{(k-1)((n+k)a_{n+k} - na_n)}{k}$$

for all positive integer n.

Thus, $(na_n)_1^\infty$ is a k-convex sequence.

Next, we suppose that if $(n^{p-1}a_n)_1^\infty$ is non-decreasing then $(n^p a_n)_1^\infty$ is a k-convex sequence, where p is a positive integer. Assume that $(n^p a_n)_1^\infty$ is non-decreasing. Then $(n^{p-1}a_n)_1^\infty$ is non-decreasing. Thus, $(n^p a_n)_1^\infty$ is a k-convex sequence.

Let $b_n = n^p a_n$ for all positive integer n. Then $n^{p+1}a_n = nb_n$ for all positive integer n. Thus, $(b_n)_1^\infty$ is non-decreasing and k-convex.

By the k-convexity of $(b_n)_1^\infty$, we have

$$b_{n+1} \le b_n + \frac{b_{n+k} - b_n}{k} \text{ and } b_{n+k-1} \le b_n + \frac{(k-1)(b_{n+k} - b_n)}{k}$$

for all positive integer n.

By the assumption of $(b_n)_1^\infty$, we have

$$(n+1)b_{n+1} \le (n+1)b_n + \frac{(n+1)(b_{n+k} - b_n)}{k}$$

$$= nb_n + \frac{(n+k)b_{n+k} - nb_n + (k-1)(b_n - b_{n+k})}{k}$$

$$\le nb_n + \frac{(n+k)b_{n+k} - nb_n}{k}$$

and

$$(n+k-1)b_{n+k-1} \le (n+k-1)b_n + \frac{(n+k-1)(k-1)(b_{n+k} - b_n)}{k}$$

$$= nb_n + \frac{(k-1)((n+k)b_{n+k} - nb_n + b_n - b_{n+k})}{k}$$

$$\le nb_n + \frac{(k-1)((n+k)b_{n+k} - nb_n)}{k}$$

for all positive integer n.

Thus, $(nb_n)_1^\infty$ is a k-convex sequence.Hence, $(n^{p+1}a_n)_1^\infty$ is a k-convex sequence.By the mathematical induction, we are done.

Corollary 2.2. [5] Let$k$ be an integer such that $k \ge 2$, and let $(a_n)_1^\infty$ be a k-convex sequence. It follows that if $(a_n)_1^\infty$ is non-decreasing then $(na_n)_1^\infty$ is a k-convex sequence.

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