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Integration of Sensitizing Runs Rules and Tukey's Control Chart in Monitoring Gamma Distribution

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Abstract

The objective of this study was to enhance the performance of Tukey's control chart based on the out-of-control average run length or ARL_1 values in monitoring gamma distribution. The sensitizing runs rules had been proposed and then integrated to improve ability of Tukey's control chart. The study result indicated Tukey's control chart with sensitizing runs rules was more substantially effective than without sensitizing runs rules for detecting process mean shift as considering from smaller values of ARL_1 at every mean shift sizes.

Keywords: Tukey's control chart, Sensitizing runs rules, ARL_1

Mathematics Subject Classification: 62-07, 62G35

Introduction

A use of control chart for investigating change of manufacturing process is one of the important powerful tools in statistical process control or SPC. The Shewhat chart for an individual observation or X chart is extensively utilized in monitoring process mean. Two major characteristics of Shewhart individual control chart are as follows: (1) It requires large numbers of observations to set up the control limits in phase I [1] and (2) a quality characteristic of process is assumed normally distributed. In order to fix these advantages of Shewhart individual control chart, Alemi [2] and Borckardt et al. [3] had been presented Tukey's control chart so it is not affected by the unusual observations or outliers. Later, Torng et al. [4] indicated a good performance of Tukey's control chart in monitoring gamma distribution and short run process. Some researchers, [5], [6] and [7], applied technique of supplemental rules to improve performance of Shewhart control chart since it is insensitive to small shift.

Therefore, this study was not only purposed integration of sensitizing runs rules to Tukey's control chart but also indicated improvement of Tukey's control chart performance in monitoring process mean shift for skewed distribution like gamma distribution through a simulation.

Materials And Methods

A simulation data of this study provided with Minitab Macro by assuming the quality characteristic x_i under gamma distribution which has the probability density function as of Equation 1.

$$f(x_i \mid \alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x_i^{\alpha-1} e^{-x_i/\beta} \quad ; x_i>0,\ \alpha>0,\ \beta>0 \tag{1}$$

where α and β be respectively the shape and scale parameter. This study chose $\alpha = 4$ and $\beta = 1$ referred as [8], [9], [10] and [11]. The in-control state then defined $x_i \sim G(4,1)$; $i = 1,2,\ldots,20$ while the out-of-control state ($i = 21,22,\ldots,30$) shifted in the unit of standard deviation ($\delta\sigma$).

Given the out-of-control conditions followed [4], the process mean shifted to small shift ($\delta = 1$), medium shift ($\delta = 2$) and large shift ($\delta = 3$). The run length was computed at each of process iteration. When all iterations completely run, the ARL_1 could be determined.

A Box plot procedure is considered to construct the control limits of Tukey's control chart formulated as of Equation 2.

$$\begin{aligned} &\text{Upper Control Limit } (UCL.) = Q_1 - k(Q_3 - Q_1) \\ &\text{Lower Control Limit } (LCL.) = Q_3 + k(Q_3 - Q_1) \end{aligned} \tag{2}$$

where Q_1 and Q_3 be the first and third quartile, respectively and k be the parameter denoted the control limit coefficient. This study defined $k = 1.5$ as of its default [2] and [3].

The definition of with sensitizing runs rules in this study only counted on adding one more rules out of the Western Electric Rules [12]. That meant if either two successive points fell outside the upper control limit (UCL.) or the lower control limit (LCL.), that process was said to be the out-of-control state. On the other hand, the meaning of without sensitizing runs rules defined as if any point plotted between the upper and the lower control limit, that process would be pointed in statistical control.

Results

To compare the performance of Tukey's control chart between with and without sensitizing runs rules, the values of ARL_1 were determined as displayed in Table 1. This table illustrated that Tukey's control chart with sensitizing runs rules gave smaller ARL_1 values than of the Tukey's control chart without sensitizing runs rules in every process shift size. Moreover, Tukey's control chart without sensitizing runs rules obtained the smallest ARL_1 values when the process mean shifted to the large shift. While the process mean changed to the medium shift, it showed smaller ARL_1 values than changing to the small shift. These results were in accordance with [4]. It is also getting the same conclusion of Tukey's control chart with sensitizing runs rules.

Table 1: ARL_1 values of Tukey's control charts

Shift size	With Sensitizing Runs Rules	Without Sensitizing Runs Rules
1σ	17.78	83.55
2σ	4.45	20.89
3σ	2.23	5.98

Discussion

The sensitizing runs rules was beneficial to improve the performance of Tukey's control chart for monitoring process mean shift under a skewed distribution, like gamma distribution, as providing the smaller ARL_1 values compared to Tukey's control chart without sensitizing runs rules.

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