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# On Symmetric Properties Of Generalized Sasakian Space Form

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**Abstract:**

In the present paper, we have studied locally and globally, φ-quasiconformally symmetric and φ-concircularly symmetric generalized Sasakian space form and obtained some interesting results.

**Key Words:** Generalized Sasakian space form, φ-quasiconformally symmetric manifold, φ-concircularly symmetric manifold, Einstein manifold.

**AMS Subject Classification (2000):** 53D10, 53D15, 53C25.

## Introduction

A Sasakian manifold with constant φ-sectional curvature is a Sasakian space form and it has a specific form of its curvature tensor. Similar notion also holds for Kenmostu and Cosymplectic space form. In order to generalize such space form in a common frame, P. Alegre, D. E. Blair and A. Carriazo [1, 3], introduced the notion of generalized Sasakian space form. It is defined as an almost Contact metric manifold (M, φ, ξ, η, g) whose curvature tensor
is given by,

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z) \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned}$$

for any vector fields *X, Y,Z* on *M*, where $f_1$, $f_2$, $f_3$ are differentiable function on M. In [1, 3], authors have given several examples of such manifolds. If $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, then generalized Sasakian space form with Sasakian structure becomes Sasakian space form. Generalized Sasakian space form and Sasakian space form have been studied by several authors, viz., [3], [2], [5], [9], [14].
The notion of the quasi-conformal curvature tensor was given by Yano and Sawaki [16]. According to them a quasi-conformal curvature tensor $C^*$ is defined by

$$C^*(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{2n+1}\left[\frac{a}{2n} + 2b\right][g(Y,Z)X - g(X,Z)Y] \quad (1.1)$$

where *a* and *b* are constants and *R, S,Q* and *r* are the Riemannian curvature tensor of type (1, 3), the Ricci tensor of type (0, 2), the Ricci operator defined by *g(QX, Y ) = S(X, Y )* andthe scalar curvature of the manifold respectively. From (1.1) we obtain

$$\nabla_W C^*(X,Y)Z = a(\nabla_W R)(X,Y)Z + b[(\nabla_W S)(Y,Z)X - (\nabla_W S)(X,Z)Y + g(Y,Z)(\nabla_W Q)(X) - g(X,Z)(\nabla_W Q)(Y)] - \frac{dr(W)}{2n+1}\left[\frac{a}{2n} + 2b\right][g(Y,Z)X - g(X,Z)Y]. \quad (1.2)$$

In the paper [13], U. C. De et al. introduced and studied φ-quasiconformally symmetric on Sasakian manifolds. A Contact metric manifold *(M, g)* is called locally φ–quasiconformally symmetric if the condition

$$\varphi^2((\nabla_W C^*)(X,Y,Z)) = 0.$$

holds on *M*, where *X, Y,Z* and *U* are horizontal vectors. If *X, Y, Z* and *U* are arbitrary vectors then the manifold is called globally φ-quasiconformally symmetric.

A transformation of an *(2n + 1)*-dimensional Riemannian manifold *M*, which transforms
every geodesic circle of *M* into a geodesic circle is called a Concircular transformation [6]. The interesting invariant of a Concircular transformation is the Concircular curvature tensor $\tilde{C}$ which is defined by [6].

$$\tilde{C}(Y,Z,U) = R(Y,Z,U) - \frac{r}{2n(2n+1)}[g(Z,U)Y - g(Y,U)Z], \quad (1.3)$$

where *r* is the scalar curvature of the manifold. From (1.3) we obtain

$$(\nabla_X \tilde{C})(Y,Z,U) = (\nabla_X R)(Y,Z,U) - \frac{dr(W)}{2n(2n+1)}[g(Z,U)Y - g(Y,U)Z] \quad (1.4)$$

In a recent paper, U. C. De and Krishnendu De [11] introduced and studied φ-concircularly Symmetry on Kenmotsu Manifolds. A Contact metric manifold *(M, g)* is called locally φ-concircularly symmetric if the condition

$$\varphi^2((\nabla_W C^*)(X,Y,Z)) = 0.$$

holds on *M*, where *X, Y, Z* and *U* are horizontal vectors. If *X, Y, Z* and *U* are arbitrary vectors then the manifold is called globally φ-concircularly symmetric. The present paper is organized as follows:

In section 2, we recall some preliminary results. In section 3, we studied globally φ-concircularly symmetric and φ-quasiconformally symmetric generalized Sasakian space form. And in section 4, we studied 3-dimensional locally φ-concircularly symmetric and φ-quasiconformally symmetric generalized Sasakian space form.

**Preliminaries**

An odd-dimensional Riemannian manifold (M, g) is called an almost Contact manifold if there exists on *M*, a (1, 1) tensor field φ, a vector field ξ and a 1-form η [7] such that,

$$\phi^2(X) = -X + \eta(X)\xi, \;\; \eta(\xi) = 1, \;\; \phi(\xi) = 0, \; \eta \circ \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \;\; g(X,\xi) = \eta(X), \tag{2.2}$$

for any vector fields *X*, *Y* on *M*.

If in addition, ξ is a Killing vector field then *M* is said to be a K-Contact manifold. It is well known that a Contact metric manifold is a K-Contact manifold if and only if $(\nabla_X \xi) = -\varphi X$ for any vector field *X* on *M*.

Given an almost Contact metric manifold (M, φ, ξ, η, g) we say that *M* is an generalized
Sasakian space form [1], if there exists three functions $f_1$, $f_2$ and $f_3$ on *M* such that

$$R(X,Y)Z = f_1\{g(Y,Z)X - g(X,Z)Y\}$$
(2.3)
$$+ f_2\{g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z$$
$$+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)$$
$$+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}$$

for any vector fields *X*, *Y,Z* on *M*, where *R* denotes the curvature tensor of *M*. This kind of manifold appears as a natural generalization of the well-known Sasakian space form *M(C)*, which can be obtained as particular cases of generalized Sasakian space form by taking $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$.
Further in a *(2n + 1)*-dimensional generalized Sasakian space form, we have [1]

$$(\nabla_X \varphi)Y = (f_1 - f_3)g(X,Y)\xi - \eta(Y)X, \tag{2.4}$$

$$(\nabla_X \xi) = -(f_1 - f_3)\varphi X, \tag{2.5}$$

$$(\nabla_X \eta)(Y) = g((\nabla_X \xi), Y) = -(f_1 - f_3)g(\varphi X, Y) = (f_1 - f_3)g(X, \varphi Y), \tag{2.6}$$

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi, \tag{2.7}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y)$$
$$- (3f_2 + (2n-1)f_3)\eta(X)\eta(Y), \tag{2.8}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{2.9}$$

$$R(X,Y)\xi = (f_1 - f_3)[\eta(Y)X - \eta(X)Y], \tag{2.10}$$

$$R(\xi,X)Y = (f_1 - f_3)[g(X,Y)\xi - \eta(Y)X], \tag{2.11}$$

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X), \tag{2.12}$$

$$S(\varphi X, \varphi Y) = S(X,Y) - 2n(f1 - f3)\eta X)\eta(Y). \tag{2.13}$$

Using (2.5), (2.6) and (2.10) we have

$$(\nabla_W R)(X,Y)\xi = (f_1 - f_3)[(f_1 - f_3)\{g(\varphi Y,W)X - g(\varphi X,W)Y\} - R(X,Y)\varphi W]. \tag{2.14}$$

Analogous to the definitions of globally φ-concircularly symmetric Kenmotsu manifolds
[11] and globally φ-quasiconformally symmetric Sasakian manifolds [13], here we define the following:

**Definition 2.1.** A generalized Sasakian space form $M$ is said to be globally φ-concircularly symmetric if the Concircular curvature tensor $\tilde{C}$ satisfies

$$\varphi^2\left(\left(\nabla_W \tilde{C}\right)(X,Y,Z)\right) = 0, \tag{2.15}$$

for all vector fields $X$, $Y$, $Z$ and $W$ in $M$.

**Definition 2.2.** A generalized Sasakian space form M is said to be globally φ-quasiconformally symmetric if the quasiconformal curvature tensor $C^*$ satisfies

$$\varphi^2\left((\nabla_W C^*)(X,Y,Z)\right) = 0, \tag{2.16}$$

for all vector fields $X$, $Y$, $Z$ and $W$ in $M$.

**Globally φ-Quasiconformally Symmetric and φ-Concircularly Symmetric Generalized Sasakian Space Form**

Let us suppose that M is a globally φ-quasiconformally symmetric generalized Sasakian space form. Then by definition

$$\varphi^2\left((\nabla_W C^*)(X,Y,Z)\right) = 0,$$

From (2.1) we have,

$$-(\nabla_W C^*)(X,Y,Z) + \eta((\nabla_W C^*)(X,Y,Z))\xi = 0. \tag{3.1}$$

Using (1.2) in (3.1) we have,

$$\begin{aligned} &- ag((\nabla_W R)(X,Y)Z,U) - b[(\nabla_W S)(Y,Z)g(X,U) - (\nabla_W S)(X,Z)g(Y,U) \\ &- (\nabla_W S)(Y,Z)\eta(X)\eta(U) + (\nabla_W S)(X,Z)\eta(Y)\eta(U) + g(Y,Z)g((\nabla_W Q)X,U) \\ &- g(X,Z)g((\nabla_W Q)Y,U) - g(Y,Z)((\nabla_W Q)X)\eta(U) + g(X,Z)\eta((\nabla_W Q)Y)\eta(U)] \\ &+ \frac{dr(W)}{2n+1}\left[\frac{a}{2n} + 2b\right][g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] + a\eta\left((\nabla_W R)(X,Y)Z\right)\eta(U) \\ &- \frac{dr(W)}{2n+1}\left[\frac{a}{2n} + 2b\right][g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\eta(U) = 0. \end{aligned} \tag{3.2}$$

Let $\{e_i\}$, i = 1, 2, ..., 2n + 1 be an orthonormal basis of the tangent space at any point of the space form. Then putting $X = U = e_i$, in (3.1) and taking summation over $i$, $1 \le i \le 2n+1$, and then using (2.1), we get

$$\begin{aligned} &- (a + (2n-1)b)(\nabla_W S)(Y,Z) - bdr(W)g(Y,Z) - b(\nabla_W S)(Z,\xi)\eta(Y) \\ &+ ag\left((\nabla_W R)(\xi,Y)Z,\xi\right) + \frac{2ndr(W)}{(2n+1)}\left[\frac{a}{2n} + 2b\right]g(Y,Z) \\ &+ \frac{dr(W)}{2n+1}\left[\frac{a}{2n} + 2b\right][g(Y,Z)\eta(X) - \eta(Z)\eta(Y)] = 0. \end{aligned} \tag{3.3}$$

Using (2.10) and (2.14) and the relation $g((\nabla_W R)(X,Y)Z,U) = -g((\nabla_W R)(X,Y)U,Z)$, we have

$$g(\nabla_W R(\xi,Y)Z,\xi) = 0. \tag{3.4}$$

Putting $Z = \xi$ in (3.3) and then using (3.4), we get

$$(\nabla_W S)(Y,\xi) = \frac{dr(W)}{(2n+1)}\eta(Y), \tag{3.5}$$

Provided $a-(2n-1)b \neq 0$. If $a-(2n-1)b = 0$ then from (1.1), it follows that $C^* = aC$

Putting $Y = \xi$ in (3.4), we get $dr(W) = 0$. This implies $r$ is constant. So from (3.5) we have

$$(\nabla_W S)(Y,\xi) = 0. \tag{3.6}$$

We know that

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi), \tag{3.7}$$

By virtue of (2.5), (2.6) and (3.6), it follows from (3.7) that

$$S(\phi W,Y) = 2n(f1 - f3)g(\phi W,Y). \tag{3.8}$$

Replacing $Y$ by $\phi Y$ in (3.8) and using (2.13), we get

$$S(W,Y) = 2n(f1 - f3)g(W,Y). \tag{3.9}$$

**Theorem 3.1.** If a generalized Sasakian space form is globally $\phi$-quasiconformally symmetric, then the manifold is an Einstein manifold, provided $f_1 - f_3 \neq 0$.

Suppose we have $S(Y,Z) = \alpha g(Y,Z)$, then from (1.4) we have

$$(\nabla_X \tilde{C})(Y,Z,U) = (\nabla_X R)(Y,Z,U). \tag{3.10}$$

Applying $\phi^2$ on both sides of the equation (3.9), we get

$$\phi^2(\nabla_X \tilde{C})(Y,Z,U) = \phi^2(\nabla_X R)(Y,Z,U) \tag{3.11}$$

This leads to the following:

**Theorem 3.2.** A globally $\phi$-quasiconformally symmetric generalized Sasakian space form is
globally symmetric.

Let us suppose that M is a globally $\phi$-concircularly symmetric generalized Sasakian space form. Then by definition

$$\phi^2((\nabla_X \tilde{C})(X,Y,Z)) = 0.$$

Using (2.1) we have,

$$-(\nabla_X \tilde{C})(X,Y,Z) + \eta((\nabla_X \tilde{C})(X,Y,Z))\xi = 0, \tag{3.12}$$

using (1.4) in (3.12) we have,

$$-g((\nabla_W R)(X,Y,Z),U) + dr(W)2n(2n+1)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \tag{3.13}$$
$$+\eta\big((\nabla_W R)(X,Y,Z)\big)\eta(U) - dr(W)2n(2n+1)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\eta(U) = 0.$$

Let $\{e_i\}$, $i = 1, 2, \ldots, 2n + 1$ be an orthonormal basis of the tangent space at any point of the space form. Then putting $X = U = e_i$ , in (3.13) and taking summation over $i$, $1 \le i \le 2n + 1$,
and then using (2.1), we get

$$-(\nabla_W S)(Y,Z) + \frac{dr(W)}{(2n+1)} g(Y,Z) + g((\nabla_W R)(\xi,Y,Z),\xi) - \frac{dr(W)}{2n(2n+1)}[g(Y,Z) - \eta(Y)\eta(Z)] = 0. \tag{3.14}$$

Using (2.10) and (2.14) and the relation $g((\nabla_W R)(X,Y)Z,U) = -g((\nabla_W R)(X,Y)U,Z)$,
We have

$$g(\nabla_W R(\xi,Y)Z,\xi) = 0. \tag{3.15}$$

Putting $Z = \xi$ in (3.14) and using (3.15), we get

$$(\nabla_W S)(Y,\xi) = \frac{dr(W)}{(2n+1)}\eta(Y) \tag{3.16}$$

And putting $Y = \xi$ in (3.4), we get $dr(W) = 0$. This implies $r$ is constant. Then from (3.16) we have

$$(\nabla_W S)(Y,\xi) = 0 \tag{3.17}$$

We know that

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi), \tag{3.18}$$

By virtue of (2.5), (2.6) and (3.17), it follows from (3.18) that

$$S(\phi W,Y) = 2n(f1 - f3)g(\phi W,Y). \tag{3.19}$$

Replacing $Y$ by $\phi Y$ in (3.19) and then using (2.13), we get

$$S(W,Y) = 2n(f1 - f3)g(W,Y). \tag{3.20}$$

**Theorem 3.3.** If a generalized Sasakian space form is globally $\phi$-concircularly symmetric, then the manifold is an Einstein manifold, provided $f_1 - f_3 \neq 0$.

Suppose $S(Y,Z) = \alpha g(Y,Z)$, that is, the manifold is an Einstein manifold. Then from (1.4) we have

$$(\nabla_X \tilde{C})(Y,Z,U) = (\nabla_X R)(Y,Z,U). \tag{3.21}$$

Applying $\phi^2$ on both sides of equation (3.21) we have

$$\phi^2\left(\nabla_X \tilde{C}\right)(Y,Z,U) = \phi^2(\nabla_X R)(Y,Z,U). \tag{3.22}$$

This leads to the following:

**Theorem 3.4.** A globally $\phi$-concircularly symmetric generalized Sasakian space form is globally symmetric.

### 4. 3-Dimensional Generalized Sasakian Space Form

For a three dimensional generalized Sasakian space form the function $f_2 = 0$ [1]. Using (2.3) in (1.1), we obtain the quasiconformal curvature tensor for a 3-dimensional generalized Sasakian space form as

$$C^*(X,Y)Z = af_1\{g(Y,Z)X - g(X,Z)Y\} + af_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{3}\left[\frac{a}{2}+2b\right][g(Y,Z)X - g(X,Z)Y], \tag{4.1}$$

Using (2.7) and (2.8), we obtain from (4.1) that

$$C^*(X,Y)Z = \left\{af_1 + 2b(2f_1 - f_3) - \frac{r}{3}\left[\frac{a}{2}+2b\right]\right\}[g(Y,Z)X - g(X,Z)Y] + \{af_3 + bf_3\}[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi]. \tag{4.2}$$

Taking covariant differentiation on both sides of (4.2), we get

$$(\nabla_W C^*)(X,Y)Z = \{adf_1(W) + 2b\left(2df_1(W) - df_3(W)\right) - \frac{dr(W)}{3}\left[\frac{a}{2}+2b\right]\}[g(Y,Z)X - g(X,Z)Y] + \{adf_3(W) + bdf_3(W)\}\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} + \{af_3 + bf_3\}\{\eta(X)\nabla_W\eta(Z)Y + \nabla_W\eta(X)\eta(Z)Y - \nabla_W\eta(Y)\eta(Z)X - \eta(Y)\nabla_W\eta(Z)X + g(X,Z)\nabla_W\eta(Y)\xi + g(X,Z)\eta(Y)\nabla_W\xi - g(Y,Z)\nabla_W\eta(X)\xi - g(Y,Z)\eta(X)\nabla_W\xi\} \tag{4.3}$$

If we consider $X, Y, Z$ orthogonal to $\xi$ then (4.3) reduces to

$$(\nabla_W C^*)(X,Y)Z = \{adf_1(W) + 2b\left(2df_1(W) - df_3(W)\right) - \frac{dr(W)}{3}\left[\frac{a}{2}+2b\right]\}[g(Y,Z)X - g(X,Z)Y] + \{af_3 + bf_3\}\{g(X,Z)\eta(Y)\nabla_W\xi - g(Y,Z)\nabla_W\eta(X)\xi\} \tag{4.4}$$

Applying $\phi^2$ on (4.4), we get

$$\phi^2((\nabla_W C^*)(X,Y)Z) = \{adf_1(W) + 2b\left(2df_1(W) - df_3(W)\right) - \frac{dr(W)}{3}\left[\frac{a}{2}+2b\right]\}[g(Y,Z)X - g(X,Z)Y] \tag{4.5}$$

Using (2.9) in (4.5), we get

$$\phi^2((\nabla_W C^*)(X,Y)Z) = \frac{dr(W)}{3}[a+b][g(X,Z)Y - g(Y,Z)X] \tag{4.6}$$

Assume $\phi^2(\nabla_W C^*)(X,Y)Z = 0$, if $a+b=0$ then substituting $a=-b$ in (1.1) we find

$$(\nabla_W C^*)(X,Y)Z = (\nabla_W C)(X,Y)Z, \tag{4.7}$$

where $C$ is the Weyl conformal curvature tensor. But for a 3-dimensional Riemannian manifold

C = 0, this implies $C^* = 0$. Therefore $a+b \neq 0$, then the equation (4.6) implies $df_3(W) = 0$.

This leads to the following theorem:

**Theorem 4.5.** A 3-dimensional generalized Sasakian space form is locally $\phi$-quasiconformally symmetric if and only if $f_3$ is constant.

Using (2.3) in (1.3), we get for a 3-dimensional generalized Sasakian space the Concircular curvature tensor is given by

$$\begin{aligned}\tilde{C}(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} + f_3\{\eta(X)\eta(Z)Y \\ & - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \\ & -\frac{r}{2}[g(Y,Z)X - g(X,Z)Y].\end{aligned} \tag{4.8}$$

Taking the covariant differentiation on both sides of (4.8), we have

$$\begin{aligned}(\nabla_W\tilde{C})(X,Y)Z = {} & df_1(W)\{g(Y,Z)X - g(X,Z)Y\} + df_3(W)\{\eta(X)\eta(Z)Y \\ & - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \\ & + f_3(W)\{(\nabla_W\eta(X)\eta(Z)Y + (\nabla_W\eta(X)\eta(Z)Y \\ & - \nabla_W\eta(Y)\eta(Z)X - \eta(Y)(\nabla_W\eta(Z)X + g(X,Z)\nabla_W\eta(Y)\xi \\ & + g(X,Z)\eta(Y)\nabla_W\xi - g(Y,Z)\nabla_W\eta(X)\xi - g(Y,Z)\eta(X)\nabla_W\xi\} \\ & - \frac{dr(W)}{6}[g(Y,Z)X - g(X,Z)Y].\end{aligned} \tag{4.9}$$

If we consider $X, Y, Z$ orthogonal to $\xi$ then (4.9) reduces to

$$\begin{aligned}(\nabla_W\tilde{C})(X,Y)Z = {} & df_1(W)\{g(Y,Z)X - g(X,Z)Y\} + f_3(W)g(X,Z)\nabla_W\eta(Y)\xi \\ & - g(Y,Z)\nabla_W\eta(X)\xi\} - \frac{dr(W)}{6}[g(Y,Z)X - g(X,Z)Y].\end{aligned} \tag{4.10}$$

Applying $\phi^2$ to (4.10), we get

$$\phi^2(\nabla_W\tilde{C})(X,Y)Z = \left[\frac{dr(W)}{6} - df_1(W)\right]\{g(Y,Z)X - g(X,Z)Y\} \tag{4.11}$$

This leads to the following:

**Theorem 4.6.** A 3-dimensional generalized Sasakian space form is locally $\phi$-concircularly symmetric if and only if $f_3$ is constant.

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