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EDGE-ODD GRACEFULNESS OF GRAPHS $(P_3 x P_n) \nabla P_n$, $(P_3 x P_n) \nabla 2P_n$ and $(P_3 x P_n) \oplus P_n$

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Abstract

In this article, the edge-odd gracefulness of $(P_3 x P_n) \nabla P_n$ $(P_3 x P_n) \nabla P_n$ and $(P_3 x P_n) \oplus P_n$ are got.

Keywords: Graceful Graph, Edge-odd graceful labeling, Edge-odd Graceful Graph.

Introduction

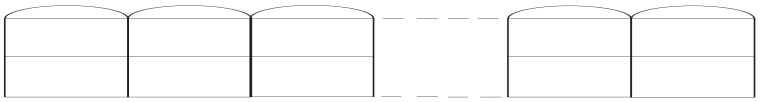
Solairaju, Sasikala, and Vimala [2009] got the edge-odd graceful labeling for a spanning tree of the cartesian product of P_2 and C_n. Solairaju, Vimala and Sasikala [2009] obtained the even edge gracefulness labeling for a spanning tree of the cartesian product of S_3 and S_n. Solairaju and Kavitha Devi [2009] found the graceful labeling for the graph $K(P_{2n+1} * P_k)$. Solairaju and Antony Arockiasamy [2010] proved the gracefulness for tree $E*3S_n$.

Edge –odd graceful of $(P_3 \times P_n) \nabla P_n$

The following definition is now given for the discussions:

Definition2.1: Edge-odd Graceful Graph: A *(p, q)* connected graph is edge-odd graceful if there exists an injection map *f :* E(G) *{1,3,.., 2q-1}* so that induced map f_+:V(G) *{0,1,2,..,(2k-1)}* defined by $f_+(x) \equiv \sum f(x, y)$ (mod 2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge-odd graceful.

Definition 2.2: $(P_3 \times P_n) \nabla P_n$ is a connected graph defined by the following figure 1:



Theorem 2.3: The connected graph $(P_3 \times P_n) \nabla P_n$ (P_n -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph $(P_3 \times P_n) \nabla P_n$

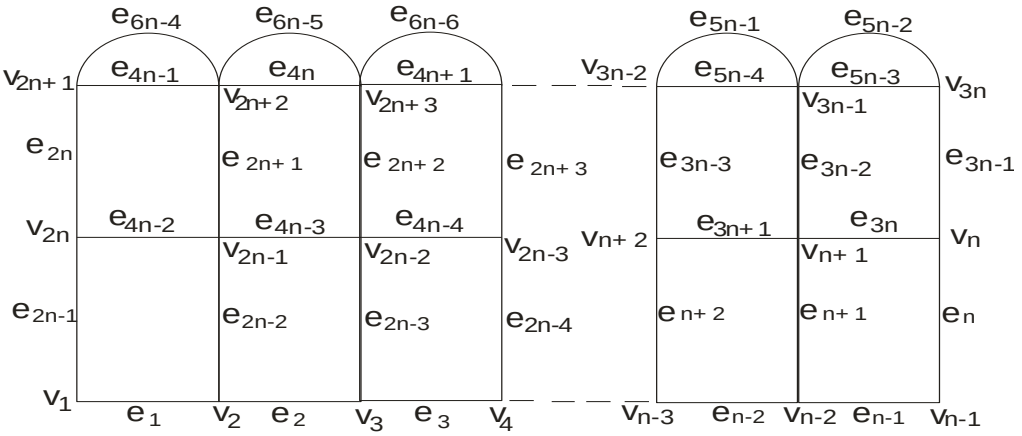


Figure 2: Edge-odd Graceful Graph $(P_3 \times P_n)$ ∇ P_n

To find edge-odd graceful, define f :E(G)→ {1,3,5,.., 2q} by f($\mathbf{e_i}$) = (2i-1), i = 1,2,3, …, (2n-3), (2n-1),…, (4n-3),(4n-1),…,(6n-4) →(1);f(e_{2n-2}) = 8n-5; f(e_{4n-2}) = 4n-5 →(2). The induced map f_+: V(G) → {1,2,3,…, 2q} by $f_+(v) \equiv \sum f(uv)$ (mod2q) where this sum run over all edges through v. Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder $(P_3 \times P_n) \nabla P_n$ is obtained.

Example 2.4: The connected graph $(P_3 \times P_{12}) \nabla P_{12}$ is edge –odd graceful.

Due to the rules (1) and (2) in (2.3), edge odd graceful labeling of the graph $(P_3 \times P_{12}) \nabla P_{12}$ is obtained as follows:

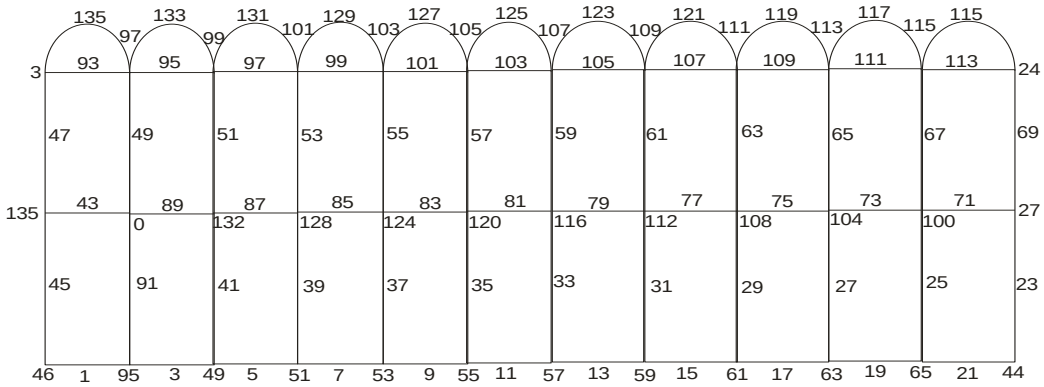
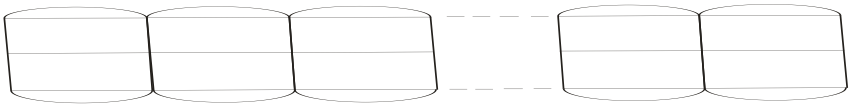


Figure 3: Edge-odd Graceful Graph $(P_2 \times P_{12})$ ∇ P_{12}

Edge-odd graceful of $(P_3 x P_n) \nabla 2P_n$

Definition 3.1: $(P_3 \times P_n) \nabla 2P_n$ is a connected graph defined by the following figure 4.



Theorem 3.2: The connected graph $(P_3 \times P_n) \nabla 2P_n$ (P_n -a path with 'n' vertices) is

edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph $(P_3 \times P_n) \nabla 2P_n$ is follows:

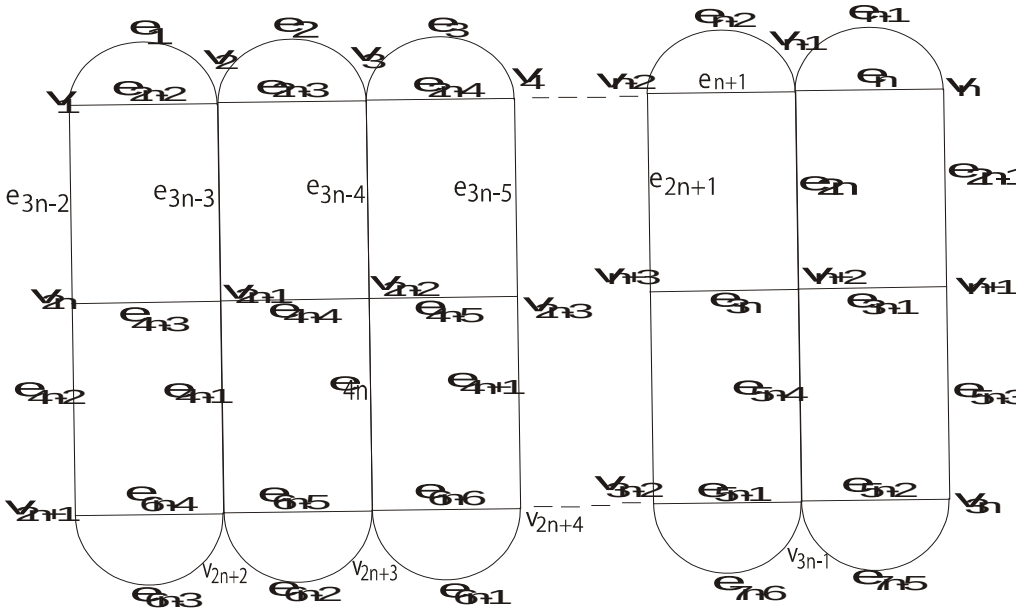


Figure 5: Edge-odd Graceful Graph $(P_3 \times P_n) \nabla 2P_n$

T**o** find edge-odd graceful, define f :E(G)→ {1,3, 5,…, 2q} by $f(\mathbf{e_i}) = (2i-1)$, i = 1,2,3,…, (2n-2),2n,…, (3n-3),(3n-1),…, (7n-5)→(1); $f(e_{2n-1})$= (6n-5); $f(e_{3n-2})$ = 4n-3 →(2)

The induced map f_+: V(G) → {1,2,3,…, 2q} by $f_+(v) \equiv \sum f(uv)$ (mod2q) where this sum run over all edges through v. Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder $(P_3 \times P_n) \nabla 2P_n$ is obtained.

Example 3.3: The connected graph $(P_3 \times P_{12}) \nabla 2P_{12}$ is edge –odd graceful.

Due to the rules (1) and (2) in (3.2), edge odd graceful labeling of $(P_3 \times P_{12}) \nabla 2P_{12}$ is obtained as follows:

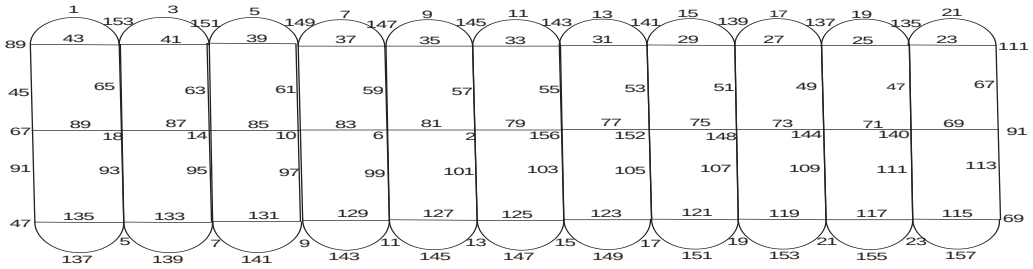
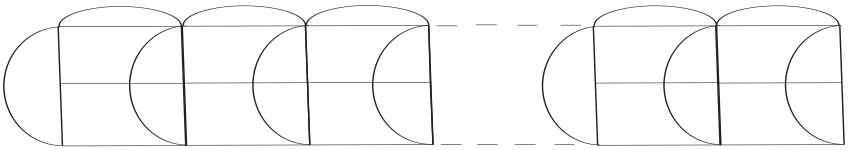


Figure 6: Edge-odd Graceful Graph $(P_2 \times P_{12})$ **∇** P_{12}

Definition 3.4: $(P_3 \times P_n) \oplus P_n$ is a connected graph defined by the following: figure 7.



Theorem 3.5: The connected graph $(P_3 \times P_n) \oplus P_n$ (P_n -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph $(P_3 \times P_n) \oplus P_n$ is as follows:

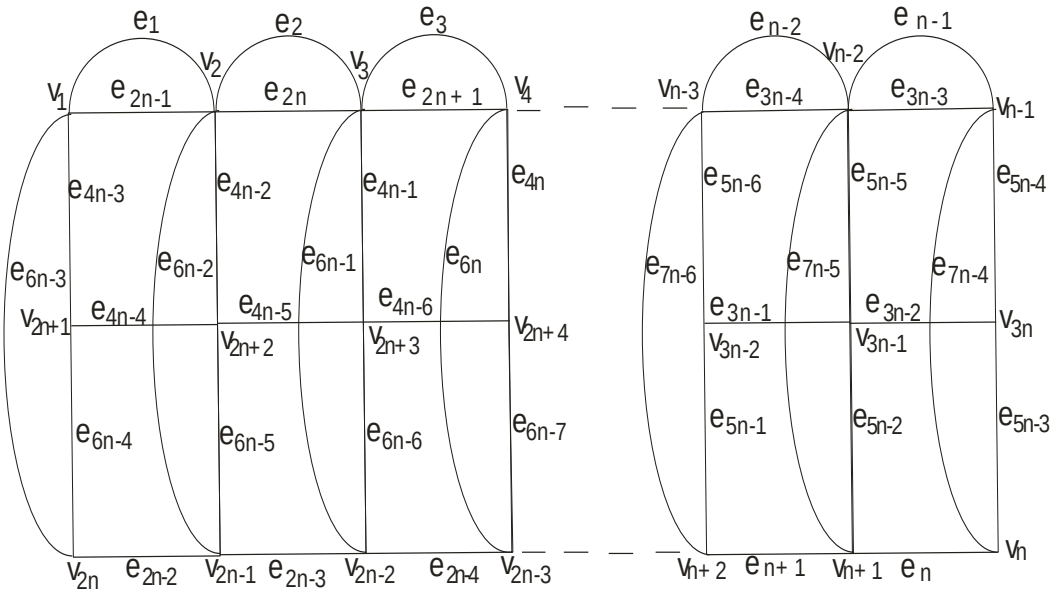


Figure 8: Edge-odd Graceful Graph $(P_3 \times P_n) \oplus P_n$

To find edge-odd graceful , define f :E(G)→ {1,3,5,…,2q} by f ($\mathbf{e_i}$) = (2i-1), i = 1,2,3, …, (2n-3),(2n-1),…,(6n-5),(6n-6),…,(7n-4)→(1); f(e_{6n-4}) = (4n-5); f(e_{2n-2}) = (12n-9) →(2)

The induced map f_+: V(G) $\rightarrow$ {1,2,3,…, 2q} by $f_+(v) \equiv \sum f(uv)$ (mod2q) where this sum run over all edges through v. Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder $(P_3 \times P_n) \oplus P_n$ is obtained.

Example 3.7: The connected graph $(P_3 \times P_8) \oplus P_8$ is edge –odd graceful.

Due to the rules (1) and (2) in (3.5), edge odd graceful labeling of $(P_3 \times P_8) \oplus P_8$ is obtained as follows:So the connected graph $(P_2 \times P_n) \oplus 2P_n$ and $(n\text{-}1)P_3 \times P_n) \lozenge P_n$.

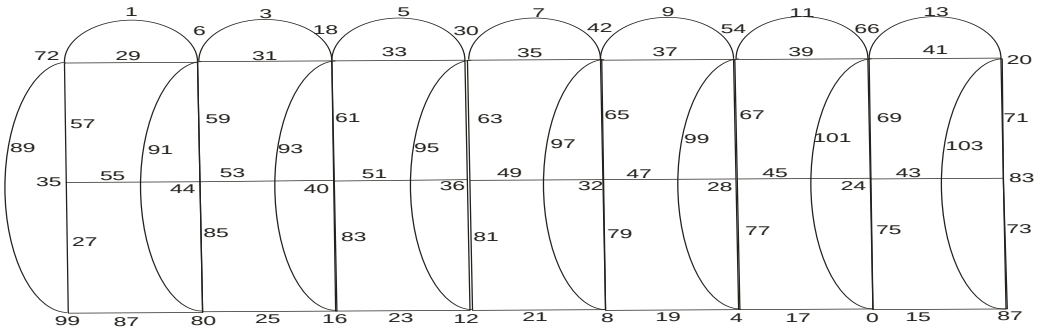
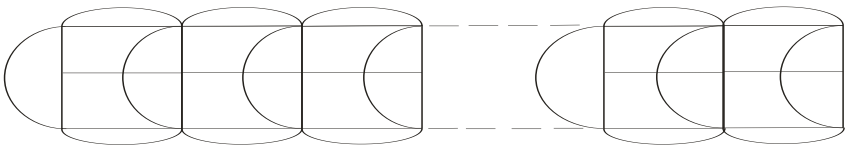


Figure 9: Edge-odd Graceful Graph $(P_3 \times P_8) \oplus P_8$

Definition 3.8: $(P_2 \times P_n) \oplus 2P_n$ is a connected graph defined by the following: figure 10.



Theorem 3.9: The connected graph $(P_2 \times P_n) \oplus 2P_n$ (P_n -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph $(P_2 \times P_n) \oplus 2P_n$ is as folloes:

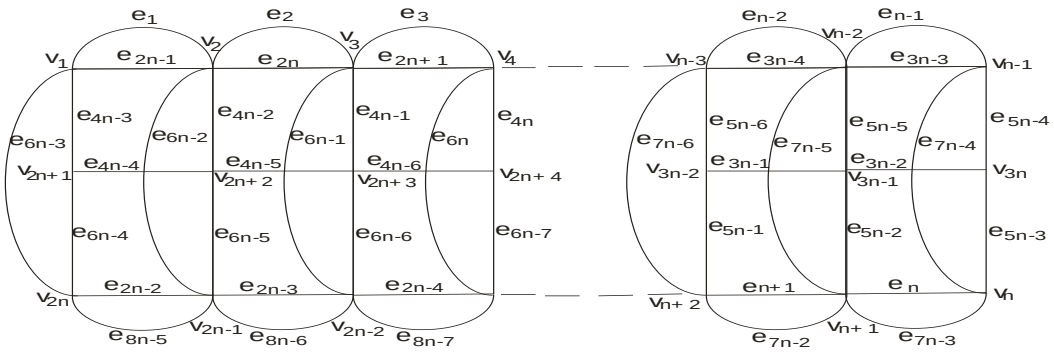


Figure 11: Edge-odd Graceful Graph $(P_2 \times P_n) \oplus 2P_n$

To find edge-odd graceful, define f:E(G)→ {1,3,5,…,2q} by f($\mathbf{e_i}$) = (2i-1), i = 1 to (8n-5)→(1).

The induced map f_+: V(G) → {1,2,3,…,2q} by $f_+(v) \equiv \sum f(uv) \pmod{2q}$ where this sum run over all edges through v …(2).Both of f and f_+ find the distinct labels for vertices and also the edge labelings. Here the edge –odd graceful labeling of ladder $(P_2 \times P_n) \oplus 2P_n$ is obtained.

Example 3.10: The connected graph $(P_3 \times P_9) \oplus 2P_9$ is edge –odd graceful.

Due to the rules (1) and (2) in (3.9), edge odd graceful labeling of $(P_3 \times P_9) \oplus 2P_9$ is obtained as follows:

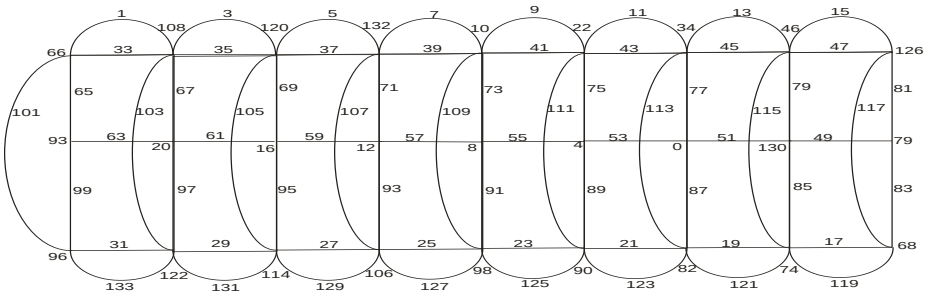
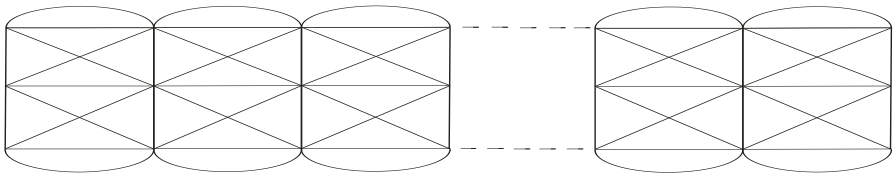


Figure 12: Edge-odd Graceful Graph $(P_3 \times P_9) \oplus 2P_9$

Definition 3.12: $(n-1)P_3 \times P_n) \lozenge P_n$ is a connected graph defined by the following figure 13.



Theorem 3.13: The connected graph $(n-1)P_3 \times P_n) \lozenge P_n$ (P_n -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph $(n-1)P_3 \times P_n) \lozenge P_n$ is as follows:

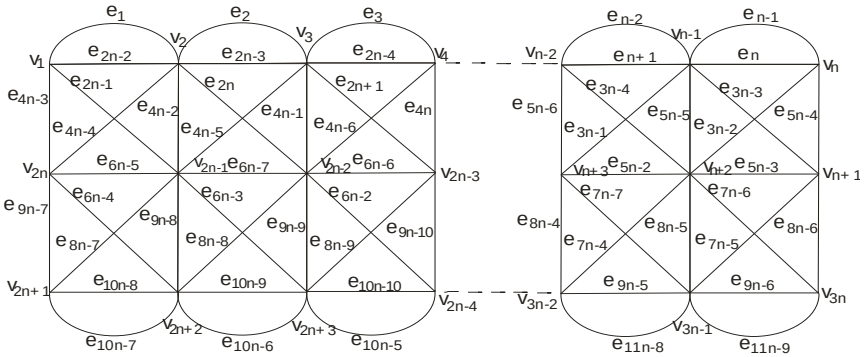


Figure 14: Edge-odd Graceful Graph $(n-1)P_3 \times P_n) \lozenge P_n$

To find edge-odd graceful, define f:E(G)→ {1,3,5,…, 2q} by f($\mathbf{e_i}$) = (2i-1), i = 1 to (11n – 9) →(1). The induced map f_+: V(G) → {1,2,3,…,2q} by f_+ (v) ≡ ∑f(uv) (mod2q) where this sum run over all edges through v. Both of f and f_+ find the distinct labels for vertices and also the edge labeling is distinct. Here the edge – odd graceful labeling of ladder (n-1)$P_3 \times P_n$) ◊ P_n is obtained.

Example 3.14: The connected graph ($11k_4P_3 \times P_{12}$) ◊ P_{12} is edge –odd graceful. Due to the rules (1) in (3.13), edge odd graceful labeling of ($P_2 \times P_{12}$) ◊ P_{12} is obtained as follows:

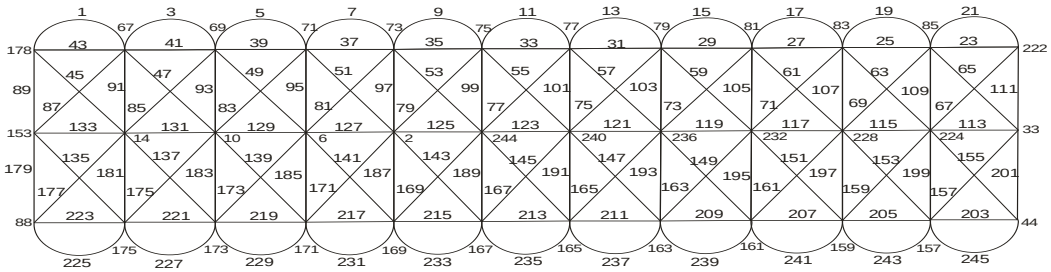


Figure 15: Edge-odd Graceful Graph ($11\mathbf{k_4}P_3 \times P_{12}$) ◊ P_{12}

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