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Algorithm To Construct Reverse Super Vertex Magic Labeling Of Complete Graphs

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ABSTRACT

The study of graph labeling has focused on finding classes of graphs which admits a particular type of labeling. In this paper we consider a particular class of graph which demonstrates Reverse super vertex magic labeling. The classes we consider here is a complete graph K_n . The reverse vertex magic labeling on a graph with v vertices and e edges will be defined as a one to one map taking the vertices and edges on to the integers 1,2,3,…..v + e with the property that the difference of the labels of vertex incident edges and the label on a vertex is constant independent of the choice of the vertex. The reverse super vertex magic labeling of a graph is the reverse vertex magic labeling with the condition that all the vertices of the graph takes the labels 1,2,3,---- , v . We use the magic square of the order n to construct reverse super vertex magic labeling for K_n.

Key Words: Magic Squares, reverse magic constant , complete graphs, reverse vertex magic labeling, reverse super vertex magic labeling.

1.INTRODUCTION

Let G=(V,E) be a graph which is finite , simple and undirected . The graph G has a vertex set V = V(G) and edge set E=E(G) . We denote an e = $|E| and v = |V|$. A standard graph theoretic notation is followed . In this paper we deal only with complete graphs. The labeling of agraph is a map that takes graph elements V or E or $V \cup E$ to the numbers. In this paper , the domain is a set of all vertices and edges giving rise to labeling . The most complete recent survey of graph labeling [1].

Sedlacek introduced the magic labeling concept in 1963. The notion of reverse super magic graph was introduced by S.Venkataramana in 2007[5].Formal definitions of reverse magic labeling , reverse vertex magic labeling , reverse edge magic labeling , reverse super vertex magic labeling of graphs are as follows.

In [5], we defined a reverse magic labeling of a graph G(V,E) as a bijection $f: V \cup E \to \{1,2,3,.....,v+\varepsilon\}$ such that for all edges xy, f(xy)-{f(x)+f(y)} are the same where v and ε denote the order and size of the graph G. A graph is said to be A reverse edge-magic labeling f is called reverse super edge-magic if f(V) = {1,2,3,….., v} and f(E)={ v+1, v+2, v+3,……….. $V+\varepsilon$ }. A graph G is called reverse super edge-magic if there exists a reverse super edge-magic labeling of G. In [4] the notion of a vertex –magic total labeling was introduced . This is an assignment of the integers from 1 to $v + \varepsilon$ to the vertices and edges of G so that at each vertex label and the label on the edges incident at that vertex , add to a fixed constant . More formally , a one -to –one map f from $V \cup E$ onto the integers {1,2,3,……, $v+\varepsilon$} is a vertex – magic labeling if there is a constant k and so that for every vertex u,

$$f(u) + \sum f(uv) = k$$

Where the sum runs over all vertices v adjacent to u .

In [4] , V.Swaminathan et al presented super vertex-magic labeling of a graph ie the vertex - magic labeling f is called super vertex-magic if f(E) ={ 1,2,3,……, ε} and f(V) = { ε+1, ε+2, ε+3,……, $\varepsilon+v$} . A graph G is called super vertex-magic if there exists a super vertex-magic labeling of G.

In [5], S.VenkataRamanaetalintroduced the concept of reverse super vertex-magic labeling of a graph. A reverse vertex-magic labeling f is a bijection f from $V \cup E$ onto the integers {1,2,3,……, $v+\varepsilon$} such that for all vertex u, f[N(u)] - f(u) is a constant.

A reverse vertex-magic labeling f is called reverse super vertex-magic labeling if f(E) ={ 1,2,3,……, ε} and f(V) = { ε+1, ε+2, ε+3,……, $\varepsilon+v$} . A graph G is called reverse super vertex-magic if there exists a reverse super vertex-magic labeling of G.

In this paper , we have proposed an algorithm to construct reverse supervertex magic labeling of complete graphs K_n where n is odd and $\boldsymbol{n \equiv 0(mod4), where\ n} > 4$. Also we proved that K_n , where $\boldsymbol{n \equiv 2(mod4)}$ does not have RSVML.

2. Construction:

The process of constructing RSVML of complete graph is carried out as follows:

- K_n, where n is odd.
- K_n , where n $\equiv o(mod4)$ and n> 4.
- K_n , where n $\equiv 2(mod4)$ are not RSVML.

Theorem2.1:- The reverse magic constant k for the given K_n lieswith in the range

$$\boldsymbol{\frac{n(n-1)(n-3)}{4} - 1 \text{to} \frac{n(n-1)(n+3)}{4} - 1.}$$

Proof:-Case(i)

The complete graph K_n has n vertices and n(n-1)/2 edges . Therefore to label the graph elements , we have to choose numbers from the set {1,2,3,…….[(n(n-1)/2)+n]} . If choose {1,2,3,…(n(n-1)/2)} numbers to label the edges and $\{\frac{n(n-1)}{2}+1, \frac{n(n-1)}{2}+2, \dots\dots\dots, \frac{n(n-1)}{2}+n\}$ to label the vertices , we will get the minimum reverse magic constant is $\frac{n(n-1)(n-3)}{4}-1$.

Let $m=\frac{n(n-1)}{2}$.$K_{min}=\frac{2\sum_{j=1}^{m}(n+j)-\sum_{i=1}^{n} i}{n}$

$$\therefore 2\sum_{i=1}^{m} i = 2[1+2+3+\cdots\dots\dots\dots..+m] = 2\frac{m(m+1)}{2}$$

$$= \frac{n(n-1)}{2}\left[\frac{n(n-1)}{2}+1\right] \quad = \frac{n(n-1)}{4}[n^2-n+2]$$

$$= \frac{(n^2-n)}{4}[n^2-n+2] = \frac{n^4-n^3+2n^2-n^3+n^2-2n}{4}$$

$$= \frac{n^4-2n^3+3n^2-2n}{4}.$$

$$\sum_{j=1}^{n}(m+j) = (m+1)+(m+2)+\ (m+3)+\cdots\dots\dots.\dots\dots..+(m+n)$$

$$= \text{mn} + \frac{n(n+1)}{2} = \frac{n(n-1)}{2}n + \frac{n(n+1)}{2}$$

$$= \frac{n}{2}[n(n-1)+(n+1)] = \frac{n(n^2+1)}{2} = \frac{n^3+n}{2}.$$

$$\text{So } k_{min} = \frac{\frac{n^4-2n^3+3n^2-2n}{4}-\frac{(n^3+n)}{2}}{n} = \frac{n^4-2n^3+3n^2-2n-2n^3-2n}{4n}$$

$$= \frac{n^4-4n^3+3n^2-4n}{4n} = \frac{n[n^3-4n^2+3n-4]}{4n}$$

$$= \frac{n(n^2-4n+3)}{4}-1 = \frac{n(n-1)(n-3)}{4}-1.$$

Case(ii) :- If we choose {1,2,3,…………,n} to label the vertices of K_n and {n+1, n+2, n+3,……(n+$\frac{n(n-1)}{2}$)} to label the edges of K_n , we will get the maximum reverse magic constant $\frac{n(n-1)(n-3)}{4}-1$.

Let $m=\frac{n(n-1)}{2}$.$K_{min}=\frac{2\sum_{j=1}^{m}(n+j)-\sum_{i=1}^{n} i}{n}$

$$2\sum_{j=1}^{m}(n+j) = 2\{(n+1)+(n+2)+\ (n+3)+\cdots\dots\dots.\dots\dots..+(n+m)$$

$$=2\{\text{nm} + \frac{m(m+1)}{2}\} = 2\{n\frac{n(n-1)}{2} + \frac{\frac{n(n-1)}{2}(\frac{n(n-1)}{2}+1)}{2}$$

$$= 2\{\frac{n^3-n}{2} + \frac{(n^2-n)}{8}[n^2-n+2]\} = 2\{\frac{n^3-n^2}{2} + \frac{n^4-n^3+2n^2-n^3+n^2-2n}{8}\}$$

$$= 2\left\{\frac{4n^3-4n^2+n^4-2n^3+3n^2-2n}{8}\right\} = \frac{n^4+2n^3-n^2-2n}{4}.$$

$$\sum_{i=1}^{n} i = [1 + 2 + 3 + \cdots \ldots \ldots \ldots .. + n] = \frac{n(n+1)}{2}$$

So k_{max} $= \dfrac{\frac{n^4+2n^3-n^2-2n}{4} - \frac{(n^2+n)}{2}}{n} = \dfrac{n^4+2n^3-n^2-2n-2n^2-2n}{4n}$

$= \dfrac{n^4+2n^3-3n^2-4n}{4n} = \dfrac{n[n^3+2n^2-3n-4]}{4n}$

$= \dfrac{n(n^2+2n-3)}{4} - 1 = \dfrac{n(n-1)(n+3)}{4} - 1.$

2.2. Algorithm to Construct RSVML for K_n , Where n is odd.

This algorithm takes an integer parameter n , the number of vertices of the complete graph K_n. This algorithm construct an nXn matrix , the construction is similar to the construction of magic square of order n. Let M denotes such a nXn matrix and i,j are row and column indices respectively. The array S contains numbers from 1 to n+m where $m = \frac{n(n-1)}{2}$.

Step1: [Initialization] i→ $1; j \rightarrow 1;\ m \rightarrow \frac{n(n-1)}{2}$.
Repeat for k→ 1 to n+m do S[k]→k;

Step 2: Fill the matrix M starting with [i,j] in such a way that the filling process proceeds in down – right direction using the numbers of S. Assume that the rows and columns are cyclically around. If the present position is already filled then get back to the previous position and start filling from its down position . Repeat this until all the numbers from S are filled in M.

Step3: At this stage n+$\frac{n(n-1)}{2}$ cells of M are filled and the remaining $\frac{n(n-1)}{2}$ cells are empty. Fill these empty cells by symmetry ie M[i,j] = M[j,i].

Step4: Now use the entries of M to label the vertices and edges of K_n as follows. The entries across the diagonal are used to label the vertices and left out entries to label the edges of the complete graph K_n.

Step5: Stop.
Applying this algorithm to K_3 and K_5 are illustrated below.
Example 1: K_3 , Here n=3 and m= $\frac{3(3-1)}{2} = 3$ and so S = {1,2,3,4,5,6}.

1		

1		
	2	

1		
	2	
		3

1		
	2	**6**
		3

1		
	2	**6**
5		**3**

1	**4**	
	2	**6**
5		**3**

By symmetry fill the remaining cells

1	4	5
4	2	6
5	6	3

The reverse magic constant is $8 = \frac{n(n-1)(n+3)}{4} - 1 = \frac{3(3-1)(3+1)}{4} - 1$

Mapping of the matrix entries to vertices and edges of K_3 is as shown below in Fig 1.

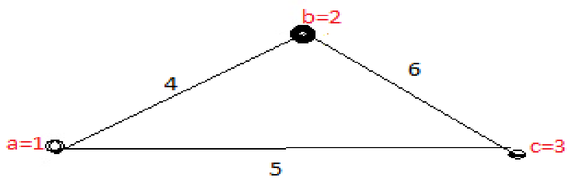

Fig: Reverse Labeled graph K_3

Example2:- K_5 , here n =5 and m = 5(5-1)/2 = 10 and so S = { 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15} .

1				
	2			
		3		
			4	
				5

1			11	
	2			15
14		3		
	13		4	
		12		5

1	7		11	
	2	6		15
14		3	10	
	13		4	9
8		12		5

1	7	14	11	8
7	2	6	13	15
14	6	3	10	12
11	13	10	4	9
8	15	12	9	5

The Reverse magic constant is $39 = \frac{n(n-1)(n+3)}{4} - 1 = \frac{5(5-1)(5+3)}{4} - 1.$

Mapping of the matrix entries to vertices and edges of K_5 is as shown in fig2 below.

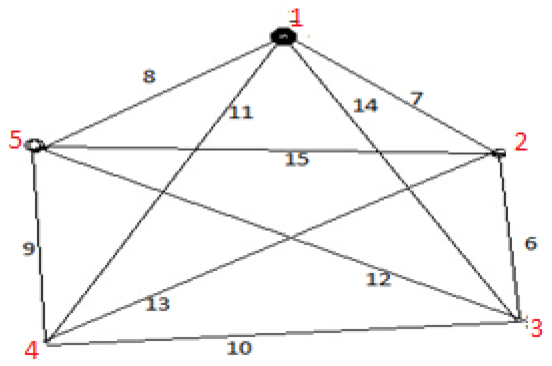

Fig2: Reverse Labeled graph K_5

2.3. There exists no RVSML for K_n , where n ≡ 2(mod4).
From Theorem 2.1 , it is known that the maximum reverse magic constant for K_n is
For any positive integer n. If n is even and n≡ 2(mod4) . The equation
Evaluates to a non integer , hence it is not possible to obtain RSVML for K_n , where n ≡2(mod4) the equation $\frac{n(n-1)(n+3)}{4} - 1$ evaluates to a non integer , hence it is not possible to obtain RSVML for K_n , where $\boldsymbol{n \equiv 2(mod4)}$.

Example :- If n = 6, then $\frac{n(n-1)(n+3)}{4} - 1 = \frac{6(6-1)(6+3)}{4} - 1$=66.5 becomes

2.4 . There exists RSMVL for K_n , where $\boldsymbol{n \equiv 0(mod4)}$.
From Theorem 2.1, it is known that the maximum reverse magic constant for K_n is for any +ve integer n. If n is even and $\equiv 0(mod4)$, the equation evaluates to an integer; hence it is possible to obtain RSVML f, where possible$n \equiv 0(mod4)$.

Example4:- For K_n here n = 8 and the reverse magic constant is -153.
Using the algorithm in [ ] , the 8x8 matrix which realizes the RSVML for K_8 is shown below.

	1	2	3	4	5	6	7	8
1	--	30	26	29	21	10	14	24
2	30	---	22	34	33	15	12	9
3	26	22	---	25	18	35	17	13
4	29	34	25	---	11	16	23	19
5	21	33	18	11	---	20	27	28
6	10	15	35	16	20	---	31	32
7	14	12	17	23	27	31	---	36
8	24	9	13	19	28	32	36	---
k	153	153	153	153	153	153	153	153

Conclusion

From section 2.1 ,section 2.2 and section 2.3 , it follows that all complete graphs K_n for n > 2 there is a reverse super vertex magic labeling (RSVML) except for K_4 and K_n, $n \equiv 0(mod4)$.

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