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Nanogeneralized-closed sets and Slightly NanoSeparation Axioms

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Abstract

A new kind of sets callednanogeneralized-closed (briefly N g -closed) sets is introduced and studied in a nano topological space.The class of all N g -closed sets is strictly larger than the class of all nano -closed sets. Furthermore, prove that g -closed sets is a special type of N g -closed sets in a nano topological space. Some of their properties are investigated. Finally, some characterirzations of nano -regular and nano -normal spaces have been given.

Keyword: nano topological space**,** N g -closed, nano -regular , nano -normal

Introduction

LellisThivagar [1] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He has also studied nano closure and nano interior of a set. Njastad [5], Levine [2] and Mashhour [3] respectively introduced the notions of - open, semi-open and pre-open sets.Since then these concepts have been widelyinvestigated. It was made clear that each -open set is semi-open and pre-open but the converse of each is not true. Njastad has shown that the family of - open sets is a topology on X satisfying . The families SO(X,τ) of all semi–open sets and PO(X,τ) of all preopen sets in (X,τ) are not topologies. It was proved that both SO(X,τ) and PO(X,τ) are closed under arbitrary unions but not under finite intersection.

In this paper A new kind of sets calledN g -closed sets is introduced and studied in a nano topological space [1](by short, **NANO TOP**).The class of all N g -closed sets is strictly larger than the class of all nano -closed sets. Furthermore, prove that g -closed sets is a special type of N g-closed sets in a nano topological space. Some of their properties are investigated. Finally, some characterizations of nano -regular and nano -normal spaces have been given.

Definition 1.1 [1] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. If $\tau_R(X)$ satisfies the following axioms:

1. $U, \phi \in \tau_R(X)$
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$

Then $\tau_R(X)$ is a topology on U called the nanotopology on with respect to X. We call as the **NANO TOP**space The elements of $\tau_R(X)$ are called as nano-open sets.

Definition 1.2[1] If $(U, \tau_R(X))$ is **NANO TOP** space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $N\,\mathrm{int}(A)$. That is, $N\,\mathrm{int}(A)$ is the largest nano-open subset of A. The nano closure of A is defined as theintersection of all nano closed sets containing A and it is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallestnano closed set containing A.

Definition 1.3[1] A **NANO TOP**space $(U, \tau_R(X))$ is said to be extremally disconnected, if the nanoclosure of each nano-open set is nano-open.

Definition 1.4 [1] Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. A subset A of U is said to be nanosemi-open [2](by short $N\,SO$-open) (resp. nanopreopen [3](by short $N\,PO$-open), α-open [4](by short $N\,\alpha O$-open), nano- β-open (by short $N\,\beta O$-open) or nanosemi-preopen (by short $N\,SPO$-open), nano- b-open(by short $N\,bO$-open)) if $A \subset cl(int(A))$

(resp. $A \subset int(cl(A)), A \subset int(cl(int(A))), A \subset cl(int(cl(A))), A \subset cl(int(A)) \cup int(cl(A))$). A subset A is said to be nan-δ-open(by short $N\,\delta O$-open) if for each $x \in A$, there exists a Nano regular open(by short $N\,rO$-open) set G such that $x \in G \subset A$. The complement of a nano-δ-open set is said to be nano-δ-closed.

Definition 1.5 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. A subset A of U is said to be nano-δ-semi-open (resp. nano-δ-preopen) if $A \subset cl(Int_\delta(A))$ (resp. $A \subset int(cl_\delta(A))$).

The family of all nanosemi-open (resp. nanopreopen, nano-α-open, nano-β-open, nano-b-open, nano-δ-open, nano-δ-semi-open, nano-δ-preopen) sets in X

is denoted by $NSO(X)$ (resp. $NPO(X)$, $N\alpha(X)$, $N\beta(X)$, $NBO(X)$, $N\delta O(X)$, $N\delta SO(X)$, $N\delta PO(X)$).

The following lemma is useful in the sequel:

Lemma 1.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X and $A,B \in U$, then

1. $N\operatorname{int}(A) \subseteq A \subseteq Ncl(A))$.
2. $A \subseteq B$ implies that $N\operatorname{int}(A) \subseteq N\operatorname{int}(B)$ and $Ncl(A) \subseteq Ncl(B)$
3. $N\operatorname{int}(N\operatorname{int}(A)) = N\operatorname{int}(A)$ and $Ncl(Ncl(A)) = Ncl(A)$
4. $N\operatorname{int}(U-A) = U - Ncl(A)$ and $Ncl(U-A) = U - N\operatorname{int}(A)$

Lemma 1.2 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X, then

1. $x \in N\operatorname{int}(A)$ if and only if there is a nano-open set $w \subseteq A$ such that $x \in w$
2. $x \in Ncl(A)$ if and only if $w \cap A \neq \phi$ whenever $x \in w \in \tau_R(X)$.
3. If $A \in \tau_R(X)$, then $A = N\operatorname{int}(A)$ and if A is nano-closed, then $A = Ncl(A)$

Remark 1.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X, *then*

1. $N\operatorname{int}(\phi) = \phi$ and $Ncl(X) = X$
2. $N\operatorname{int}(X)$ is the union of all nano -open sets in X
3. $Ncl(\phi)$ is the intersection of all nano -closed sets in X

Properties of Ng-Closed Set

Definition 2.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X then, $A \subseteq U$ is called a Ng-closed set if $Ncl(A) \subseteq B$ whenever $A \subseteq B \in \tau_R(X)$. The complement of a N$g$-closed set is called a Ng-open set

Example 2.1 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$ Then the $\tau_R(X) = \{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$ **NANO TOP** on U with respect to X. $A = \{a,b\}$ is a Ng-closed set

Theorem 2.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X then. If $A \subseteq U$ is Ng-closed set, then $Ncl(A) - A$ does not contain any non-empty closed set.

Proof. Let F be a N g -closedsubset of U such that $F \subseteq Ncl(A) - A$, where A is N g -closed. Since $U - F$ is open, $A \subseteq U - F$ and A is N g -closed, $Ncl(A) \subseteq U - F$ and thus $F \subseteq U - Ncl(A)$. Thus $F \subseteq (U - Ncl(A)) \cap Ncl(A) = \phi$ and hence $F = \phi$.

If $Ncl(A) - A$ does not contain any non-empty closed subset of U, then A need not be N g -closed in general.

Corollary 2.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X and $A \subseteq U$ be a N g -closed set. Then $Ncl(A) = A$ if and only if $Ncl(A) - A$ is closed.

Proof. Let A be a N g -closed set. If $Ncl(A) = A$, then $Ncl(A) - A = \phi$, and $Ncl(A) - A$ is a closed set. Conversely, let $Ncl(A) - A$ be a closed set, where A is N g -closed. Then by Theorem 2.1, $Ncl(A) - A$ does not contain any non empty closed set. Since $Ncl(A) - A$ is a closed subset of itself, $Ncl(A) - A = \phi$ and hence $Ncl(A) = A$.

Theorem 2.2 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X and $A \subseteq B \subseteq Ncl(A)$ where A is N g -closed. Then B is N g -closed.

Proof. Let $B \subseteq V \in \tau_R(X)$. Since A is N g -closed and $A \subseteq V$, $Ncl(A) \subseteq V$. Now, $B \subseteq Ncl(A)$, $Ncl(B) \subseteq Ncl(A)$ and hence $Ncl(B) \subseteq V$.

Theorem 2.3 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. Then A is N g -open if and only if $F \subseteq N\text{int}(A)$ wherever $F \subseteq A$ and F is closed.

Proof. Let A be a N g -open set and $F \subseteq A$, where F is closed. Then $U - A$ is a N g -open set contained in an open set $U - F$. Hence $Ncl(U - A) \subseteq U - F$, i.e. $U - N\text{int}(A) \subseteq U - F$. So $F \subseteq N\text{int}(A)$. Conversely, suppose that $F \subseteq N\text{int}(A)$ for any closed set F whenever $F \subseteq A$. Let $U - A \subseteq V$, where $V \in \tau_R(X)$. Then $U - V \subseteq A$ and $U - V$ is closed. By assumption, $U - V \subseteq N\text{int}(A)$ and hence $Ncl(U - A) = U - N\text{int}(A) \subseteq V$. Therefore $(U - A)$ is N g -closed and hence A is N g -open.

Theorem 2.4 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. Then the following are equivalent:

1. For every N g -open set V of U, $Ncl(V) \subseteq V$.
2. Every subset of U is N g -closed.

Proof.

1. $\Rightarrow$ (2). Let A be any subset of U and $A \subseteq V \in \tau_R(X)$. Then by (1) $Ncl(V) \subseteq V$ and hence $Ncl(A) \subseteq Ncl(V) \subseteq V$. Thus A is Ng-closed.
2. $\Rightarrow$ (1). Let $V \in \tau_R(X)$. Then by (2), V is Ng-closed and hence $Ncl(V) \subseteq V$.

Theorem 2.5 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. If a subset A of U is Ng-open and $N\text{int}(A) \subseteq B \subseteq A$, then B is Ng-open.

Proof. We have $U - A \subseteq U - B \subseteq U - N\text{int}(A) = Ncl(U - A)$. Since $U - A$ is Ng-closed, it follows from Theorem 2.2 that $U - B$ is Ng-closed and hence B is Ng-open.

Now we introduce the following new definition:

Definition 2.2 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X then $\tau_R(X)$ is called a nano $-T_{1/2}$-space (by short $NT_{1/2}$) if for every N g-closed A of U, $Ncl(A) = A$.

Theorem 2.6 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. Then the implication (1) $\Rightarrow$ (2) holds. If $N\text{int}(x) \in \tau_R(X)$ for every $x \in U$, then the following statements are equivalent:

1. U is a $NT_{\frac{1}{2}}$-space.
2. Every singleton is either N-closed or $x = N\text{int}(x)$.

Proof

1. $\Rightarrow$ (2). Suppose x is not a closed subset for some $x \in U$. Then $U - x$ is not N-open and hence U is the only open set containing $U - x$. Therefore $U - x$ is N g -closed. Since U is a $NT_{\frac{1}{2}}$ -space, $(U - x) = U - N\text{int}(x) = U - x$ and thus $x = N\text{int}(x)$.
2. $\Rightarrow$ (1). Let A be a Ng-closed subset of U and $x \in Ncl(A)$. We show that $x \in A$. If x is closed and $x \notin A$, then $x \in (Ncl(A) - A)$. Then $x \in U - A$ and hence $A \subseteq U - x$. Since A is a Ng-closedset and $U - x$ is an open subset of U, $Ncl(A) \subseteq U - x$ and hence $x \in U - N\text{int}(A)$. Therefore $x \in Ncl(A) \cap (U - Ncl(A)) = \phi$. This is a contradiction. Therefore, $x \in A$. If $x = N\text{int}(x)$, since $x \in Ncl(A)$, then for every N-open set V containing x, we have $V \cap A \neq \phi$. But $x = N\text{int}(x)$ is N-open and $x \cap A \neq \phi$. Hence

$x \in A$. Therefore, in both cases we have $x \in A$. Therefore, $A = Ncl(A)$ and hence U is a $NT_{\frac{1}{2}}$-space.

Nano-Regular Spaces And Nano-Normal Spaces

Definition 3.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. Then $\tau_R(X)$ is said to be nano-regular (by short NR) if for each N-closed F subset of U and each $x \notin F$, there exist disjoint N-opensets A and B such that $x \in a$ and $F \subseteq B$.

Theorem 3.1 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X. Consider the following statements:

1. X is NR.
2. For each $x \in U$ and each $A \in \tau_R(X)$ with $x \in A$, there exists $B \in \tau_R(X)$ such that $x \in B \subseteq c_{\omega}(B) \subseteq A$.

Then the implication (1) $\Rightarrow$ (2) holds. If $N\,\text{int}(A) \in \tau_R(X)$ for every N-closed A of U, then the statements are equivalent.

Proof.

1. $\Rightarrow$ (2). Let $x \notin (U - A)$, where $A \in \tau_R(X)$. Then there exist disjoint G, $B \in \tau_R(X)$ such that $(U - A) \subseteq G$ and $x \in B$. Thus $B \subseteq U - G$ and hence $x \in B \subseteq Ncl(B) \subseteq Ncl(U - G) = U - G \subseteq A$.
2. $\Rightarrow$ (1). Let F be a N-closedand $x \notin F$. Then $x \in U - F \in \tau_R(X)$ and hence there exists $B \in \tau_R(X)$ such that $x \in B \subseteq Ncl(B) \subseteq U - F$. Therefore, $F \subseteq U - Ncl(B) = N\,\text{int}(U - B) \in \tau_R(X)$.

Theorem 3.2 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X, and consider the following statements:

1. X is NR.
2. For each closed set F and $x \notin F$, there exists $A \in \tau_R(X)$ and N g-open set B such that $x \in A$, $F \subseteq B$ and $A \cap B = \phi$.
3. For each $V \subseteq U$ and each closed set F with $V \cap F = \phi$, there exist $A \in \tau_R(X)$ and a N g-open set B such that $V \cap A \neq \phi$, and $A \cap B = \phi$. Then the implications (1) $\Rightarrow$ (2)$\Rightarrow$ (3) hold. If $N\,\text{int}(V) \in \tau_R(X)$ for every N g-openset V of U, then the statements are equivalent.

Proof.

1. $\Rightarrow$ (2). Obvious.
2. $\Rightarrow$ (3). Let $V \subseteq U$ and F be a closed set with $V \cap F = \phi$. Then for $x \in V$, $x \notin F$, and hence by (2), there exist $A \in \tau_R(X)$ and a N g-open set B such

that $x \in A$, $F \subseteq B$ and $A \cap B = \phi$. Hence $V \cap A \neq \phi$, $F \subseteq B$ and $A \cap B = \phi$.

3. $\Rightarrow$ (1). Let $x \notin F$, where F is N-closed in U . Since $F \cap \{x\} = \phi$, by (3) there exist A and a N g-closed set W such that $x \in A$, $F \subseteq W$ and $A \cap W = \phi$. Then by Theorem 2.3 we have $F \subseteq i_{\omega}(W) = V \in \tau_R(X)$ and hence $A \cap B = \phi$.

Definition 3.2 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X . Then $\tau_R(X)$ is said to be nano-normal (by short NL)if for any two disjoint N-closed sets A and B there exist two disjoint N -open sets F_1 and F_2 such that $A \subseteq F_1$ and $B \subseteq F_2$.

Theorem 3.3 Let $\tau_R(X)$ be a **NANO TOP** on U with respect to X , and consider the following statements:

1. U is NL.
2. For any pair of disjoint closed sets A and B of U , there exist disjoint N g-open sets F_1 and F_2 of U such that $A \subseteq F_1$ and $B \subseteq F_2$.
3. For each closed set A and each open set B containing A, there exists a N g-open set F_1 such that $A \subseteq F_1 \subseteq Ncl(F_1) \subseteq F_2$.

Then the implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) hold. If $N\operatorname{int}(F_1) \in \tau_R(X)$ and $Ncl(F_1)$ is N -open for every N g-open F_1 of U , then the statements are equivalent.

Proof.

1. $\Rightarrow$ (2). F_1 and F_2 be a pair of disjoint N -closedof U . Then by (1) there exist disjoint N g-opensets F_1 and F_2 of U such that $A \subseteq F_1$ and $B \subseteq V$. Then (2) follows
2. $\Rightarrow$ (3). Let A be a closed set and B be an open set containing A. Then A and $U - B$ are two disjoint N -closedsets. Hence by (2) there exist disjoint N g-open F_1 of U such that $A \subseteq F_1$ and $U - B \subseteq F_2$. Since F_2 is N g-openand $U - B$ is a N -closed set with $U - B \subseteq F_1$, by Theorem 2.5, $U - B \subseteq N\operatorname{int}(F_1)$. Hence $Ncl(U - F_2) = U - N\operatorname{int}(F_2) \subseteq B$. Thus $A \subseteq F_1 \subseteq Ncl(F_1) \subseteq Ncl(U - F_2) \subseteq B$.
3. $\Rightarrow$ (1). Let A and B be two disjoint closed subsets of U . Then A is a N -closed set and $F_1 - B$ is an N -open set containing A. Thus by (3) there exists a N g-openset F_1 such that $A \subseteq F_1 \subseteq Ncl(F_1) \subseteq U - B$. Thus by Theorem 2.5 $A \subseteq N\operatorname{int}(F)$ and $B \subseteq U - Ncl(F)$, where $N\operatorname{int}(F_1)$ and $U - Ncl(F) = N\operatorname{int}(U - F_1)$ are disjoint sets. Since F_1 is N g-open, $N\operatorname{int}(F_1) \in \tau_R(X)$ and $N\operatorname{int}(U - F_1) \in \tau_R(X)$. Hence U is NL.

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