Advances in Theoretical and Applied Mathematics (ATAM).
ISSN 0973-4554 Volume 19, Number 1 (2024), pp. 1-23

http://www.ripublication.com/atam.htm

An Adjusted Borda Rule Based Multi-Criteria Decision-Making Method

Abdulai Inusah[*]

Department of Mathematics, University of Tampere,
Hervanta Campus, Tampere, Finland.
E-mail: inusah.abdulai@tuni.fi

Abstract

In this article, we propose a method for resolving choice and ranking problems in both fuzzy and non-fuzzy environments. One of the advantages of this new method is its ability to resolve choice and ranking problems emanating from both fuzzy and non-fuzzy fields. Hence, unlike the usual methods such as PROMETHEE, AHP, TOPSIS and ELECTRE; there is no need to extend this method to fuzzy fields and vice versa. Another advantage of the method is that, contrary to some outranking methods such as PROMETHEE I, II; the issue of incomparability between pairs of options does not arise at all, and so the problem of loss of information due to the presence of incomparable alternatives is avoided. This simple and user-friendly method makes use chiefly of an adapted Borda rule. To illustrate the use of the method, four data sets have been analysed. The first data set is on electricity generation sources for Turkey, the second one is on the economical growth of the Baltic countries and Poland, the third one is on car model selection, and the fourth data set is about ranking five mobile telecommunication networks from the best to the worst. Further, to show the effectiveness of the novel method, a popular traditional outranking approach called Preference Ranking Organisation MeTHod for Enrichment Evaluation I, II (PROMETHEE I, II) has been applied to the four data sets and the results compared with the results of the novel method introduced herein. Ultimately, it is established that the rankings and choices of the two methods are similar.

Keywords: Borda rule; Fuzzy; Non-fuzzy; Decision alternatives; Performance difference; Ranking; Rank reversal; Borda mark; Preferable.

---

[*] Corresponding author

1. INTRODUCTION

Decision-making involving quite a significant number of alternative courses of action in relation to numerous conflicting criteria is usually a difficult task for all decision-makers Chen et al. (1992) . Depending on the environment as to whether it is a fuzzy or a non-fuzzy one, several decision support models have been advanced by academics and decision science professionals to assist decision-makers deliver on their mandate of making logical and well informed decisions for the achievement of their individual, collective, organisational or institutional goals. This paper introduces a new multi-criteria Decision-making method to help decision-makers resolve their decision problems in both fuzzy and non-fuzzy domains. In this new model, just like every other existing multi-attribute decision making approach, every decision alternative is evaluated on a finite set of criteria. This set of criteria most of the time is a combination of both maximising and minimising criteria. So, in this novel method, the approach used to assign Borda marks to minimising criteria is quite different from that of the normal Borda method. Another distinguishing feature of this method is the calculation of deviations between pairs of alternatives on each criterion. So, on the whole, the method is simple, user-friendly and largely dependent on an adapted Borda rule. Aside the simplicity of this new method and its ability to solve decision problems arising from both fuzzy and non-fuzzy enviroments, it can handle and resolve large size decision problems without much stress. Moreover, under this method, all alternatives in any given set are comparable. This means the loss of vital information as a result of the existence of incomparable alternatives is totally absent here. Another advantage is that, almost every decision maker can understand and use it as it does not involve complex mathematical calculations. However, a major drawback of this method is that it has not been able to satisfy the principle of independence of irrelevant alternatives (IIA) espoused in Arrow's impossibility theorem, and as a result it is not immune to rank reversals. Given for instance three alternatives say x, y and z, the ranking of x, y is said to be *independent of irrelevant alternatives* if the order of x, y does not change when the third alternative, z, is either removed or added to this ranking Dym et al. (2002). On the contrary, a rank reversal is the case where the relative order of two alternatives in a linear order changes when an alternative is either removed or added to the ranking Wang and Luo (2007). Nonetheless, except for theoretical considerations, in practical applications, this consequence does not significantly undermine the validity of the rankings that are derived from the Borda rule Wang and Luo (2007).

Whereas fuzzy fields are distinguished by the availability of imprecise, vague, incomplete, uncertain, and subjective information, non-fuzzy domains are precisely the direct opposite. That is, they are characterised by the presence of crisp, precise, complete and certain information. In each of these two domains, a number of single and hybrid multi-criteria methods have been developed and applied with varied degrees of success. Some of the popular single models advanced in the literature for the resolution of non-fuzzy multi-criteria decision-making problems are the Analytic Hierarchy Process (AHP) proposed by Chen et al. (1992), Saaty et al. (2013), Saaty (1986); the Preference Ranking Organisation MeTHod for Enrichment Evaluation (PROMETHEE) introduced by Brans Saaty (1990) Brans and Vincke (1985), Brans et al. (1986); the Elimination and Choice Translating Reality (ELECTRE) originally introduced in Banayoun et al. (1966); and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) developed by Huang et al. (1981), Hwang and Lin (2012). The rest include the weighted sum model (WSM) Handoko (2017), the weighted product model (WPM), Goswami (2020), Aminudin (2018), the Simple Multi Attribute Rating Technique (SMART) Triantaphyllou et al. (1998), Analytic Network Process (ANP) Triantaphyllou (2000), Decision-Making Trial and Evaluation Laboratory (DEMATEL) Risawandi and Rahim (2016), and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) which stands for Multicriteria Optimization and Compromise Solution, commonly referred to as VIKOR Saaty et al. (2013). Moreover, the most frequently used famous methods to develop hybrid methods are AHP, DEMATEL, TOPSIS and VIKOR Thakkar (2021), Umamaheswari and Kumari (2014). For instance, AHP and PROMETHEE have been integrated as hybrid models for use in various decision-making settings Dağdeviren (2008), Zavadskas et al. (2016), Tzeng and Huang (2011), Babic and Plazibat (1998),Wang et al. (2006), Wang and Yang (2007), Akincilar and Dagdeviren (2014), Goswami and Behera (2021). For fuzzy problems, the single models include fuzzy PROMETHEE Goumas and Lygerou (2000), Geldermann et al. (2000), fuzzy DEMATEL, fuzzy TOPSIS, fuzzy VIKOR, fuzzy ELECTRE, fuzzy AHP, fuzzy ANP, fuzzy WSM, fuzzy WPM methods Umamaheswari and Kumari (2014), Wang and Yang (2007), Rouhani et al. (2014) and many more. The hybrid includes fuzzy DEMATEL based ANP (fuzzy DANP) and VIKOR by Wang and Yang (2007), Arjomandi et al. (2021), fuzzy DEMATEL combined with fuzzy ANP by Lu et al. (2013) , fuzzy DEMATEL combined with fuzzy ANP and multi-objective linear programming, fuzzy ANP and multi-objective linear programming (MOLP) by Rostamkhani and Abbasi (2021), Bakeshlou et al. (2016), Lin (2016), fuzzy DEMATEL-ANP-TOPSIS by Gupta and Jayant (2021), fuzzy ANP and PROMETHEE by Samanlioglu and Ayağ (2016), fuzzy AHP and PROMETHEE by Avikal et al. (2014), Goswami and Behera (2021), Kabir and Sumi (2014). Fuzzy TOPSIS and fuzzy VIKOR Kumar and Barman (2021), Sakthivel et al. (2018), Alpay and Iphar (2018). Another development worthy of mention is the introduction of a hybrid fuzzy N-soft PROMETHEE method by Akram et al. (2023) for the selection of the best robot house keepers. In this hybrid method, AHP was used to establish the weights of criteria. Akram M. et al also combined AHP and PROMETHEE to develop a hybrid fuzzy PROMETHEE method called an m-Polar fuzzy PROMETHEE for group decision making Akram et al. (2020). In this model too, the authors used AHP to determine the weight of each criterion, and many more.

In this paper, we advance a single multi-attribute decision-making method that is capable of addressing choice and ranking problems emanating from both fuzzy and non-fuzzy domains. As previously mentioned, this proposed method mainely depends on an adapted Borda rule.

The rest of the paper is organised as follows: in Section 2, we state the decision problem, in section 3, we recall the concept of the Borda rule; in section 4, we present the algorithm of the proposed method; in section 5, we apply the method to real life multi-attribute decision-making cases; in section 6, we discuss the outcome of the analysis and in section 7, we conclude the study.

2. DECISION PROBLEM

Suppose $A = \{\beta_i = 1, \cdots, m\}$ is a set of options and $B = \{c_j : j = 1, \cdots, n\}$ is a set of criteria. If a set $W = \{w_j \in (0, 1] : j = 1, \cdots, n\}$ denotes the corresponding set of weights of the criteria in B, rank the options in A from the most preferable option to the least preferable one.

3. THE BORDA RULE

The method of choice often referred to as “ the method of marks” was introduced into the system by the French Mathematician and Physicist, Jean-Charles de Borda in Paris in 1770 Black (1976), Hwang and Lin (2012). Borda method is the rank-order method. Let us assume that there are k candidates in a finite set A participating in an election. Then, the application of the Borda rule is dependent on the assumption that the preference order of every voter is offering a utility index of a kind and that every voter's utility is the same as another voter's. So, with k candidates participating in an election, the most preferred candidate is assigned (k-1) marks, the second most preferred candidate is assigned (k-2) marks, the next candidate is given (k-3) marks, the next candidate is assigned (k-4) marks and the least preferred candidate is assigned (k-k) marks or 0. This means among the k candidates in A, the most preferred candidate is the one with the highest number of marks and invariably is the candidate to grand voters the greatest utility.

The steps involved in the implementation of the Borda rule are as follows

1) Rank the decision alternatives or candidates (in case this is an election) on each criterion ordinally as $1st, 2nd, 3rd, \cdots, kth$.

2) Assign to the ranking-oder: $1st, 2nd, 3rd, \cdots, kth$ the marks $k-1, k-2, k-3, \cdots, k-k=0$ respectively.

3) (i) Calculate the Borda score (F_B) for each decision alternative of the set A by finding the sum of the separate marks of each decision alternative over all the criteria. This can be done via the formula
   $F_B(a) = \sum_{b \in A} number(i : aP_ib)$,
   where i denotes a decision-maker or a voter; P denotes preference; a, b are any pair of decision alternatives from A and aP_ib denotes the decision-maker's or voter's preference of a over b. The notation $number(i : aP_ib)$ represents the marks of the decision-maker or voter i preference of a over b, and that of $\sum_{b \in A} number(i : aP_ib)$ denotes the sum of the marks of voters' or decision makers' preference of a to b.

   (ii) In case there is a tie in rank between two or more decision alternatives, assign to each decision alternative or candidate in the tie the average of the marks they would have been assigned if they were to occupy different ranks, and after this you then determine the value of F_B as given in 3) (i)Black (1976) Kendall (1962).

4) Regard the decision alternative with the highest F_B score as the best or the most preferred decision alternative and the one with the lowest F_B score as the worst or the least preferred one. For further details see Black (1976), Hwang and Lin (2012).

4. THE ALGORITHM OF THE PROPOSED METHOD

Using the same decision problem defined in section 2, the step-by-step implementation of the multi-criteria decision-making method introduced herein to solve this kind of decision problems is as follows

Step 1 (i) For all maximising criteria, the bigger the score value the better the decision alternative. Hence, rank the decision alternatives on each maximising criterion as $1st, 2nd, 3rd, \cdots, kth$ such that the alternative with the biggest score value on a maximising criterion is assigned the position $1st$, the second biggest is assigned the position $2nd$, the third biggest is given the position 3rd and so continue this way up to the alternative with the smallest score value and then you assign it the position kth.

(ii) For all minimising criteria, the smaller the score value of a decision alternative on a minimising criterion, the better the alternative. So, rank the decision alternatives on each minimising criterion as $1st, 2nd, 3rd, \cdots, kth$ so that the decision alternative with the smallest score value on a criterion is assigned the position $1st$, the second smallest is assigned the position $2nd$, the third smallest is assigned the position $3rd$ and it continuous this way upto the alternative with the biggest score value and then assign it the position kth.

Step 2 (i) For each maximising criterion, assign to the ranking-oder $1st$, $2nd$, $3rd$, $\cdots$, kth the *Borda marks* $k-1$, $k-2$, $k-3$,$\cdots$, $k-k=0$, respectively.

(ii) For every minimising criterion, assign to the ranking-oder $1st$, $2nd$, $3rd$, $\cdots$, kth the Borda marks 0, 1, 2, 3, $\cdots$, $k-1$, respectively.

(iii) In case there is a tie in rank between two or more decision alternatives, assign to each decision alternative in the tie the average of the marks they would have been assigned if they were to occupy different ranks.

Step 3 Calculate what is called the *weighted Borda marks* by finding the product of the Borda marks of each decision alternative on a criterion and the weight of that criterion.

Step 4 Calculate the performance difference between each pair of alternatives for each criterion c_j, that is, for all $\beta_k, \beta_l \in A$ and $c_j \in C$, where $j = 1, \cdots, n$, calculate $\beta_k - \beta_l$.

Step 5 Determine the sum of the performance difference between each pair of alternatives over all the criteria c_j, that is, $\sum_{j=1}^{n}(\beta_k - \beta_l)$.

Step 6 Rank the alternatives $\beta_i \in A$ in accordance with Definition 1 and Theorem 1.

Definition 4.1. Let $A = \{\beta_i : i = 1, \cdots, m\}$ be a set of options and $B = \{c_j : j = 1, \cdots, n\}$ a set of criteria and the weight of a criterion $c_j \in B$ denoted by w_j. An alternative $\beta_k \in A$ is preferable to another alternative $\beta_l \in A$ written as $\beta_k \succeq \beta_l$ if $\sum_{j=1}^{n}(\beta_k - \beta_l) > \sum_{j=1}^{n}(\beta_l - \beta_k)$, and β_k is equally preferable to β_l denoted by $\beta_k \equiv \beta_l$ if $\sum_{j=1}^{n}(\beta_k - \beta_l) = \sum_{j=1}^{n}(\beta_l - \beta_k) = 0$.

Theorem 1. The relation $\equiv$ is an equivalence relation on A whereas the relation $\succeq$ is not an equivalence relation with respect to A.

Proof of Theorem 1. (1) $\equiv$:

i. For all $\beta_k \in A$, $\sum_{j=1}^{n}(\beta_k - \beta_k) = \sum_{j=1}^{n}(\beta_k - \beta_k) = 0$. Hence, $\beta_k \equiv \beta_k$, and so, $\equiv$ is reflexive.

ii. For all $\beta_k, \beta_l \in A$, such that $\beta_k \equiv \beta_l$, we have $\sum_{j=1}^{n}(\beta_l - \beta_k) = \sum_{j=1}^{n}(\beta_k - \beta_l) = 0$, and so $\beta_l \equiv \beta_k$, and the relation $\equiv$ is symmetric.

iii. For all $\beta_k, \beta_l, \text{and } \beta_z \in \text{A}$, suppose that $\beta_k \equiv \beta_l$, and $\beta_l \equiv \beta_z$. This means from $\sum_{j=1}^{n}(\beta_k - \beta_l) = \sum_{j=1}^{n}(\beta_l - \beta_k) = 0$, and $\sum_{j=1}^{n}(\beta_l - \beta_z) = \sum_{j=1}^{n}(\beta_z - \beta_l) = 0$, we have $\sum_{j=1}^{n} \beta_k = \sum_{j=1}^{n} \beta_l = 0$ and $\sum_{j=1}^{n} \beta_l = \sum_{j=1}^{n} \beta_z = 0$. Thus, $\sum_{j=1}^{n}(\beta_k - \beta_z) = \sum_{j=1}^{n}(\beta_z - \beta_k) = 0$. So, $\beta_k \equiv \beta_z$, and the relation $\equiv$ is transitive.

(2) $\succeq$:

i. For all $\beta_k \in A$, $\sum_{j=1}^{n}(\beta_k - \beta_k) \not\succ \sum_{j=1}^{n}(\beta_k - \beta_k)$. Hence, the relation $\succeq$ is irreflexive, and the proof ends here.

□

5. CASE STUDY 1: RANKING OF SEVEN ELECTRICITY SOURCES FOR TURKEY

(1) Application of the PROMETHEE method. We have applied the novel method and the PROMETHEE approach to the electricity data obtained and evaluated by Topcu and Ulengin (2004) to identify a suitable electricity generation alternative for Turkey.

The data acquired from Turkey and analysed by the authors is shown in Table 1.

Table 1: Electricity generation sources for Turkey

Attributes	**suitability**	**sustainability**	**stability**	**external cost estimate**	**levelized cost**
	max	max	max	min	min
Weight	0.2	0.2	0.2	0.2	0.2
hydro	5	1	7	7.2	5
wind	6	1	7	3.6	45
pv	7	1	10	8.1	250
biomass	3	1	10	32	82
fossil fuels	3	0	6	204	40
natural gas	8	0	4	80	30
nuclear	3	0	10	7.3	75.

Applying the Preference Ranking Organisation Method for Enrichment Evaluation model, we selected the usual-preference function from among the six recommended functions Brans et al. (1986) as the right preference function for this analysis. Eventually, the calculation generated the aggregated preference indices $\pi(\beta_k, \beta_l)$ as in the matrix below

$$\begin{array}{c|ccccccc} & \text{h} & \text{w} & \text{p} & \text{b} & \text{f} & \text{ng} & \text{n} \\ \hline \text{h} & 0 & 0.2 & 0.4 & 0.6 & 1 & 0.8 & 0.8 \\ \text{w} & 0.4 & 0 & 0.4 & 0.6 & 0.8 & 0.4 & 0.8 \\ \text{p} & 0.4 & 0.4 & 0 & 0.4 & 0.8 & 0.6 & 0.4 \\ \text{b} & 0.2 & 0.2 & 0.2 & 0 & 0.6 & 0.6 & 0.2 \\ \text{f} & 0 & 0.2 & 0.2 & 0.2 & 0 & 0.2 & 0.2 \\ \text{ng} & 0 & 0.2 & 0.4 & 0.4 & 0.6 & 0 & 0.6 \\ \text{n} & 0.2 & 0.2 & 0.2 & 0.4 & 0.4 & 0.4 & 0 \end{array}$$

, where the lower case letters h, w, p, b, f, ng and n represent hydro, wind, pv, biomass, fossil fuel, natural gas, and nuclear, respectively. From this matrix of the aggregated preference indices, the values of the leaving, entering and net flows of the decision alternatives have been determined as presented in Table 2.

Table 2: Values of the leaving, entering and net flows of the decision alternatives

α_i	$\phi^+(\beta_i)$	$\phi^-(\beta_i)$	$\phi(\beta_i)$
h	0.63	0.2	0.43
w	0.57	0.23	0.34
p	0.50	0.30	0.20
b	0.33	0.43	-0.10
f	0.17	0.53	-0.36
ng	0.37	0.50	-0.13
nuc	0.23	0.50	-0.27.

Based on Table 2, the complete ranking of the seven electricity sources is as follows, hydro $\succeq$ wind $\succeq$ pv $\succeq$ biomass $\succeq$ natural gas $\succeq$ nuclear $\succeq$ fossil fuels. Therefore, ranking the seven electricity sources in respect of the five criteria for Turkey, hydro has emerged the optimal source of electricity whereas fossil fuels is found to be the worst source.

(2) Application of the new method. We have also applied the new method we have introduced in this study to the electricity data in Table 1 as follows.

   (i) We ranked ordinally the seven alternatives on each criterion according to their performance values.

Table 3: Ordinal ranking of Table 1

Attributes	**suitability**	**sustainability**	**stability**	**external cost estimate**	**levelized cost**
	max	max	max	min	min
hydro	4th	1st	2nd	2nd	1st
wind	3rd	1st	2nd	1st	4th
pv	2nd	1st	1st	4th	7th
biomass	5th	1st	1st	5th	6th
fossil fuels	5th	7th	3rd	7th	3rd
natural gas	1st	7th	4th	6th	2nd
nuclear	5th	7th	1st	3rd	5th.

(ii) We assigned the corresponding marks to the ordinal rankings in Table 3 as shown in Table 4.

Table 4: Marks assigned to the Ordinal rankings in Table 3

Attributes	**suitability**	**sustainability**	**stability**	**external cost estimate**	**levelized cost**
	max	max	max	min	min
hydro	3	4.5	4.5	1	0
wind	4	4.5	4.5	0	3
pv	5	4.5	5.5	3	6
biomass	1	4.5	5.5	4	5
fossil fuels	1	0	4	6	2
natural gas	6	0	3	5	1
nuclear	1	0	5.5	2	4.

(iii) The weighted marks are shown in Table 5.

Table 5: Weighted marks

Attributes	**suitability**	**sustainability**	**stability**	**external cost estimate**	**levelized cost**
	max	max	max	min	min
hydro	0.6	0.9	0.9	0.2	0
wind	0.8	0.9	0.9	0	0.6
pv	1	0.9	1.1	0.6	1.2
biomass	0.2	0.9	1.1	0.8	1
fossil fuels	0.2	0	0.8	1.2	0.4
natural gas	1.2	0	0.6	1	0.2
nuclear	0.2	0	1.1	0.4	0.8

(iv) Let the electricity sources: hydro, wind, pv, biomass, fossil fuels, natural gas, and nuclear be denoted by h, w, pv, b, f, ng, and n, respectively. The performance difference between each pair of alternatives over each criterion is shown in table 6

Table 6: performance difference

Attributes	**suitability** max	**sustainability** max	**stability** max	**external cost estimate** min	**levelized cost** min
h - h	0	0	0	0	0
h - w	-0.2	0	0	-0.2	0.6
h - pv	-0.4	0	-0.2	0.4	0.6
h - b	0.4	0	-0.2	0.6	1
h - f	0.4	0.9	0.1	1	0.4
h - ng	-0.6	0.9	0.3	0.8	0.2
h - n	0.4	0.9	-0.2	0.2	0.8.
w - w	0	0	0	0	0
w - pv	-0.2	0	-0.2	0.6	0.6
w - b	0.6	0	-0.2	0.8	0.4
w - f	0.6	0.9	0.1	1.2	-0.2
w - ng	-0.4	0.9	0.3	1	-0.4
w - n	0.6	0.9	-0.2	0.4	0.2
pv - pv	0	0	0	0	0.
pv - b	0.8	0	0	0.2	-0.2
pv - f	0.8	0.9	0.3	0.6	-0.8
pv - ng	-0.2	0.9	0.5	0.4	-1
pv - n	0.8	0.9	0	-0.2	-0.4
b - b	0	0	0	0	0
b - f	0	0.9	0.3	0.4	-0.8
b - ng	-1	0.9	0.5	0.2	-0.8.
b - n	0	0.9	0	-0.4	-0.2
f - f	0	0	0	0	0
f - ng	-1	0	0.2	-0.2	-0.2
f - n	0	0	-0.3	-0.8	-0.4
ng - ng	0	0	0	0	0
ng - n	1	0	-0.5	-0.6	0.6
n - n	0	0	0	0	0.

(v) Sum of the performance difference between each pair of alternatives over all the criteria in C is in Table 7.

Table 7: Sum of performance difference

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{h}) = 0;$ $\sum_{j=1}^{5}(\mathrm{h}-\mathrm{h}) = 0,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{w}) = 0.2;$ $\sum_{j=1}^{5}(\mathrm{w}-\mathrm{h}) = -0.2,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{pv}) = 0.4;$ $\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{h}) = -0.4,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{b}) = 1.8;$ $\sum_{j=1}^{5}(\mathrm{b}-\mathrm{h}) = -1.8,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{f}) = 2.8;$ $\sum_{j=1}^{5}(\mathrm{f}-\mathrm{h}) = -2.8,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{ng}) = 1.6;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{h}) = -1.6,$

$\sum_{j=1}^{5}(\mathrm{h}-\mathrm{n}) = 2.1;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{h}) = -2.1,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{w}) = 0;$ $\sum_{j=1}^{5}(\mathrm{w}-\mathrm{w}) = 0,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{pv}) = 0.8;$ $\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{w}) = -0.8,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{b}) = 1.6;$ $\sum_{j=1}^{5}(\mathrm{b}-\mathrm{w}) = -1.6,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{f}) = 2.6;$ $\sum_{j=1}^{5}(\mathrm{f}-\mathrm{w}) = -2.6,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{ng}) = 1.4;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{w}) = -1.4,$

$\sum_{j=1}^{5}(\mathrm{w}-\mathrm{n}) = 1.9;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{w}) = -1.9,$

$\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{pv}) = 0;$ $\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{pv}) = 0,$

$\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{b}) = 0.8;$ $\sum_{j=1}^{5}(\mathrm{b}-\mathrm{pv}) = -0.8,$

$\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{f}) = 1.8;$ $\sum_{j=1}^{5}(\mathrm{f}-\mathrm{pv}) = -1.8,$

$\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{ng}) = 0.6;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{pv}) = -0.6,$

$\sum_{j=1}^{5}(\mathrm{pv}-\mathrm{n}) = 1.1;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{pv}) = -1.1,$

$\sum_{j=1}^{5}(\mathrm{b}-\mathrm{b}) = 0;$ $\sum_{j=1}^{5}(\mathrm{b}-\mathrm{b}) = 0,$

$\sum_{j=1}^{5}(\mathrm{b}-\mathrm{f}) = 0.8;$ $\sum_{j=1}^{5}(\mathrm{f}-\mathrm{b}) = -0.8,$

$\sum_{j=1}^{5}(\mathrm{b}-\mathrm{ng}) = -0.2;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{b}) = 0.2,$

$\sum_{j=1}^{5}(\mathrm{b}-\mathrm{n}) = 0.3;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{b}) = -0.3,$

$\sum_{j=1}^{5}(\mathrm{f}-\mathrm{f}) = 0;$ $\sum_{j=1}^{5}(\mathrm{f}-\mathrm{f}) = 0,$

$\sum_{j=1}^{5}(\mathrm{f}-\mathrm{ng}) = -1.2;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{f}) = 1.2,$

$\sum_{j=1}^{5}(\mathrm{f}-\mathrm{n}) = -1.5;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{f}) = 1.5,$

$\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{ng}) = 0;$ $\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{ng}) = 0,$

$\sum_{j=1}^{5}(\mathrm{ng}-\mathrm{n}) = 0.5;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{ng}) = -0.5,$

$\sum_{j=1}^{5}(\mathrm{n}-\mathrm{n}) = 0;$ $\sum_{j=1}^{5}(\mathrm{n}-\mathrm{n}) = 0.$

(vi) Ranking the summations in tandem with Definition 1, Theorem 1, the following is observed

h $\succeq$ w, pv, b, f, ng, n,

w $\succeq$ pv, b, f, ng, n,

pv $\succeq$ b, f, ng, n,

ng $\succeq$ b, f, n,

b $\succeq$ f, n,

n $\succeq$ f.

> From the above comparisons, it is clear that no two electricity sources are equally preferable. That is, there are no $\beta_k, \beta_l \in A$ in which $\beta_k \equiv \beta_l$ holds. Therefore, we have studied the relation β_k is preferable to β_l denoted by $\beta_k \succeq \beta_l$ and obtained the following ranking
> hydro $\succeq$ wind $\succeq$ pv $\succeq$ natural gas $\succeq$ biomass $\succeq$ nuclear $\succeq$ fossil fuels.
> Hence, hydro is the most preferable source of electricity while fossil fuels is the worst or least preferable source.

Summary of the results of the two methods are in Table 8

Table 8: Ranking the seven sources of electricity in descending order from the most preferable source to the least preferable one by the two models

Model	Ranking
PROMETHEE	hydro $\succeq$ wind $\succeq$ pv $\succeq$ biomass $\succeq$ natural gas $\succeq$ nuclear $\succeq$ fossil fuels
New method	hydro $\succeq$ wind $\succeq$ pv $\succeq$ natural gas $\succeq$ biomass $\succeq$ nuclear $\succeq$ fossil fuels

5.1. Case study 2: Criteria values of economical growth of Baltic countries and Poland

Let c_1, c_2, c_3, c_4, c_5 denote the criteria: annual growth of the GDP, %; annual growth of production,%; average annual salary in euro,%; unemployment rate,% and export/import ratio,%, respectively. Then, the Criteria values of economical growth of different countries and the weight of each criterion are as presented Podvezko and Podviezko (2010) in Table 9.

Table 9: Criteria values of economical growth of Estonia, Latvia, Lithuania and Poland

Countries	c_1 max	c_2 max	c_3 max	c_4 min	c_5 max
Weight	0.28	0.19	0.15	0.18	0.20
Estonia	5.1	9.8	430	9.3	0.70
Latvia	7.5	6.5	298	10.3	0.55
Lithuania	9.7	16.1	306	11.6	0.73
Poland	3.8	8.4	501	19.3	0.79

Now, using the usual-preference function under the PROMETHEE model, we have induced the aggregated preference indices $\pi(\beta_k, \beta_l)$ as found in the following matrix

$$\begin{array}{l} \\ \text{Estonia} \\ \text{Latvia} \\ \text{Lithuania} \\ \text{Poland} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Estonia} & \text{Latvia} & \text{Lithuania} & \text{Poland} \end{array} \\ \begin{pmatrix} 0 & 0.72 & 0.33 & 0.65 \\ 0.28 & 0 & 0.18 & 0.46 \\ 0.67 & 0.82 & 0 & 0.65 \\ 0.35 & 0.36 & 0.35 & 0 \end{pmatrix} \end{array},$$

and the corresponding values of the leaving, and entering flows of the four countries are in Table 10.

Table 10: Values of the leaving, and entering flows of countries

β_i	$\phi^+(\beta_i)$	$\phi^-(\beta_i)$
Estonia	0.57	0.43
Latvia	0.31	0.63
Lithuania	0.71	0.29
Poland	0.35	0.59

From Table 10, the ranking of the four countries is the following
Lithuania $\succeq$ Estonia $\succeq$ Poland $\succeq$ Latvia.
Hence, Lithuania has the highest growth and Latvia has the lowest growth.

Also, following the same procedure as applied in case study 1, the novel method is applied as follows

(i) We ranked ordinally the four countries on each criterion according to their performance values displayed in Table 9.

Table 11: Ordinal ranking of Table 9

Countries	c_1 max	c_2 max	c_3 max	c_4 min	c_5 max
Weight	0.28	0.19	0.15	0.18	0.20
Estonia	3rd	2nd	2nd	1st	3rd
Latvia	2nd	4th	4th	2nd	4th
Lithuania	1st	1st	3rd	3rd	2nd
Poland	4th	3rd	1st	4th	1st

(ii) The corresponding marks to the ordinal rankings in Table 11 is shown in Table 12.

Table 12: Marks assigned to the Ordinal rankings in Table 11

Countries	c_1 max	c_2 max	c_3 max	c_4 min	c_5 max
Weight	0.28	0.19	0.15	0.18	0.20
Estonia	1	2	2	0	1
Latvia	2	0	0	1	0
Lithuania	3	3	1	2	2
Poland	0	1	3	3	3

(iii) The weighted marks of table 12 are shown in Table 13.

.

Table 13: Weighted marks

Countries	c_1 max	c_2 max	c_3 max	c_4 min	c_5 max
Weight	0.28	0.19	0.15	0.18	0.20
Estonia	0.28	0.38	0.30	0	0.20
Latvia	0.56	0	0	0.18	0
Lithuania	0.84	0.57	0.15	0.36	0.40
Poland	0	0.19	0.45	0.54	0.60

(iv) Let Es, La, Li, and Po denote Estonia, Latvia, Lithuania and Poland, respectively. The performance difference between each pair of alternatives over each criterion is shown in table 14.

Table 14: performance difference

Countries	c_1 max	c_2 max	c_3 max	c_4 min	c_5 max
Es - Es	0	0	0	0	0
Es - La	-0.28	0.38	0.30	0.18	0.0.20
Es - Li	-0.56	-0.19	0.15	0.36	-0.20
Es - Po	0.28	0.19	-0.15	0.54	0.20
La - La	0	0	0	0	0
La - Li	0.56	-0.57	-0.15	0.18	- 0.40
La - Po	0.56	-0.19	-0.45	0.36	-0.40.
Li - Li	0	0	0	0	0
Li - Po	0.84	0.38	-0.30	0.18	-0.20
Po - Po	0	0	0	0	0

(v) Sum of the performance difference between each pair of countries over all the five criteria is in Table 15

Table 15: Sum of performance difference

$$\begin{array}{rr}
\sum_{j=1}^{5}(\mathrm{Es}-\mathrm{Es}) = 0; & \sum_{j=1}^{5}(\mathrm{Es}-\mathrm{Es}) = 0, \\
\sum_{j=1}^{5}(\mathrm{Es}-\mathrm{La}) = 0.78; & \sum_{j=1}^{5}(\mathrm{La}-\mathrm{Es}) = -0.78, \\
\sum_{j=1}^{5}(\mathrm{Es}-\mathrm{Li}) = -0.44; & \sum_{j=1}^{5}(\mathrm{Li}-\mathrm{Es}) = 0.44, \\
\sum_{j=1}^{5}(\mathrm{Es}-\mathrm{Po}) = 1.06; & \sum_{j=1}^{5}(\mathrm{Po}-\mathrm{Es}) = -1.06, \\
\sum_{j=1}^{5}(\mathrm{La}-\mathrm{La}) = 0; & \sum_{j=1}^{5}(\mathrm{La}-\mathrm{La}) = 0, \\
\sum_{j=1}^{5}(\mathrm{La}-\mathrm{Li}) = -0.38; & \sum_{j=1}^{5}(\mathrm{Li}-\mathrm{La}) = 0.38, \\
\sum_{j=1}^{5}(\mathrm{La}-\mathrm{Po}) = -0.12; & \sum_{j=1}^{5}(\mathrm{Po}-\mathrm{La}) = 0.12, \\
\sum_{j=1}^{5}(\mathrm{Li}-\mathrm{Li}) = 0; & \sum_{j=1}^{5}(\mathrm{Li}-\mathrm{Li}) = 0, \\
\sum_{j=1}^{5}(\mathrm{Li}-\mathrm{Po}) = 0.9; & \sum_{j=1}^{5}(\mathrm{Po}-\mathrm{Li}) = -0.9, \\
\sum_{j=1}^{5}(\mathrm{Po}-\mathrm{Po}) = 0; & \sum_{j=1}^{5}(\mathrm{Po}-\mathrm{Po}) = 0,
\end{array}$$

(vi) Ranking the summations in line with Definition 1, Theorem 1, this is what follows
Li $\succeq$ Es, La, Po,
Es $\succeq$ La, Po,
Po $\succeq$ La.

From this analysis, it is obvious that no two contries have the same level of economical growth. Therefore, we have analysed the relation β_k is preferable to β_l and had the following ranking
Lithuania $\succeq$ Estonia $\succeq$ Poland $\succeq$ Latvia.
Therefore, Lithuania has been found to be the country with the highest economical growth, whereas Latvia is the least one.
In fact, summary of the two rankings are in Table 16

Table 16: Ranking the four countries in descending order from the highest economical growth country to the lowest one by the two models

Model	Ranking
PROMETHEE	Lithuania $\succeq$ Estonia $\succeq$ Poland $\succeq$ Latvia
New method	Lithuania $\succeq$ Estonia $\succeq$ Poland $\succeq$ Latvia

.

5.2. Case study 3: Ranking four cars

Performance data of four car models too have been studied and ordered linearly from the most preferable car model to the least preferable model via the PROMETHEE method and our new method.
Table 17 shows the original data and weights of attributes, and Table 18 displays the complete orders of the two models.

Table 17: The performance of four cars over four criteria

Alternatives	style max	reliability max	fuel eco. max	cost min
weight	0.1	0.4	0.3	0.2
civic	7	9	9	8
saturn	8	7	8	7
ford	9	6	8	9
mazda	6	7	8	6

Table 18: linear orders of the four cars in descending order from the most preferable car to the least preferable one by the two models

Method	Ranking
PROMETHEE	civic $\succeq$ mazda, saturn $\succeq$ ford
New method	civic $\succeq$ mazda, saturn $\succeq$ ford

5.3. Case study 4: providing a complete order for five mobile phone networks in the Greater Accra region of Ghana

Information on the five telcommunication networks was acquired from the National Communication Authority of the country (NCA).

- **Alternatives**: we denote the five networks by β_1, β_2, β_3, β_4, and β_5. This means, $A = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\}$.
- **Criteria**: the NCA uses four main criteria to determine the quality of service each of the five mobile networks delivers to their customers. These four attributes are namely,
  Call set up time (denoted by c_1), Call completion rate (c_2), Call congestion rate (c_3), and Call drop rate (c_4).

- **Weight of criteria**: the weight of each criterion as determined by the NCA is 0.25.
- **Decision Table**: the score values of each network over the four criteria are displayed in Table 19.

Table 19: A table containing the score values of the five operators over the four criteria

Options	c_1	c_2	c_3	c_4
	min	max	min	min
α_1	15.12	80	17	3
α_2	12.09	96	3	1
α_3	11.67	41	27	32
α_4	13.86	81	12	8
α_5	15.28	88	10	2

.

Applying each of the two methods to this data induced the rankings in Table 20.

Table 20: ordering the options in descending order from the most preferable to the least preferable via the two methods

Method	Ranking
PROMETHEE	$\beta_2 \succeq \beta_5 \succeq \beta_4 \succeq \beta_1 \succeq \beta_3$
New method	$\beta_2 \succeq \beta_5 \succeq \beta_4 \succeq \beta_1 \succeq \beta_3$

.

6. DISCUSSION

From this study, it is apparent that the two methods - the traditional PROMETHEE method using the usual generalised preference function, and the new method proposed here - have induced almost the same rankings over all the four cases of study we have analysed. In fact, all decision alternatives in each case study have been established to be in a linear order.

In case study 1, the summary of which is in Table 7, the novel method and the traditional PROMETHEE method have provided almost the same ranking. For they have both selected hydro power as the optimal electricity source for Turkey and fossil fuels as the worst source. However, there is a swap in positions between biomass and natural gas in the two rankings. Whereas biomass is preferred to natural gas in the usual PROMETHEE method, natural gas is preferred to biomass in the new method. The possible reason for this swap is the fact that the new model is more compensatory than the PROMETHEE model. Compensatory in this sense simply means major gains on some criteria offset minor losses on others.

Considering the three remaining cases of study whose results are captured in Tables 14, 16, and 18; both methods have delivered precisely the same rankings on each one of them. In Table 14, both methods have settled on Lithuania as the country with the highest economical growth, and Latvia as the country with least economical growth. In Table 16, both methods have selected civic car to be the most preferable car model, and ford to be the least preferable one. Similarly, in Table 18, the two methods have identified the telecommunication network denoted by β_2 to be the best or most preferable network among the five mobile networks and the network represented by β_3 as the least preferable one.

7. CONCLUSIONS

We have introduced in this study a mathematical technique for resolving multi-criteria decision-making problems (ranking and choice problems). A given outranking problem is addressed by first of all ranking the score values of options on each criterion ordinally. After this ranking, we determine the corresponding Borda marks of each ordinal number. There after, we establish the product of the weight of the corresponding criterion and the Borda marks to get what is called the *weighted Borda marks*. We then compute the performance difference between every pair of alternatives for each criterion from the weighted Borda marks. Having established these performance differences, we calculate the sum of the performance difference between every pair of options with respect to all the criteria. So, given two alternatives say β_k, β_l in the set of alternatives say A, β_k, is said to be preferable to β_l written as $\beta_k \succeq \beta_l$ if the sum of the performance difference, $\sum_{j=1}^{n}(\beta_k-\beta_l)$, over all the criteria is greater than the sum of the performance difference, $\sum_{j=1}^{n}(\beta_l-\beta_k)$, [that is, $\sum_{j=1}^{n}(\beta_k-\beta_l) > \sum_{j=1}^{n}(\beta_l-\beta_k)$]. However, β_k, β_l are equally preferable expressed as $\beta_k \equiv \beta_l$ if the sums of the performance difference are equal and are equal to zero, [that is, $\sum_{j=1}^{n}(\beta_k-\beta_l) = \sum_{j=1}^{n}(\beta_l-\beta_k) = 0$]. Hence, ranking by this method, a linearly ordered ranking for every set of alternatives is always obtained.

Frankly, the method we have proposed in this study has an upper hand over some traditional outranking methods because it is less laborious and has the ability to solve both fuzzy and non-fuzzy multi-criteria decision problems. Moreover, it is able to cope with large size decision problems more efficiently than some methods such ELECTRE I, II in the literature.

To demonstrate the effectiveness and efficiency of the new model, it has been used to analyse four sets of data. The first data set is about the selection of the best electricity generation source out of seven different sources in relation to five criteria for Turkey. The second data has to do with ranking the Baltic countries and Poland based on the level of economic growth. The third one is about identifying the best car model out of four different models, and the fourth data set is to rank five telecommunication networks from the best to the worst within the region of Accra.

In fact, the results of the new method over all the four cases of study have been compared with that of the PROMETHEE I, II method and they have been found to be similar.

Eventually, the novel method selected hydro power as the best or optimal energy source for Turkey and fossil fuels as the worst one. In terms of economic growth, Lithuania has emerged as the country with the highest economic growth followed by Estonia, Poland and then Latvia. On the car models, the method has settled on the civic car as the best car model and ford as the least best, while mazda and saturn are on par. On the telecommunication networks, the network identified as β_2 is the best network and that denoted by β_3 is the worst one.

REFERENCES

Cheng, S. J., & Hwang, C. L. (1992). ‘Fuzzy multiple attribute decision making: Methods and applications’, *Lecture Notes in Economics and Mathematical Systems, Springer.*

Dym, C. L., Wood, W. H., & Scott, M. J. (2002). ‘Rank ordering engineering designs: pairwise comparison charts and Borda counts’,*Research in Engineering Design, 13, 236-242.*

Wang, Y. M., & Luo, Y. (2009). ‘On rank reversal in decision analysis’. *Mathematical and Computer Modelling*, 49(5-6), 1221-1229.

Saaty, T. L., Vargas, L. G., Saaty, T. L., & Vargas, L. G. (2013). ‘The analytic network process’ (pp. 1-40). Springer US.

Saaty, T. L. (1986). ‘Axiomatic foundation of the analytic hierarchy process’. *Management science*, 32(7), 841-855.

Saaty, T. L. (1990). ‘An exposition of the AHP in reply to the paper’ “remarks on the analytic hierarchy process”. *Management science*, 36(3), 259-268.

Brans, J. P., & Vincke, P. (1985). ‘Note—A Preference Ranking Organisation Method: (The PROMETHEE Method for Multiple Criteria Decision-Making)’. *Management science*, 31(6), 647-656.

Brans, J. P., Vincke, P., & Mareschal, B. (1986). ‘How to select and how to rank projects: The PROMETHEE method’. *European journal of operational research*, 24(2), 228-238.

Banayoun, R., Roy, B., & Sussman, N. (1966). ‘Manual de Reference du Programme Electre, Note de Synthese et Formation 25’. *Direction Scientifique SEMA.*

Hwang, C. L., Yoon, K., Hwang, C. L., & Yoon, K. (1981). ‘Methods for multiple attribute decision making. Multiple attribute decision making: methods and applications a state-of-the-art survey’, 58-191.

Hwang, C. L., & Lin, M. J. (2012). ‘Group decision making under multiple criteria: methods and applications’ (Vol. 281). *Springer Science & Business Media.*

Handoko, D., Mesran, M., Nasution, S.D., Yuhandri, Y. and Nurdiyanto, H., 2017. Application of Weight Sum Model (WSM) in determining special allocation funds recipients. The IJICS (International Journal of Informatics and Computer Science), 1(2).

Goswami, S.S., Behera, D.K. and Mitra, S., 2020. A comprehensive study of weighted product model for selecting the best product in our daily life. Brazilian Journal of Operations & Production Management, 17(2), pp.1-18.

Aminudin, N., Sundari, E., Shankar, K., Deepalakshmi, P., Fauzi, R.I. and Maseleno, A., 2018. Weighted Product and its application to measure employee performance. International Journal of Engineering & Technology, 7(2.26), pp.102-108.

Triantaphyllou, E., Shu, B., Sanchez, S. N., & Ray, T. (1998). ‘Multi-criteria decision making: an operations research approach’. *Encyclopedia of electrical and electronics engineering, 15(1998)*, 175-186.

Triantaphyllou, E., & Triantaphyllou, E. (2000). ‘Multi-criteria decision making methods’ (pp. 5-21). *Springer US.*

Risawandi, R. R., & Rahim, R. (2016). ‘Study of the simple multi-attribute rating technique for decision support’. *Decision-making, 4, C4.*

Thakkar, J. J., & Thakkar, J. J. (2021). ‘Decision-making trial and evaluation laboratory (DEMATEL)’. *Multi-Criteria Decision Making*, 139-159.

Umamaheswari, A., & Kumari, P. (2014). ‘Fuzzy TOPSIS and fuzzy VIKOR methods using the triangular fuzzy hesitant sets’. *Int J Comput Sci Eng Inf Technol Res, 4(3)*, 15-24.

Zavadskas, E. K., Govindan, K., Antucheviciene, J., & Turskis, Z. (2016). ‘Hybrid multiple criteria decision-making methods: A review of applications for sustainability issues’.*Economic research-Ekonomska istraživanja, 29(1)*, 857-887.

Tzeng, G. H., & Huang, J. J. (2011). ‘Multiple attribute decision making: methods and applications’. *CRC press.*

Babic, Z., & Plazibat, N. (1998). ‘Ranking of enterprises based on multicriterial analysis’. *International journal of production economics, 56,* 29-35.

Wang, J. J., & Yang, D. L. (2007). ‘Using a hybrid multi-criteria decision aid method for information systems outsourcing’. *Computers & Operations Research, 34(12)*, 3691-3700.

Dağdeviren, M. (2008). ‘Decision making in equipment selection: an integrated approach with AHP and PROMETHEE’. *Journal of intelligent manufacturing, 19*, 397-406.

Wang, J. J., Wei, C. M., & Yang, D. L. (2006). ‘A decision method for vendor selection based on AHP/PROMETHEE/GAIA’. *JOURNAL-DALIAN UNIVERSITY OF TECHNOLOGY, 46(6)*, 926.

Rouhani, S., Ashrafi, A., & Afshari, S. (2014, April). ‘Fuzzy DEMATEL model for evaluation criteria of business intelligence’. *In International Conference on Enterprise Information Systems (Vol. 2, pp. 456-463).* SciTePress.

Goumas, M., & Lygerou, V. (2000). ‘An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects’.*European Journal of Operational Research, 123(3),* 606-613.

Geldermann, J., Spengler, T., & Rentz, O. (2000). ‘Fuzzy outranking for environmental assessment. Case study: iron and steel making industry’. *Fuzzy sets and systems, 115(1),* 45-65.

Lu, M. T., Tzeng, G. H., & Tang, L. L. (2013). 'Environmental strategic orientations for improving green innovation performance in fuzzy environment-Using new fuzzy hybrid MCDM model'. *International Journal of Fuzzy Systems, 15(3).*

Arjomandi, M. A., Dinmohammadi, F., Mosallanezhad, B., & Shafiee, M. (2021). 'A fuzzy DEMATEL-ANP-VIKOR analytical model for maintenance strategy selection of safety critical assets'. *Advances in Mechanical Engineering, 13(4),* 1687814021994965.

Samanlioglu, F., & Ayağ, Z. (2016). 'Fuzzy ANP-based PROMETHEE II approach for evaluation of machine tool alternatives'. *Journal of Intelligent & Fuzzy Systems, 30(4),* 2223-2235.

Bakeshlou, E. A., Khamseh, A. A., Asl, M. A. G., Sadeghi, J., & Abbaszadeh, M. (2017). 'Evaluating a green supplier selection problem using a hybrid MODM algorithm'. *Journal of Intelligent Manufacturing, 28,* 913-927.

Lin, R. H. (2009). 'An integrated FANP–MOLP for supplier evaluation and order allocation'. *Applied Mathematical Modelling, 33(6),* 2730-2736.

Gupta, V., & Jayant, A. (2021). 'A novel hybrid mcdm approach followed by fuzzy dematel-anp-topsis to evaluate low carbon suppliers'.

Avikal, S., Mishra, P. K., & Jain, R. (2014). 'A Fuzzy AHP and PROMETHEE method-based heuristic for disassembly line balancing problems'.*International Journal of Production Research, 52(5),* 1306-1317.

Goswami, S. S., & Behera, D. K. (2021). 'Evaluation of the best smartphone model in the market by integrating fuzzy-AHP and PROMETHEE decision-making approach'. *Decision, 48,* 71-96.

Kabir, G., & Sumi, R. S. (2014). 'Power substation location selection using fuzzy analytic hierarchy process and PROMETHEE: A case study from Bangladesh'. *Energy, 72,* 717-730.

Kumar, S., & Barman, A. G. (2021). 'Fuzzy TOPSIS and fuzzy VIKOR in selecting green suppliers for sponge iron and steel manufacturing'. *Soft Computing, 25*, 6505-6525.

Sakthivel, G., Saravanakumar, D., & Muthuramalingam, T. (2018). 'Application of failure mode and effect analysis in manufacturing industry-an integrated approach with FAHP-fuzzy TOPSIS and FAHP-fuzzy VIKOR'. *International Journal of Productivity and Quality Management, 24(3)*, 398-423.

Alpay, S., & Iphar, M. (2018). 'Equipment selection based on two different fuzzy multi criteria decision making methods: Fuzzy TOPSIS and fuzzy VIKOR'. *Open Geosciences, 10(1),* 661-677.

Akram, M., Sultan, M., Alcantud, J. C. R., & Al-Shamiri, M. M. A. (2023). 'Extended fuzzy N-Soft PROMETHEE method and its application in robot butler selection'. *Math. Biosci. Eng, 20*, 1774-1800.

Akram, M., Shumaiza, & Alcantud, J. C. R. (2020). 'An m-polar fuzzy PROMETHEE approach for AHP-assisted group decision-making'. *Mathematical and Computational Applications, 25(2),* 26.

Akincilar, A., & Dagdeviren, M. (2014). 'A hybrid multi-criteria decision making model to evaluate hotel websites'. *International Journal of Hospitality Management, 36*, 263-271.

Black, D. (1976). 'Partial justification of the Borda count'. *Public Choice*, 1-15.

Kendall, M. G. (1962). 'Rank Correlation Methods (London: Griffin)'. Kendall3Rank Correlation Methods1962.

Topcu, Y. I., & Ulengin, F. (2004). 'Energy for the future: An integrated decision aid for the case of Turkey'. *Energy, 29(1),* 137-154.

Podvezko, V., & Podviezko, A. (2010). 'Use and choice of preference functions for evaluation of characteristics of socio-economical processes'.

Rostamkhani, R., & Abbasi, M. (2021). 'AN INTEGRATED FUZZY ANP-FUZZY DEMATEL-FUZZY MULTIOBJECTIVE LINEAR PROGRAMMING APPROACH FOR WEAPON SYSTEMS ACQUISITION'. *International Journal of Industrial Engineering, 28(2).*