Intuitionistic Fuzzy Two Stage Multiobjective Transportation Problems

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Abstract

Present paper proposes a new method to find the intuitionistic fuzzy optimal solution of two stage multiobjective transportation problems (MOTP). There are several methods are available in literature for the solution of such problem. But, there is no any methods are available for the solution of two stage MOTP with intuitionistic fuzzy (IF) parameters yet. Here, we have considered MOTP with triangular intuitionistic fuzzy numbers (TIFN) parameters. In this method problem is completed in two stages. Present method is very simple and easy to apply in real life transportation problem (TP).

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Key Words: Intuitionistic Fuzzy Numbers, Triangular, Intuitionistic Fuzzy Numbers, Interval Numbers.

1. INTRODUCTION

Transportation problem is one of the best optimization method applicable in various fields of human activity. TP deals with transportation of goods from a set of supply to a set of demand points so as minimize total transportation cost. Hitchcock [14] initiated and modelled basic transportation in form of standard linear programming problem. In beginning of decision making parameters of MOTP are assumed to be fixed in values. But due many uncertain situation like road conditions, traffic conditions, variation in diesel prices etc. and some other unpredicted factors like weather condition. Therefore

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Definition 1. [2] Let $X$ be universal sets. An intuitionistic fuzzy sets $\tilde{A}$ in $X$ is a sets of the form

$$\tilde{A} = \{(x, \mu_{\tilde{A}} (x), \nu_{\tilde{A}} (x)) : x \in X\},$$

where $\mu_{\tilde{A}} : X \to [0, 1]$ and $\nu_{\tilde{A}} : X \to [0, 1]$, define degrees of membership and non-membership of the element $x \in X$, respectively and for every $x \in X, 0 \leq \mu_{\tilde{A}} (x) + \nu_{\tilde{A}} (x) \leq 1$.

The value of $\pi_{\tilde{A}} (x) = 1 - \mu_{\tilde{A}} (x) - \nu_{\tilde{A}} (x)$, is called the degree of non-determinacy (or uncertainty) of the element $x \in X$ to the intuitionistic fuzzy set $\tilde{A}$.

Definition 2. An intuitionistic fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}} (x), \nu_{\tilde{A}} (x)) : x \in \mathbb{R}\}$ is called intuitionistic fuzzy numbers if

(i) There exists a real number $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}} (x) = 1$ and $\nu_{\tilde{A}} (x) = 0$,

(ii) Membership $\mu_{\tilde{A}}$ of $\tilde{A}$ is convex and non-membership $\nu_{\tilde{A}}$ is concave.

(iii) $\mu_{\tilde{A}}$ is upper semi-continuous and $\nu_{\tilde{A}}$ is lower semi-continuous.

(iv) Support($\tilde{A}$) = $\{x \in \mathbb{R} : \nu_{\tilde{A}} (x) \leq 1\}$.

Definition 3. Let $X$ and $Y$ be two universal sets, $f : X \to Y$ be a function. Extension principle for intuitionistic fuzzy sets find membership and non-membership value $f(\tilde{A})$ where $\tilde{A}$ is an intuitionistic fuzzy sets on $X$:

$$\mu_{f(\tilde{A})} (y) = \begin{cases} \text{Sup}\{\mu_{\tilde{A}} (x), x \in f^{-1} \{y\}\} & \text{if } y \in \text{Range}(f) \\ 0, & \text{if } y \notin \text{Range}(f) \end{cases}$$

$$\nu_{f(\tilde{A})} (y) = \begin{cases} \text{Inf}\{\nu_{\tilde{A}} (x), x \in f^{-1} \{y\}\} & \text{if } y \in \text{Range}(f) \\ 1, & \text{if } y \notin \text{Range}(f) \end{cases}$$

Definition 4. A triangular intuitionistic fuzzy number be given by $\tilde{A} = \{(a_1, a_2, a_3), (b_1, b_2, b_3)\}$ where $a_1, a_2, a_3, b_1, b_2, b_3 \in \mathbb{R}$ such that $b_1 \leq a_1 \leq a_2 = b_2 \leq a_3 \leq b_3$. Its membership and non-membership are given as:

$$\mu_{\tilde{A}} (x) = \begin{cases} 1, & x = a_2 \\ 0, & x \geq a_3 \land x \leq a_1 \\ \phi(x), & a_1 < x < a_2 \end{cases} \quad \nu_{\tilde{A}} (x) = \begin{cases} 0, & x = a_2 \\ 1, & x \geq b_3 \land x \leq b_1 \\ \eta(x), & b_1 < x < a_2 \\ \xi(x), & a_2 < x < b_3 \end{cases}$$

where: $(a_1, a_2) \to [0, 1], \phi: (a_2, a_3) \to [0, 1], \eta: (b_1, a_2) \to [0, 1], \xi: (a_2, b_3) \to [0, 1]$.

Definition 5. Let triangular intuitionistic fuzzy number be given by $\tilde{A} = \{(a_1, a_2, a_3), (b_1, b_2, b_3)\}$ where $b_1 \leq a_1 \leq a_2 = b_2 \leq a_3 \leq b_3$. Then its parametric form are

$$\{ [a_1 + \alpha(a_2 - a_1), \ a_3 - \alpha(a_3 - a_1)], \ [a_2 + (1-\alpha)(b_3 - a_2), \ a_2 - (1-\alpha)(a_2 - b_1)] \}.$$
Definition 6. [12, 13] Let \( \tilde{a} = [a_L, a_R] \) be an interval. The minimization problem with the interval-valued objective function is expressed as \( \min \{ \tilde{a} : \tilde{a} \in S \} \), where \( S \) is a set of constraints, which is equal to bi-objective mathematical programming \( \min \{a_R : m(\tilde{a}) \in S \} \).

2. THEORETICAL DEVELOPMENT

Let minimum IF requirement of a homogeneous product at the destination \( j \) be \( \tilde{b}_j \), IF availability be \( \tilde{a}_i \) of the same at source \( i \) and \( T^k(x) \) is a vector of \( K \) objectives. The TS-MOTP with intuitionistic fuzzy time deals with supplying the destination their minimum IF requirement is stage-I and IF quantity \( \sum_i \tilde{a}_i - \tilde{b}_j \) is supplied to the destination is stage-II from the sources which have surplus IF quantity left after the completion of stage-I, Mathematically, stated the stage-I problem is:

\[
\min T^k_1(X) = \min \left[ \max_{i \times j} \left( t^k_{ij}(x_{ij}) \right) \right], k = 1, 2, \ldots, K
\]

Where

\[
S = \begin{cases} 
\sum_{j=1}^{n} x_{ij} \leq \tilde{\varphi}_i, & i \in I, \\
\sum_{j=1}^{n} x_{ij} \leq \tilde{\phi}_j, & j \in J, \\
x_{ij} \geq 0, & \forall (i, j) \in I \times J,
\end{cases}
\]

where \( \tilde{\varphi}_i, \tilde{\phi}_j \) and \( l_{ij} \) are TIFNs.

Corresponding to a feasible solution \( \tilde{X} = (x_{ij}) \) of the stage-I problem, the set \( S(X) = \tilde{x}_{ij} \) of the feasible solutions of stage-II problem is given by

\[
S = \begin{cases} 
\sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{\varphi}_i, & i \in I, \\
\sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{\phi}_j, & j \in J, \\
\tilde{x}_{ij} \geq 0, & \forall (i, j) \in I \times J,
\end{cases}
\]

where \( \tilde{\varphi}_i \) is the quantity available at the \( i^{th} \) source on the completion of the stage-I, that is \( \tilde{\varphi}_i = \varphi_i - \sum_j x_{ij} \).

Clearly \( \sum_i \tilde{\varphi}_i = \sum_j \varphi_j \). Thus the TS-MOTP would be mathematically formulated as:

\[
\min_{X \in S(X)} T^k_2(X) = \min \tilde{X} \in S(X) [\max_{i \times j} \left( t^k_{ij}(x_{ij}) \right)]
\]

In this paper we have found feasible schedule \( X = (x_{ij}) \) of the stage-I problem corresponding to which the optimal time for stage-II is such that the sum of the shipment times is the least. The TS-MOTP can, therefore, be stated as:
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\[ \text{Minimize } z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \frac{1-a}{2a} c_{ij_1} + \frac{1-a}{2a} c_{ij_2} + \frac{1-a}{2a} c_{ij_3} \right) x_{ij} \]

Such that

\[ \sum_{j=1}^{n} x_{ij} = \{(a_{ij_1}, a_{ij_2}, a_{ij_3}, a_{ij_4}, a_{ij_5}) \}, i = 1, 2, ..., m, \]

\[ \sum_{i=1}^{m} x_{ij} = \{(b_{ij_1}, b_{ij_2}, b_{ij_3}, b_{ij_4}, b_{ij_5}) \}, j = 1, 2, ..., n, \]

\[ x_{ij} \geq 0, i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n. \]

\[ \text{Minimize } z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{1-a}{2a} c_{ij_1} + \frac{1-a}{2a} c_{ij_2} + \frac{1-a}{2a} c_{ij_3} + \frac{1-a}{2a} c_{ij_4} + \frac{1-a}{2a} c_{ij_5} \right] x_{ij} \]

Such that

\[ \sum_{j=1}^{n} x_{ij} = \left[ \frac{1-a}{2a} (a_{ij_1} + c_{ij_3}) + \frac{1-a}{2a} (a_{ij_3} + a_{ij_5}) + \frac{1-a}{2a} (a_{ij_1} + a_{ij_5}) \right], i = 1, 2, ..., m, \]

\[ \sum_{i=1}^{m} x_{ij} = \left[ \frac{1-a}{2a} (b_{ij_1} + b_{ij_3}) + \frac{1-a}{2a} (b_{ij_1} + b_{ij_5}) + \frac{1-a}{2a} (b_{ij_3} + b_{ij_5}) \right], j = 1, 2, ..., n, \]

\[ x_{ij} \geq 0, \ 0 < a < 1, \ i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n. \]

\[ \text{Minimize } z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{1-a}{2a} c_{ij_1} + \frac{1-a}{2a} c_{ij_2} + \frac{1-a}{2a} c_{ij_3} + \frac{1-a}{2a} c_{ij_4} + \frac{1-a}{2a} c_{ij_5} \right] x_{ij} \]

Such that

\[ \sum_{j=1}^{n} x_{ij} = \left[ \frac{1-a}{2a} (a_{ij_1} + c_{ij_3}) + \frac{1-a}{2a} (a_{ij_3} + a_{ij_5}) + \frac{1-a}{2a} (a_{ij_1} + a_{ij_5}) \right], i = 1, 2, ..., m, \]
\[
\sum_{i=1}^{m} x_{ij} = \left[ \frac{1-a(b_{ij1}^k + b_{ij2}^k)}{2} + \frac{1+a(b_{ij1}^k + a(b_{ij3}^k + b_{ij1}^k))}{2} + \frac{1-a(b_{ij3}^k + b_{ij1}^k)}{4} \right], j = 1, 2, ..., n,
\]

\[
x_{ij} \geq 0, \ 0 < \alpha < 1, \ i = 1, 2, ..., m \ \text{and} \ j = 1, 2, ..., n.
\]

Minimize \( z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{3-3a \mu_k}{a} x_{ij} + \frac{1+5a \mu_k}{2a} x_{ij1} + \frac{1-a \nu_k}{2a} x_{ij2} + \frac{1-a \nu_k}{a} x_{ij3} + \frac{3-3a \nu_k}{a} x_{ij1} \right] x_{ij} \)

Such that

\[
\sum_{j=1}^{n} x_{ij} = \frac{3-3a \mu_k}{a} x_{ij3} + \frac{1+5a \mu_k}{2a} x_{ij1} + \frac{1-a \nu_k}{2a} x_{ij2} + \frac{1-a \nu_k}{a} x_{ij3} + \frac{3-3a \nu_k}{a} x_{ij1}, i = 1, 2, ..., m,
\]

\[
\sum_{i=1}^{m} x_{ij} = \frac{3-3a \mu_k}{a} x_{ij3} + \frac{1+5a \mu_k}{2a} x_{ij1} + \frac{1-a \nu_k}{2a} x_{ij2} + \frac{1-a \nu_k}{a} x_{ij3} + \frac{3-3a \nu_k}{a} x_{ij1}, j = 1, 2, ..., n,
\]

\[
x_{ij} \geq 0, \ 0 < \alpha < 1, \ i = 1, 2, ..., m \ \text{and} \ j = 1, 2, ..., n.
\]

3. COMPUTATIONAL ALGORITHM

A two stage multiobjective transportation problem can be solved in following manner:

**Step 1**
Construct the multiobjective intuitionistic fuzzy transportation problems.

**Step 2**
Convert all corresponding objectives and constraints into its crisp form.

**Step 3**
Take one objective function out of given \( k \) objectives and solve it as a single objective subject to the given constraints. Form obtained solution vectors find the values of remaining \( (k-1) \) objective functions.

**Step 4**
Continue the step 3 for remaining \( (k-1) \) objective functions. If all the solutions are same, then one of them is the optimal compromise solution.

**Step 5**
Tabulate the solutions thus obtained in step 3 and step 4 to construct the Positive Ideal Solution (PIS) as given below.
Step 6
From PIS, obtain the lower bounds and upper bounds for each objective functions, where \( f_k^* \) and \( f_k' \) are the maximum, minimum values respectively.

Step 7
Set upper and lower bounds for each objective for degree of acceptance and degree of rejection corresponding to set of solutions obtained in step 4.

For membership functions: Upper and lower bound for membership functions
\[
U_k^\mu = \max(Z_k(X_r)), \quad L_k^\mu = \min(Z_k(X_r)), \quad 0 \leq r \leq K
\]

For non-membership functions:
\[
U_k^\nu = U_k^\mu - \lambda(U_k^\mu - L_k^\mu), \quad U_k^\nu = L_k^\mu, \quad 0 < \lambda < 1.
\]

Step 8
Consider the membership functions \( \mu_k(f_k(x)) \) and non-membership functions \( \nu_k(f_k(x)) \)
As following linear functions:
\[
\mu_k(f_k(x)) = \begin{cases} 
1, & f_k(x) \leq L_k^\mu \\
\frac{u_k^\mu - f_k(x)}{u_k^\mu - l_k^\mu}, & L_k^\mu \leq f_k(x) \leq U_k^\mu \\
0, & f_k(x) \geq U_k^\mu 
\end{cases}
\]

\[
\nu_k(f_k(x)) = \begin{cases} 
1, & f_k(x) \geq U_k^\nu \\
\frac{f_k(x) - L_k^\nu}{u_k^\nu - l_k^\nu}, & L_k^\nu \leq f_k(x) \leq U_k^\nu \\
0, & f_k(x) \leq L_k^\nu 
\end{cases}
\]
Step 9
An intuitionistic fuzzy optimization technique for TS-MOLP problem with such membership and non-membership functions can be written as:

Maximize $\alpha - \beta$

Such that

$\mu_k(f_k(x)) \geq \alpha$,
$\nu_k(f_k(x)) \leq \beta$,
$\alpha + \beta \leq 1$,
$\alpha \geq \beta$,
$\beta \geq 0$,

$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, ..., m$,
$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ..., n$,

$x_{ij} \geq 0$.

Above linear programming problem can be solved by simplex method.

Step 10
Repeat previous steps for stage-I and stage-II.

Step 11
Calculate $Z_{total} = Z_{stage-I} + Z_{stage-II}$.

Stage 12
Calculate acceptance and rejection degrees for $Z_{total}$ by using Zadeh extension principle.

4. ILLUSTRATION OF PROPOSED SOLUTION METHOD
Consider the intuitionistic fuzzy transportation problem

Min $z_1 = \bar{1}x_{11} + \tilde{2}x_{12} + \hat{7}x_{13} + \bar{7}x_{14} + \bar{1}x_{21} + \bar{5}x_{22} + \tilde{3}x_{23} + \bar{4}x_{24} + \bar{6}x_{31} + \bar{9}x_{32} + \bar{9}x_{33} + \bar{6}x_{34}$

Min $z_2 = \bar{4}x_{11} + \tilde{4}x_{12} + \hat{3}x_{13} + \bar{4}x_{14} + \bar{5}x_{21} + \bar{8}x_{22} + \bar{9}x_{23} + \bar{10}x_{24} + \bar{6}x_{31} + \tilde{2}x_{32} + \tilde{5}x_{33} + \bar{1}x_{34}$

Such that

$x_{11} + x_{12} + x_{13} + x_{14} = 8$
$x_{21} + x_{22} + x_{23} + x_{24} = 19$
$x_{31} + x_{32} + x_{33} + x_{34} = 17$
$x_{11} + x_{21} + x_{31} = 11$
$x_{12} + x_{22} + x_{32} = 3$
$x_{13} + x_{23} + x_{33} = 14$
\[ x_{14} + x_{24} + x_{34} = 16 \]
\[ x_{ij} \geq 0 \]

where, \( \bar{\mathcal{I}} = \{(0.1,1,1.5), (0.5,1,1.2)\} \), \( \bar{\mathcal{Z}} = \{(1,2,2.5), (1.5,2,2)\} \)
\( \mathcal{H} = \{(3,7,9), (5,7,8)\}, \mathcal{H} = \{(4,7,9), (6,7,8)\} \)
\( \bar{\mathcal{I}} = \{(0.1,1,1.6), (0.6,1,1.5)\}, \bar{\mathcal{G}} = \{(7,9,11), (8,9,10)\} \)
\( \bar{\mathcal{Z}} = \{(1,3,5), (2,3,4)\}, \bar{\mathcal{G}} = \{(2,4,6), (3,4,5)\} \)
\( \mathcal{H} = \{(6,8,10), (7,8,9)\}, \bar{\mathcal{G}} = \{(7,9,12), (8,9,11)\} \)
\( \bar{\mathcal{I}} = \{(2,4,7), (3,4,6)\}, \bar{\mathcal{G}} = \{(4,6,9), (5,6,8)\} \)
\( \bar{\mathcal{Z}} = \{(1,3,5), (2,3,4)\}, \bar{\mathcal{G}} = \{(2,4,6), (3,4,5)\} \)
\( \bar{\mathcal{I}} = \{(2,4,6), (3,4,5)\}, \bar{\mathcal{G}} = \{(3,5,7), (4,5,6)\} \)
\( \bar{\mathcal{G}} = \{(5,8,10), (5.5,8,11)\}, \bar{\mathcal{G}} = \{(8,9,10), (8.5,9,9.5)\} \)
\( \bar{\mathcal{G}} = \{(8,10,12), (9,10,10.5)\}, \bar{\mathcal{G}} = \{(5,6,6.5), (5.5,6,6.1)\} \)
\( \bar{\mathcal{G}} = \{(1,2,3), (1.2,2,2.8)\}, \bar{\mathcal{G}} = \{(4,5,6), (4.5,5,5.5)\} \).

First stage optimal solutions are: \( x_{11} = 0.5, x_{12} = 1, x_{13} = 0, x_{14} = 0, x_{21} = 2, x_{22} = 0 \),
\( x_{23} = 0.32, x_{24} = 0.17, x_{31} = 0, x_{32} = 0, x_{33} = 0.17, x_{34} = 1.82, \text{Min} \ z_1 = 28.64, \)
\( \text{Min} \ z_2 = 34.43, \alpha = 0.78, \beta = 0. \)

Second stage optimal solutions are: \( x_{11} = 4.5, x_{12} = 2, x_{13} = 0, x_{14} = 0, x_{21} = 4, x_{22} = 0, \)
\( x_{23} = 12.06, x_{24} = 0.43, x_{31} = 0, x_{32} = 0, x_{33} = 1.43, x_{34} = 13.56, \text{Min} \ z_1 = 223.41, \)
\( \text{Min} \ z_2 = 302.74, \alpha = 0.64, \beta = 0. \)

Now using definition [3] we get total minimum values are given as:
\( \text{Min} \ Z_1 = 252.05, \text{Min} \ Z_2 = 337.17 \) with degree of acceptance \( \alpha = 0.78 \) and degree of acceptance \( \beta = 0. \)

**CONCLUSION**

In some situation due to limited capacity of storage, destinations are unable to receive the quantity in excess of their minimum demand. In this case, a single shipment is not possible. Therefore, items are shipped to destinations from the origins in two stages.
Initially, the minimum demands of the destinations are shipped from origins to the destinations. After consuming part of whole of this initial shipment, they are prepared to receive the excess quantity in the second stage. The present method is based on intuitionistic fuzzy sets. Therefore, it will be perfect to handle real transportation problems.

REFERENCES


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