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Fuzzy Incidence Graph Structures

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Abstract

The author introduced the concept of fuzzy incidence graph structure using the concept of fuzzy ρ_i-incidence in his Ph.D. thesis. Some more new concepts like fuzzy ψ_i- incidence tree, fuzzy ψ_i-incidence forest, fuzzy ψ_i-incidence cycle, fuzzy ψ_i-incidence cutpair etc. are introduced. Results related to these concepts are obtained.

Keywords: Fuzzy ψ_i-incidence, fuzzy incidence graph structure, fuzzy ψ_i-incidence cycle, fuzzy ψ_i- incidence tree, fuzzy ψ_i-incidence forest, fuzzy ψ_i-incidence cutpair.

Subject Classification : 05C72, 05C05, 05C38, 05C40

1. INTRODUCTION

The concept of fuzzy set introduced by L.A. Zadeh, in 1965, involves the concept of a membership function defined on a universal set. The value of the membership function lies in [0,1]. Fuzzy graphs were introduced by A. Rosenfeld [1].

The author in [2] introduced the concept of fuzzy incidence graph structure based on the concept of fuzzy ψ_i- incidence. Dinesh [3], Mordeson [4] and Mordeson and Mathew [5] discuss the concept of fuzzy incidence graph in detail. Sampathkumar [6] introduced the concept of a graph structure. This was extended by Dinesh and Ramakrishnan in [7] to fuzzy graph structures.

Here the extent to which a ρ_i- edge is ψ_i-incident with a vertex is also taken care of as the extent to which an edge is incident with a vertex was taken care of in [3]. The extent is represented as a membership function from $V \times R_i$ to $[0, 1]$.

Essential preliminaries are given as per necessity. The concepts from Graph Theory are based on [8] and from Fuzzy Graph Theory on [9].

2. INCIDENCE GRAPH STRUCTURE AND FUZZY INCIDENCE GRAPH STRUCTURE

In [3], the concepts of fuzzy incidence and fuzzy incidence graph were discussed. These are generalized in this paper to graph structures and fuzzy graph structures.

Definition 2.1. *Let V be a non empty set. Then $G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k)$ where $R_i \subseteq V \times V, I_i \subseteq V \times R_i$, is called an incidence graph structure, $i = 1, 2, ..., k$.*

Definition 2.2. *Let $G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k)$ be an incidence graph structure where R_i is a subset of $V \times V$ and I_i is a set of ordered pairs of elements of V and $R_i, i = 1, 2, ..., k$. $(u_i\, v_i) \in R_i$ is an R_i-edge. If $(u_i, (u_i, v_i))$ and $(v_i, (u_i, v_i))$ are in I_i, then it is said to join u_i and v_i for any i. They are I_i-adjacent vertices and they are I_i- incident with (u_i, v_i). The elements of I_i are termed I_i-pairs.*

Definition 2.3. *For an incidence graph structure $G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k)$, if $(u_i, (u_i, v_i))$, $(v_i, (u_i, v_i))$,$(v_i, (v_i, w_i))$, $(w_i, (v_i, w_i))$ are in I_i, then (u_i, v_i) and (v_i, w_i) are I_i-adjacent edges.*

Definition 2.4. *Let G be an incidence graph structure. An I_i-incidence subgraph structure H of G is itself an incidence graph structure having all of its vertices, R_i -edges and I_i-pairs in G.*

Definition 2.5. *In an incidence graph structure $G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k)$, a sequence $u_0, (u_0, (u_0, u_1)), (u_0, u_1), (u_1, (u_0, u_1)), u_1, (u_1, (u_1, u_2)), (u_1, u_2), (u_2, (u_1, u_2)), u_2, (u_2, (u_2, u_3)), (u_2, u_3), (u_3, (u_2, u_3)), u_3, ..., u_k$ is an R_i-walk. If the vertices are different, then the R_i-walk is an R_i-path. If the R_i-edges are different, then it is an R_i-trail.*

We introduce a new term I_i-incidence trail here.

Definition 2.6. *In an incidence graph structure $G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k)$, a sequence $u_0, (u_0, (u_0, u_1)), (u_0, u_1), (u_1, (u_0, u_1)), u_1, (u_1, (u_1, u_2)), (u_1, u_2), (u_2, (u_1, u_2)), u_2, (u_2, (u_2, u_3)), (u_2, u_3), (u_3, (u_2, u_3)), u_3, ..., u_k$ is an I_i-incidence trail if the I_i- pairs are distinct.*

Definition 2.7. *If an R_i-walk in an incidence graph structure G is closed, then it is called an R_i-cycle provided its vertices are distinct.*

Definition 2.8. *If all the pairs of vertices in an incidence graph structure G are joined by an R_i-path, then it is said to be R_i-connected.*

Definition 2.9. *An R_i-connected incidence graph structure G having no R_i-cycles is an R_i-tree.*

Definition 2.10. *An incidence graph structure G having no R_i-cycles is an R_i-forest.*

Definition 2.11. *In an incidence graph structure G, an R_i-edge is said to be an R_i-bridge if the removal of it R_i-disconnects G.*

In a similar way, we can define an R_i-cut vertex.

Definition 2.12. *In an incidence graph structure G, a vertex is said to be an R_i-cutvertex if the removal of it R_i-disconnects G.*

In a similar way, we introduce a new concept, namely, I_i-cutpair. This is similar to the cutpair defined in [3].

Definition 2.13. *In an incidence graph structure G, a ψ_i- pair is said to be an I_i-cutpair if the removal of it R_i-disconnects G.*

Fuzzy ρ_i-incidence is defined in [2] as stated below.

Definition 2.14. *[2] Let $G = (V, R_1, R_2, ..., R_k)$ be a graph structure and $\tilde{G} = (\mu, \rho_1, \rho_2, ...\rho_k)$ (where μ and ρ_i are fuzzy subsets of V and $R_i, i = 1, 2, ..., k$ respectively) be a fuzzy graph structure of G. Let $\psi_i : V \times R_i \to [0, 1]$ be such that $\psi_i(v_j, e_l) \leq \mu(v_i) \wedge \rho_i(e_l) \forall v_j \in V, e_l \in R_i, i = 1, 2, ..., k$. Then ψ_i is the fuzzy ρ_i-incidence of $\tilde{G}$.*

We call this fuzzy ρ_i-incidence as fuzzy ψ_i-incidence now onwards, as it is more appropriate.

3. FUZZY INCIDENCE GRAPH STRUCTURE

The concept of fuzzy ρ_i-incidence graph structure introduced in [2] is modified and restated here. We rename it as a fuzzy incidence graph structure.

Definition 3.1. *Let* $G = (V, R_1, R_2, ..., R_k, V \times R_1, V \times R_2, ..., V \times R_k)$. *Let* $\tilde{G} = (\mu, \rho_1, \rho_2, ..., \rho_k, \psi_1, \psi_2, ..., \psi_k)$ *where* $\mu : V \to [0, 1]$, $\rho_i : R_i \to [0, 1]$ *and* $\psi_i : V \times R_i \to [0, 1]$ *be fuzzy subsets of* V, R_i *and* $V \times R_i$ *respectively with* $\psi_i(v_j, e_l) \leq \mu(v_j) \wedge \rho_i(e_l)$
$\forall v_j \in V, e_l \in R_i$ *and* $\rho_i(e_l) \leq \mu(v_l) \wedge \mu(v_2) \forall v_1, v_2 \in V$. *Then* $\tilde{G}$ *is called a fuzzy incidence graph structure.*

Note that a fuzzy graph structure is itself a fuzzy incidence graph structure with fuzzy ψ_i-incidence taking only two values 0 and 1.

We introduce the concepts of ρ_i-edge, ψ_i-pair, ρ_i-path and ρ_i-connectedness in a fuzzy incidence graph structure in the same way the concepts like edge, path and connectedness are introduced in a a fuzzy graph in [8] and in a fuzzy incidence graph in [3].

In all the coming discussions, by $\tilde{G}$, we mean a fuzzy incidence graph structure $(\mu, \rho_1, \rho_2, ..., \rho_k, \psi_1, \psi_2, ..., \psi_k)$ of the incidence graph structure
$G = (V, R_1, R_2, ..., R_k, I_1, I_2, ..., I_k), I_i \subseteq V \times R_i$ unless otherwise stated.

Definition 3.2. *Let* $(x, y) \in supp(\rho_i)$ *in* $\tilde{G}$. *Then* (x, y) *is a* ρ_i*-edge of the fuzzy incidence graph structure* $\tilde{G}$ *and if*
$(x, (x, y)), (y, (x, y)) \in supp(\psi_i)$, *then* $(x, (x, y))$ *and* $(y, (x, y))$ *are* ψ_i*-pairs,* $i = 1, 2, ..., k$.

Definition 3.3. *If* (x, y) *is a* ρ_i*-edge, but either* $(x, (x, y))$ *or* $(y, (x, y))$ *is not a* ψ_i*-pair, then it is a non* ψ_i*-incident* ρ_i*-edge.*

Definition 3.4. *A sequence*
$v_0, (v_0, e_1), e_1, (v_1, e_1), v_1, (v_1, e_2), e_2, (v_2, e_2), v_2, ..., v_{n-1}, (v_{n-1}, e_n), e_n, (v_n, e_n), v_n$ *of vertices,* ρ_i*-edges and* ψ_i*-pairs in* $\tilde{G}$ *which are distinct except possibly for* $v_0 = v_n$ *such that* (v_{j-1}, v_j) *is a* ρ_i*- edge* e_j*, is a* ρ_i*-path.*

ρ_i-connectedness in $\tilde{G}$ is defined as follows.

Definition 3.5. *Two vertices* v_j *and* v_l *joined by a* ρ_i*-path in a fuzzy incidence graph structure* $\tilde{G}$ *are said to be* ρ_i*-connected.*

Definition 3.6. *The* ψ_i*-incidence strength of a fuzzy incidence graph structure* $\tilde{G}$ *is defined to be equal to*
$\wedge\{\psi_i(v_j, e_l) | (v_j, e_l) \in supp(\psi_i)\}$.

ρ_i-cycle, fuzzy ρ_i-cycle, ρ_i-tree, fuzzy ρ_i-tree, ρ_i-forest, fuzzy ρ_i-forest etc. can be defined similar to fuzzy cycle, tree, fuzzy tree, forest, fuzzy forest etc. defined in a fuzzy incidence graph. Fuzzy ψ_i- incidence cycle, fuzzy ψ_i-incidence tree and fuzzy ψ_i- incidence forest may also be defined.

Definition 3.7. *The fuzzy incidence graph structure $\tilde{G}$ is a ρ_i- cycle iff* $(supp(\mu), supp(\rho_1), supp(\rho_2), ..., supp(\rho_k), supp(\psi_1), supp(\psi_2), supp(\psi_k))$ *is an R_i-cycle.*

Definition 3.8. *$\tilde{G}$ is a fuzzy ρ_i-cycle iff* $(supp(\mu), supp(\rho_1), supp(\rho_2), ..., supp(\rho_k), supp(\psi_1), supp(\psi_2), supp(\psi_k))$ *is an R_i-cycle and there does not exist a unique* $(x, y) \in supp(\rho_i)$ *such that* $\rho_i(x, y) = \wedge\{(u, v)|(u, v) \in supp(\rho_i)\}$.

Definition 3.9. *$\tilde{G}$ is a fuzzy ψ_i-incidence cycle iff it is a fuzzy ρ_i-cycle and there exists no unique $(x, (y, z)) \in supp(\psi_i)$ such that $\psi_i(x, (y, z)) = \wedge\{\psi_i(u, (v, w))|(u, (v, w)) \in supp(\psi_i)\}$.*

Definition 3.10. *$\tilde{G}$ is a ρ_i-tree iff* $(supp(\mu), supp(\rho_1), supp(\rho_2, ..., supp(\rho_k), supp(\psi_1), supp(\psi_2), ..., supp(\psi_k)$ *is an R_i-tree. It is a ρ_i-forest iff* $(supp(\mu), supp(\rho_1), supp(\rho_2, ..., supp(\rho_k), supp(\psi_1), supp(\psi_2), ..., supp(\psi_k))$ *is an R_i- forest.*

Definition 3.11. *$\tilde{H} = (\nu, \tau_1, \tau_2, ..., \tau_k, \sigma_1, \sigma_2, ..., \sigma_k)$ is a fuzzy incidence subgraph structure of $\tilde{G}$ if $\nu \subseteq \mu, \tau_i \subseteq \rho_i, \sigma_i \subseteq \psi_i, i = 1, 2, ..., k$. It is a fuzzy incidence spanning subgraph structure if $\mu = \nu$.*

Definition 3.12. *$\tilde{G}$ is a fuzzy ρ_i-tree iff it has a fuzzy incidence spanning sub graph structure $\tilde{F} = (\mu, \tau_1, \tau_2, ..., \tau_k, \sigma_1, \sigma_2, ..., \sigma_k)$ which is also a τ_i-tree such that $\forall(u, v) \in supp(\rho_i) \setminus supp(\tau_i)$, $\rho_i(u, v) < \tau_i^\infty(u, v)$.*

It is a fuzzy ρ_i-forest iff $\tilde{G}$ has a fuzzy incidence spanning subgraph structure $\tilde{F} = (\mu, \tau_1, \tau_2, ..., \tau_k, \sigma_1, \sigma_2, ..., \sigma_k)$ which is also a τ_i- forest such that $\forall(u, v) \in supp(\rho_i) \setminus supp(\tau_i)$, $\rho_i(u, v) < \tau_i^\infty(u, v)$.

Definition 3.13. *$\tilde{G}$ is a fuzzy ψ_i-incidence tree iff it has a fuzzy incidence spanning subgraph structure $\tilde{F} = (\mu, \rho_1, \rho_2, ..., \rho_k, \sigma_1, \sigma_2, ..., \sigma_k)$ which is also a fuzzy ρ_i- tree such that for every $(u, (v, w)) \in supp(\psi_i) \setminus supp(\sigma_i)$, $\psi_i(u, (v, w)) < \sigma_i^\infty(u, (v, w))$.*

$\tilde{G}$ is a fuzzy ψ_i-incidence forest iff it has a fuzzy incidence spanning subgraph structure which is also a fuzzy ρ_i- forest $\tilde{F} = (\mu, \rho_1, \rho_2, ..., \rho_k, \sigma_1, \sigma_2, ..., \sigma_k)$ such that $\forall(u, v) \in supp(\psi_i) \setminus supp(\sigma_i), \psi_i(u, v) < \sigma_i^\infty(u, v)$.

Many results proved in Mordeson and Nair [8] and in Dinesh [3] may be extended to fuzzy incidence graph structures.

Some of them are discussed below.

Theorem 3.1. *$\tilde{G}$ is a fuzzy ψ_i-incidence forest iff in any ρ_i-cycle of $\tilde{G}$, there is $(x,(y,z))$ such that $\psi_i(x,(y,z)) < \psi_i'^{\infty}(x,(y,z))$ where $\tilde{G}' = (\mu, \rho_1, \rho_2, ..., \rho_k, \psi_1', \psi_2', ..., \psi_k')$ is the fuzzy incidence subgraph structure obtained by deletion of $(x,(y,z))$ from $\tilde{G}$.*

Proof: The result is clearly true if there are no ρ_i-cycles.

Let $(x,(y,z))$ be a ψ_i- pair in $\tilde{G}$ which belongs to a fuzzy ρ_i-cycle such that $\psi_i(x,(y,z)) < \psi_i'^{\infty}(x,(y,z))$.
Assume that $(x,(y,z))$ is the one with the least value for ψ_i among all ψ_i-pairs in $supp(\psi_i)$.
Delete $(x,(y,z))$. The resulting fuzzy incidence subgraph struture is a fuzzy ψ_i-incidence forest.

If there are other such ρ_i-cycles, remove the ψ_i- pairs in a similar manner. The ψ_i- pair thus deleted will be of lesser ψ_i-incidence strength than the previously deleted ones. After finishing the process, the remaining fuzzy incidence subgraph structure is a fuzzy ψ_i-incidence forest $\tilde{F}$.

Thus there exists a ρ_i-path $\tilde{P}$ from x to (y,z) with more ψ_i-incidence strength than $\psi_i(x,(y,z))$ and not containing $(x,(y,z))$. If there are previously deleted ψ_i- pairs in $\tilde{P}$, then we can use ρ_i-path not through them with more ψ_i-incidence strength.

Conversely, if $\tilde{G}$ is a fuzzy ψ_i-incidence forest and C any ρ_i-cycle, then there exists a ψ_i-pair $(x,(y,z))$ of C not in $\tilde{F}$ such that
$\psi_i(x,(y,z)) < \sigma_i^{\infty}(x,(y,z)) \leq \psi_i'^{\infty}(x,(y,z))$ where $\tilde{F}$ is the one mentioned in the definition for fuzzy ψ_i-incidence forest.

Theorem 3.2. *If there is at most one ρ_i-path with the most ψ_i-incidence strength between any vertex and ρ_i-edge of fuzzy incidence graph structure $\tilde{G}$, then $\tilde{G}$ is a fuzzy ψ_i-incidence forest.*

Proof: Consider $\tilde{G}$ in such a way that it is not a fuzzy ψ_i-incidence forest. Then by the theorem discussed earlier, there exists a ρ_i-cycle C in $\tilde{G}$ such that $\psi_i(x,(y,z)) \geq \psi_i'(x,(y,z))$ for every ψ_i-pair $(x,(y,z))$ of C.

Therefore $(x,(y,z))$ is the ρ_i-path with the highest ψ_i-incidence strength from x to (y,z). Suppose $(x,(y,z))$ is the ψ_i- pair with the lowest ψ_i in C. What remains is a

ρ_i- path with the highest ψ_i-incidence strength from x to (y, z). This is a contradiction. Hence $\tilde{G}$ is a fuzzy ψ_i-incidence forest.

Other results on fuzzy graph structures may be similarly proved in fuzzy incidence graph structures. Also we have the following results.

Theorem 3.3. *Let G be a ρ_i-cycle. Then $(\mu, \rho_1, \rho_2, ..., \rho_k, \psi_1, \psi_2, ..., \psi_k)$ is a fuzzy ψ_i-incidence cycle iff G is not a fuzzy ψ_i-incidence tree.*

Proof: Let $\tilde{G} = (\mu, \rho_1, \rho_2, ..., \rho_k, \psi_1, \psi_2, ..., \psi_k)$ be a fuzzy ψ_i-incidence cycle. At least two ψ_i-pairs $(x, (y, z))$ exist satisfying
$\psi_i(x, (y, z)) = \wedge\{\psi_i(u, (v, w)) : u \in V, (v, w) \in supp(\rho_i), (u, (v, w)) \in supp(\psi_i)\}$.
If $(\mu, \rho_1, \rho_2, ..., \rho_k, \sigma_1, \sigma_2, ..., \sigma_k)$ is a spanning fuzzy ψ_i-incidence tree in $\tilde{G}$, then there exists $u \in V, (v, w) \in R_i$ such that $supp(\psi_i) \setminus supp(\sigma_i) = \{(u, (v, w))\}$. So there does not exist a ρ_i- path in $(\mu, \rho_1, \rho_2, ..., \rho_k, \sigma_1, \sigma_2, ..., \sigma_k)$ between u and (v, w) of ψ_i-incidence strength greather than $\psi_i(u, (v, w))$. So $\tilde{G}$ is not a fuzzy ψ_i-incidence tree.

Conversely, suppose that $\tilde{G}$ is not a fuzzy ψ_i-incidence tree. Then it is a fuzzy ρ_i-cycle. For all $(u, (v, w))$ in $supp(\psi_i)$, we have a fuzzy incidence spanning subgraph structure $(\mu, \rho_1, \rho_2, ..., \rho_k, \sigma_1, \sigma_2, ..., \sigma_k)$ which also is a ρ_i- tree and $\sigma_i(u, (v, w)) = 0$,
$\sigma_i^{\infty}(u, (v, w)) \leq \psi_i(u, (v, w))$
and $\sigma_i(x, (y, z)) = \psi_i(x, (y, z)) \forall (x, (y, z)) \in supp(\psi_i) \setminus \{u, (v, w)\}$.

Thus $\wedge\{\psi_i(x, (y, z)) | (x, (y, z)) \in supp(\psi_i)\}$ is not uniquely attained. Therefore $\tilde{G}$ is a fuzzy ψ_i-incidence cycle.

4. ρ_I-CUTVERTEX, ρ_I-BRIDGE AND ψ_I-CUTPAIR

Similar to the ρ_i-bridge and ρ_i-cutvertex in fuzzy graph structures [10], they can be defined in fuzzy incidence graph structures.

Definition 4.1. *(x, y) is a ρ_i-bridge in a fuzzy incidence graph structure $\tilde{G}$ if for some (u, v), $\rho_i'^{\infty}(u, v) < \rho_i^{\infty}(u, v)$ for some u, v.*

Definition 4.2. *w is a ρ_i-cutvertex in a fuzzy incidence graph structure $\tilde{G}$ if $\rho_i'^{\infty}(u, v) < \rho_i^{\infty}(u, v)$ for some $u \neq w \neq v$.*

A new concept, namely, ψ_i-cutpair, may similarly be defined.

Definition 4.3. *In a fuzzy incidence graph structure $\tilde{G}$, $(x, (y, z))$ is a ψ_i-cutpair if $\psi_i'^{\infty}(u, (v, w)) < \psi_i^{\infty}(u, (v, w))$ for some pair $(u, (v, w))$ in $\tilde{G}$.*

Results on ρ_i-bridges and ρ_i-cutvertices in fuzzy graph structures introduced by Dinesh and Ramakrishnan in [10], may be extended to similar concepts in fuzzy incidence graph structures. Further, the following results are also valid in fuzzy incidence graph structures.

Theorem 4.1. *If $\psi_i'^{\infty}(x,(y,z)) < \psi_i(x,(y,z))$ in $\tilde{G}$, then $(x,(y,z))$ is a ψ_i-cutpair.*

Proof: Suppose $(x,(y,z))$ is not a ψ_i-cut pair. Then $\psi_i'^{\infty}(x,(y,z)) = \psi_i^{\infty}(x,(y,z))$ and $\psi_i(x,(y,z)) \leq \psi_i^{\infty}(x,(y,z))$. So $\psi_i(x,(y,z)) \leq \psi_i'^{\infty}(x,(y,z))$. This is a contradiction to the assumption $\psi_i'^{\infty}(x,(y,z)) < \psi_i(x,(y,z))$. Therefore $(x,(y,z))$ is a ψ_i-cut pair.

Theorem 4.2. *If $(x,(y,z))$ in $\tilde{G}$, is not the ψ_i-pair with the least value for ψ_i among all ρ_i-cycles, then $\psi_i'^{\infty}(x,(y,z)) < \psi_i(x,(y,z))$.*

Proof: Let $\psi_i'^{\infty}(x,(y,z)) \geq \psi_i(x,(y,z))$. There exists a ρ_i-path from x to (y,z) which does not involve $(x,(y,z))$ and has ψ_i-incidence strength greater than or equal to $\psi_i(x,(y,z))$. $(x,(y,z))$ and the above ρ_i-path constitute a ρ_i-cycle and $(x,(y,z))$ has the smallest ψ_i. This is a contradiction to the assumption. So $\psi_i'^{\infty}(x,(y,z)) < \psi_i(x,(y,z))$.

Theorem 4.3. *If $(x,(y,z))$ is a ψ_i-cutpair in $\tilde{G}$, then $(x,(y,z))$ is not the ψ_i-pair with the least value for ψ_i among all ρ_i-cycles.*

Proof: If in a ρ_i-cycle, $(x,(y,z))$ is the one ψ_i-pair with the least value for ψ_i, then any ρ_i-path containing it can be made a ρ_i-path without it with ψ_i-incidence strength greater than or equal to ψ_i-value of previously deleted ψ_i-pairs. So $(x,(y,z))$ is not a ψ_i-cutpair. This is a contradiction to our assumption. Hence $(x,(y,z))$ is not a ψ_i-pair with the least value for ψ_i among all ρ_i-cycles.

Theorem 4.4. *If $\tilde{G}$ is a fuzzy ψ_i-incidence forest, then the ψ_i- pairs of $\tilde{F}$ (where $\tilde{F}$ is as in the definition of fuzzy ψ_i-incidence forest) are exactly the ψ_i-cutpairs of $\tilde{G}$.*

Proof: Let $(x,(y,z))$ be not in $\tilde{F}$. Then by definition,
$\psi_i(x,(y,z)) < \sigma_i^{\infty}(x,(y,z)) \leq \psi_i'^{\infty}(x,(y,z))$ where
$(\mu,\rho_1,\rho_2,...,\rho_k,\psi_1',\psi_2',...\psi_i',\psi_{i+1}',...\psi_k')$ is the fuzzy ψ_i-incidence spanning subgraph structure of $\tilde{G}$ obtained by deleting the ψ_i-pair $(x,(y,z))$. It has ψ_i-pairs not in $\tilde{F}$ since $\tilde{F}$ is a fuzzy ψ_i-incidence forest. It has no fuzzy ψ_i-incidence cycles. Therefore $(x,(y,z))$ cannot be a ψ_i-cutpair of $\tilde{F}$.

Let $(x,(y,z))$ be a ψ_i-pair in $\tilde{F}$. Suppose that $(x,(y,z))$ is not a ψ_i-cutpair. Then there is a ρ_i-path $\tilde{P}$ from x to (y,z) not containing $(x,(y,z))$ which has ψ_i-incidence strength greater than or equal to $\psi_i(x,(y,z))$. So some ψ_i-pairs not in $\tilde{F}$ will be in $\tilde{P}$ as $\tilde{F}$ is a fuzzy ρ_i-forest. Any ψ_i-pair $(u,(v,w))$ can be removed and a ρ_i-path $\tilde{Q}$ may be found in $\tilde{F}$. This $\tilde{Q}$ will have more ψ_i-incidence strength than $\psi_i(u,(v,w))$. Also $\psi_i(u,(v,w)) \geq \psi_i(x,(y,z))$. Therefore $(x,(y,z))$ is not in $\tilde{Q}$. Replacing such $(u,(v,w))$ by such ρ_i-path in $\tilde{F}$ we will get a ρ_i-path in $\tilde{F}$ from x to (y,z) not containing $(x,(y,z))$. We have a ρ_i-cycle in $\tilde{F}$. This is a contradiction to the assumption. Hence the ψ_i-pairs of $\tilde{F}$ are the ψ_i-cutpairs of $\tilde{G}$.

Theorem 4.5. *Let* $\tilde{G}^* = (supp(\mu), supp(\rho_1), ..., supp(\rho_k), supp(\psi_1), ..., supp(\psi_k))$ *be a* ρ_i*-cycle in* $\tilde{G}$. *Then a* ρ_i*-edge is a* ρ_i*-bridge of* $\tilde{G}$ *iff it is a* ρ_i*-edge common to two* ψ_i*-cut pairs.*

Proof: Let e be a ρ_i-bridge. Then there are ρ_i-edges f and g in such a way that e lies on every ρ_i-path with the greatest ψ_i-incidence strength between f and g. Therefore there exists only one ρ_i-path with the greatest ψ_i-incidence strength joining f and g involving e. Any ψ_i-pair on this ρ_i-path will be a ψ_i-cutpair since the removal of any one of them will ψ_i-disconnect the ρ_i-path and reduce the ψ_i-incidence strength.

Conversely, let $e = (x,y)$ be common to the ψ_i-cut pairs $(x,(x,y))$ and $(y,(x,y))$. Then they are not the ψ_i-cutpairs with the smallest value for ψ_i. The ρ_i-path from f to g not containing $(x,(x,y))$ and $(y,(x,y))$ has less ψ_i-incidence strength than $\psi_i(x,(x,y)) \wedge \psi_i(y,(x,y))$. Therefore the ρ_i-path with the most ψ_i-incidence strength from f to g is $x,(x,(x,y)),(x,y),(y,(x,y)),y$. Also the ψ_i-incidence strength of ρ_i-path with the most ψ_i-incidence strength from f to g is equal to $\psi_i(x,(x,y)) \wedge \psi_i(y,(x,y))$. Therefore e is a ρ_i-bridge.

5. CONCLUSION

As in the case of fuzzy incidence graph and fuzzy graph structure, the concept of fuzzy incidence graph structure has possible applications in various fields like electrical and electronic networks where not only the edges and vertices are important. The way in which they are connected to each other through various types of relations is also important. This may be of importance in social networks where various types of relations become relevant even between the same individuals.

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