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Linear Programming Problems with coefficients as Trapezoidal Intuitionistic Fuzzy Numbers

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Abstract

This paper extends the value and ambiguity indexes for triangular intuitionistic fuzzy number (TIFN) defined in [1] to trapezoidal intuitionistic fuzzy number (TrIFN), which is used to develop a new ranking function to compare TrIFNs. The proposed ranking function is then used to solve fuzzy linear programming problem with data as TrIFNs. A numerical illustration of the proposed method is also given.

Keywords: Triangular intuitionistic fuzzy number, trapezoidal intuitionistic fuzzy number, ranking method, fuzzy linear programming.

MSC: 90C05, 90C70, 03E72.

1. INTRODUCTION

Linear programming problems have been widely used to model real world problems encountered in allocation, operations research, economic problems and so forth. However, in practical applications, the exact values of the coefficients are either vague or ambiguous due to subjective view-point of the decision maker. Such problems can be modeled using fuzzy sets. Fuzzy set theory was presented by Zadeh [2]. It was taken ahead when Bellman and Zadeh [3] proposed the concept of decision making in fuzzy environments. After the pioneering work on fuzzy linear programming by Zimmermann [4], several kinds of fuzzy linear programming problems have appeared in the literature [5, 6] and different methods have been proposed to solve such problems. Fuzzy linear programming applications in real-world situations are numerous and diverse. For example, water supply planning [7] and farm structure optimization problem in agricultural economics [8], aggregate

production planning problem [9] and machine optimization problems in manufacturing and production [10], capital asset pricing model in banking and finance [11]. The most common concept used in almost all these studies is ranking of fuzzy numbers. There exists a large amount of literature involving the ranking of fuzzy numbers [12-18].

Atanassov [19] introduced the concept of intuitionistic fuzzy sets (IFS), which is a generalization of the fuzzy set and an IFS may express and describe information in a more flexible way than a fuzzy set when uncertainity is involved. Li [20] defined the basic arithmetic operations of TIFNs using the membership and non-membership values. Mitchell [21] interpreted an IFN as an ensemble of ordinary fuzzy numbers and introduced a ranking method. Nagoorgani and Ponnalagu [22] introduced a method of scoring TIFN to rank fuzzy numbers and thereby proposed a method to solve intuitionistic fuzzy linear programming problems. Li [1] introduced a new definition of TIFN. He defined two concepts of the value and the ambiguity of a TIFN similar to those for a fuzzy number introduced by Delgado et al. [23].

In this paper we extend the value and the ambiguity index defined for TIFNs in [1] to TrIFNs. Based upon these indexes we define an ordering relation for TrIFNs, which is used to rank TrIFNs. This ordering relation is then used to solve a linear programming problem with data as TrIFNs. The fuzzy linear programming problem is converted to a crisp linear programming problem using the ordering relation. The thus obtained crisp linear programming problem is solved by using Matlab. A numerical illustration of the method is provided.

2. PRELIMINARIES

2.1 Basic Definitions

Definition 1 [19]. Given a fixed set $X = \{x_1, x_2, \dots, x_n\}$, an intuitionistic fuzzy set (IFS) is defined as $A = (x_i, \mu_A(x_i), \vartheta_A(x_i) | x_i \epsilon X)$, which assigns to each element x_i a membership degree $\mu_A(x_i)$ and a non-membership degree $\vartheta_A(x_i)$ under the condition $0 \leq \mu_A(x_i) + \vartheta_A(x_i) \leq 1$ for all $x_i \epsilon X$.

Definition 2. A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number if its membership function is as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

Definition 3. A fuzzy number $\widetilde{A^I}$ is said to be a trapezoidal intuitionistic fuzzy number if $\widetilde{A^I}$ is an IFS in $\mathbb{R}$ with the following membership function $\mu_A(x_i)$ and non-membership function $\vartheta_A(x_i)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

And,

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1{}'} & a_1{}' \le x \le a_2 \\ 0 & a_2 \le x \le a_3 \\ \dfrac{x - a_3}{a_4{}' - a_3} & a_3 \le x \le a_4{}' \\ 1 & otherwise \end{cases}$$

Where $a_1{}' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4{}'$, $\mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \le 1$.

It is denoted by $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1{}', a_2, a_3, a_4{}')\}$.

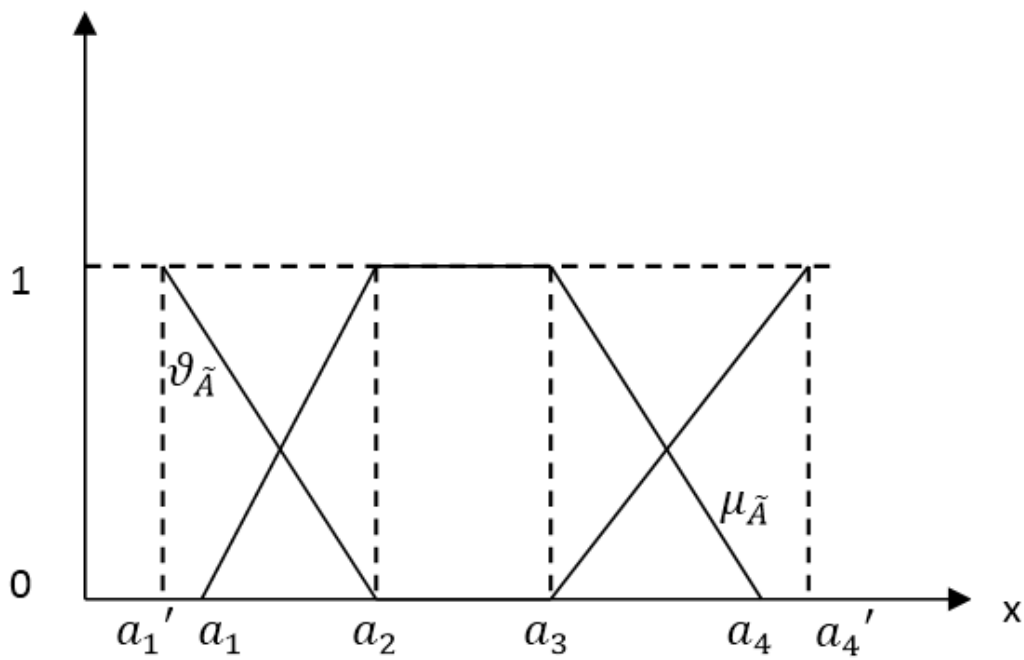

Figure 1. trapezoidal intuitionistic fuzzy number

2.2 Arithmetic operations on TrIFNs.

Let $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ and
$\widetilde{B^I} = \{(b_1, b_2, b_3, b_4); (b_1', b_2, b_3, b_4')\}$ be two TrIFNs and k be any real number. The arithmetic operations over TrIFNs are defined as follows:

$$\widetilde{A^I} + \widetilde{B^I} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); (a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4')\}$$

$$\widetilde{A^I} - \widetilde{B^I} = \{(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); (a_1' - b_4', a_2 - b_3, a_3 - b_2, a_4' - b_1')\}$$

$$\widetilde{A^I}.\widetilde{B^I} = \begin{cases} \{(a_1b_1, a_2b_2, a_3b_3, a_4b_4); (a_1'b_1', a_2b_2, a_3b_3, a_4'b_4')\} & if \quad \widetilde{A^I} > 0 \; and \; \widetilde{B^I} > 0 \\ \{(a_1b_4, a_2b_3, a_3b_2, a_4b_1); (a_1'b_4', a_2b_3, a_3b_2, a_4'b_1')\} & if \quad \widetilde{A^I} < 0 \; and \; \widetilde{B^I} > 0 \\ \{(a_4b_4, a_3b_3, a_2b_2, a_1b_1); (a_4'b_4', a_3b_3, a_2b_2, a_1'b_1')\} & if \quad \widetilde{A^I} < 0 \; and \; \widetilde{B^I} < 0 \end{cases}$$

$$\widetilde{A^I}/\widetilde{B^I}$$

$$= \begin{cases} \{(a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1); (a_1'/b_4', a_2/b_3, a_3/b_2, a_4'/b_1')\} & if \quad \widetilde{A^I} > 0 \; and \; \widetilde{B^I} > 0 \\ \{(a_4/b_4, a_3/b_3, a_2/b_2, a_1/b_1); (a_4'/b_4', a_3/b_3, a_2/b_2, a_1'/b_1')\} & if \quad \widetilde{A^I} < 0 \; and \; \widetilde{B^I} > 0 \\ \{(a_4/b_1, a_3/b_2, a_2/b_3, a_1/b_4); (a_4'/b_1', a_3/b_2, a_2/b_3, a_1'/b_4')\} & if \quad \widetilde{A^I} < 0 \; and \; \widetilde{B^I} < 0 \end{cases}$$

$$k\widetilde{A^I} = \begin{cases} \{(ka_1, ka_2, ka_3, ka_4); (ka_1', ka_2, ka_3, ka_4')\} & if \quad k > 0 \\ \{(ka_4, ka_3, ka_2, ka_1); (ka_4', ka_3, ka_2, ka_1')\} & if \quad k < 0 \end{cases}$$

2.3 Cut sets of TrIFN

According to the cut sets of the IFS defined in [19], the cut sets of TrIFN can be defined as follows

Definition 4: An (α, β)-cut set of $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ is a crisp subset of $\mathbb{R}$, which is defined as follows

$$\tilde{A}_{\beta}^{\alpha} = \{x | \mu_{\tilde{A}}(x) \geq \alpha, \vartheta_{\tilde{A}}(x) \leq \beta\}$$

Where $0 \leq \alpha + \beta \leq 1$

Definition 5: An $\alpha - cut$ set of $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ is a crisp subset of $\mathbb{R}$, which is defined as follows

$$\tilde{A}^{\alpha} = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$$

Where $0 \leq \alpha \leq 1$

From Definitions 3 and 5,

$$\tilde{A}^{\alpha} = [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$$

Definition 6: A $\beta - cut$ set of $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ is a crisp subset of $\mathbb{R}$, which is defined as follows

$$\tilde{A}_{\beta} = \{x | \vartheta_{\tilde{A}}(x) \leq \beta\}$$

Where $0 \leq \beta \leq 1$.

From Definitions 3 and 6,

$$\tilde{A}_{\beta} = [(1-\beta)a_2 + \beta a_1, (1-\beta)a_3 + \beta a_4]$$

Following on the lines of Li [1] we compute the value index and the ambiguity index for TrIFNs given in Definition 3. The detailed working of the evolution of the two indexes is based on the cut sets and is skipped. The final formulas are given in the following two definitions.

Definition 7: Let $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ be a TrIFN. Then the value and the ambiguity of $\widetilde{A^I}$ are as follows

(i) The value of the membership function of $\widetilde{A^I}$ is

$$V_{\mu}(\widetilde{A^I}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

While the value of the non-membership function is

$$V_{\vartheta}(\widetilde{A^I}) = \frac{a_1' + 2a_2 + 2a_3 + a_4'}{6}$$

(ii) The ambiguity of the membership function of $\widetilde{A^I}$ is

$$A_{\mu}(\widetilde{A^I}) = \frac{-a_1 - 2a_2 + 2a_3 + a_4}{6}$$

While the ambiguity of the non-membership function is

$$A_{\vartheta}(\widetilde{A^I}) = \frac{-a_1' - 2a_2 + 2a_3 + a_4'}{6}$$

Since $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_4'$, $A_{\mu}(\widetilde{A^I}) \leq A_{\vartheta}(\widetilde{A^I})$.

Definition 8: Let $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ be a TrIFN. Then the value and the ambiguity index of $\widetilde{A^I}$ are respectively defined as follows

$$V(\widetilde{A^I},\lambda) = V_\mu(\widetilde{A^I}) + \lambda\left(V_\vartheta(\widetilde{A^I}) - V_\mu(\widetilde{A^I})\right)$$

And

$$A(\widetilde{A^I},\lambda) = A_\vartheta(\widetilde{A^I}) - \lambda\left(A_\vartheta(\widetilde{A^I}) - A_\mu(\widetilde{A^I})\right)$$

Where $\lambda \in [0,1]$ is a weight which represents the decision maker's preference information. It helps to incorporate the subjective attitude of the decision maker in the model. For a pessimistic decision maker, $\lambda \in [0,1/2)$ and for an optimistic decision maker $\lambda \in (1/2.1]$. $\lambda = 1/2$ refers to an indifferent attitude of the decision maker.

Theorem 1: The value index is a linear transformation.

Proof: Let $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$
and $\widetilde{B^I} = \{(b_1, b_2, b_3, b_4); (b_1', b_2, b_3, b_4')\}$ be two TrIFNs. For given $\lambda \in [0,1]$,

$$V(\widetilde{A^I} + \widetilde{B^I},\lambda) = V_\mu(\widetilde{A^I} + \widetilde{B^I}) + \lambda\left(V_\vartheta(\widetilde{A^I} + \widetilde{B^I}) - V_\mu(\widetilde{A^I} + \widetilde{B^I})\right)$$

$$= \frac{a_1 + b_1 + 2(a_2 + b_2) + 2(a_3 + b_3) + a_4 + b_4}{6} + \lambda\left(\frac{a_1' + b_1' + 2(a_2 + b_2) + 2(a_3 + b_3) + a_4' + b_4'}{6} - \frac{a_1 + b_1 + 2(a_2 + b_2) + 2(a_3 + b_3) + a_4 + b_4}{6}\right)$$

$$= \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} + \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} + \lambda\left(\frac{a_1' + 2a_2 + 2a_3 + a_4'}{6} + \frac{b_1' + 2b_2 + 2b_3 + b_4'}{6} - \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} - \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}\right)$$

$$= V_\mu(\widetilde{A^I}) + \lambda\left(V_\vartheta(\widetilde{A^I}) - V_\mu(\widetilde{A^I})\right) + V_\mu(\widetilde{B^I}) + \lambda\left(V_\vartheta(\widetilde{B^I}) - V_\mu(\widetilde{B^I})\right)$$

$$= V(\widetilde{A^I},\lambda) + V(\widetilde{B^I},\lambda)$$

Let c be any scalar.

$$V(c\widetilde{A^I},\lambda) = V_\mu(c\widetilde{A^I}) + \lambda\left(V_\vartheta(c\widetilde{A^I}) - V_\mu(c\widetilde{A^I})\right)$$

$$= \frac{ca_1 + 2ca_2 + 2ca_3 + ca_4}{6} + \lambda\left(\frac{ca_1' + 2ca_2 + 2ca_3 + ca_4'}{6} - \frac{ca_1 + 2ca_2 + 2ca_3 + ca_4}{6}\right)$$

$$= c\left(\frac{a_1 + 2a_2 + 2a_3 + a_4}{6} + \lambda\left(\frac{a_1' + 2a_2 + 2a_3 + a_4'}{6} - \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}\right)\right)$$

$$= c\left(V_\mu(\widetilde{A^I}) + \lambda\left(V_\vartheta(\widetilde{A^I}) - V_\mu(\widetilde{A^I})\right)\right) = cV(\widetilde{A^I},\lambda)$$

Hence, $V(\widetilde{A^I}, \lambda)$ is a linear transformation.

Theorem 2: The ambiguity index is a linear transformation.

Proof: Similar to proof of Theorem 1.

3. PROPOSED RANKING FUNCTION

Based upon the value and ambiguity indices, a new ranking function is employed, which is as follows:

$$H(\widetilde{A^I}, \lambda) = (1-k)V(\widetilde{A^I}, \lambda) - kA(\widetilde{A^I}, \lambda)$$

Where, $k = 1 - \lambda$ is a parameter that denotes the DM's subjective attitude. An optimistic DM, corresponds to $\lambda = 1$, prefers certainty so $H(\widetilde{A^I}, \lambda) = V(\widetilde{A^I}, \lambda)$ as $k = 0$. The DM, in this case, does not rely on ambiguity for making decisions. A pessimistic DM, corresponds to $\lambda = 0$, prefers uncertainty so $H(\widetilde{A^I}, \lambda) = A(\widetilde{A^I}, \lambda)$ as $k = 1$. The DM, being pessimistic, makes a decision based upon ambiguity only. For an indifferent DM, $\lambda = 1/2$, the above function becomes $H(\widetilde{A^I}, \lambda) = \frac{1}{2}V(\widetilde{A^I}, \lambda) - \frac{1}{2}A(\widetilde{A^I}, \lambda)$ as $k = 1/2$ and the DM is neither very certain nor uncertain.

Theorem 3: The ranking function, $H(\widetilde{A^I}, \lambda)$ is a linear transformation.

Proof: Follows from Theorems 1 and 2.

The ordering relation used to compare two TrIFNs is defined as follows:

Let $\widetilde{A^I} = \{(a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4')\}$ and $\widetilde{B^I} = \{(b_1, b_2, b_3, b_4); (b_1', b_2, b_3, b_4')\}$ be two TrIFNs. For given $\lambda \in [0,1]$, we define a new ordering relation as follows

$\widetilde{A^I} \preccurlyeq \widetilde{B^I}$ if and only if $H(\widetilde{A^I}, \lambda) \leq H(\widetilde{B^I}, \lambda)$

Larger the value index and lesser the ambiguity, greater is the TrIFN.

Theorem 4: The ordering relation defined above is a partial order relation.

Proof. Follows from the definition.

4. LINEAR PROGRAMMING WITH TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS.

The general extended form of linear programming problem with TrIFN coefficients is as follows

(P1):

$$\max \sum_{j=1}^{n} \widetilde{c_j^I} x_j$$

Subject to

$$\sum_{j=1}^{n} \widetilde{a_{ij}^I} x_j \preccurlyeq \widetilde{b_i^I}, \qquad i = 1,2,\dots,m$$

$$x_j \geq 0, \qquad j = 1,2,\dots,n$$

Using the ordering relation for a predefined $\lambda \in [0,1]$, problem (P1) is equivalent to the following crisp linear programming problem

(P2):

$$\max Z = H\left(\widetilde{A^I}, \lambda\right)$$

Subject to

$$H\left(\sum_{j=1}^{n} \widetilde{a_{ij}}^I, \lambda\right) \preccurlyeq H\left(\widetilde{b_i}^I, \lambda\right), \qquad i = 1,2,\dots,m$$

$$x_j \geq 0, \qquad j = 1,2,\dots,n$$

Following theorems establish equivalence of the problems (P1) and (P2).

Theorem 5: If $x^* \in S$ is an optimal solution of (P2), then $x^* \in S$ is a fuzzy optimal solution of (P1).

Proof. Let $x^* \in S$ be an optimal solution of problem (P2). Then, for given λ

$$H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j^*, \lambda\right) \geq H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j, \lambda\right) \qquad \forall \quad x \in S$$

Let, if possible, $x^* \in S$ is not a fuzzy optimal solution of problem (P1). Then there exist an $x \in S$ such that

$$\sum_{j=1}^{n} \widetilde{c_j^I} x_j \geq \sum_{j=1}^{n} \widetilde{c_j^I} x_j^*$$

According to the definition of the ordering relation defined above, the above inequality implies that

$$H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j\,, \lambda\right) \geq H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j{}^*\,, \lambda\right)$$

Which contradicts the fact that $x^* \in S$ is an optimal solution of problem (P2).

Hence, $x^* \in S$ is a fuzzy optimal solution of problem (P1).

Theorem 6: If $x^* \in S$ is a fuzzy optimal solution of (P1), then $x^* \in S$ is an optimal solution of (P2).

Proof. Let $x^* \in S$ be a fuzzy optimal solution of (P1). Then,

$$\sum_{j=1}^{n} \widetilde{c_j^I} x_j^* \geq \sum_{j=1}^{n} \widetilde{c_j^I} x_j \qquad \forall \quad x \in S$$

Let, if possible, $x^* \in S$ is not an optimal solution of problem (P2). Then there exist an $x \in S$ such that

$$H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j\,, \lambda\right) \geq H\left(\sum_{j=1}^{n} \widetilde{c_j^I} x_j{}^*\,, \lambda\right)$$

According to the definition of the ordering relation, the above inequality implies

$$\sum_{j=1}^{n} \widetilde{c_j^I} x_j \geq \sum_{j=1}^{n} \widetilde{c_j^I} x_j^*$$

Which contradicts the fact that $x^* \in S$ is a fuzzy optimal solution of problem (P1).

Hence, $x^* \in S$ is an optimal solution of problem (P2).

Problem (P2) is a crisp linear programming problem, which can be solved by Simplex method or some mathematical software.

Proposition 1: According to the fact that each triangular intuitionistic fuzzy number $\widetilde{A^I} = \{(a_1, a_2, a_3); (a_1{}', a_2, a_3{}')\}$ can be equivalently written as TrIFN $\widetilde{A^I} = \{(a_1, a_2, a_2, a_3); (a_1{}', a_2, a_2, a_3{}')\}$, so it can be said that linear programming problem with coefficients as TIFNs is a special case of linear programming problem with coefficients as TrIFNs.

5. NUMERICAL ILLUSTRATION

Consider the following problem,

$$\max \widetilde{5^I} x_1 + \widetilde{3^I} x_2$$

Subject to

$$\widetilde{4^I} x_1 + \widetilde{3^I} x_2 \leq \widetilde{12^I}$$

$$\widetilde{1^I} x_1 + \widetilde{3^I} x_2 \leq \widetilde{6^I}$$

$$x_1, x_2 \geq 0$$

Where

$\widetilde{c_1^I} = \widetilde{5^I} = \{(4,5,5.5,6); (4,5,5.5,6.1)\}$; $\widetilde{c_2^I} = \widetilde{3^I} = \{(2.5,3,3.1,3.2); (2,3,3.1,3.5)\}$;

$\widetilde{a_{11}^I} = \widetilde{4^I} = \{(3.5,3.8,4,4.1); (3,3.8,4,5)\}$; $\widetilde{a_{12}^I} = \widetilde{3^I} = \{(2.5,3,3.1,3.5); (2.4,3,3.1,3.6)\}$;

$\widetilde{a_{21}^I} = \widetilde{1^I} = \{(0.8,1,1.2,2); (0.5,1,1.2,2.1)\}$; $\widetilde{a_{22}^I} = \widetilde{3^I} = \{(2.8,2.9,3,3.2); (2.5,2.9,3,3.2)\}$

$\widetilde{b_1^I} = \widetilde{12^I} = \{(11,11.8,12.2,13); (11,11.8,12.2,14)\}$; $\widetilde{b_2^I} = \widetilde{6^I} = \{(5.5,6,6.2,7.5); (5,6,6.2,8.1)\}$

For λ=1/2, problem (P2) becomes

$$\max Z = 2.0792x_1 + 1.2667x_2$$

Subject to the constraints

$$1.6667x_1 + 1.2999x_2 \leq 5.4917$$

$$0.2917x_1 + 1.3458x_2 \leq 2.6292$$

$$x_1, x_2 \geq 0$$

The solution to the above problem is ${x_1}^* = 3.2950$, ${x_2}^* = 0$ and $Z^* = 6.8509$.

The corresponding fuzzy optimal value is

$\widetilde{Z^*} = \{(13.036,16.475,18.1225,19.77); (13.036,16.475,18.1225,20.0995)\}$

The solution depends upon the attitude of the DM. An optimistic DM will yield a better solution than a pessimistic DM.

6. CONCLUSION

This paper extended the concept of value and ambiguity indices for triangular intuitionistic fuzzy number (TIFN) defined in [1] to trapezoidal intuitionistic fuzzy number (TrIFN), which is used to develop a new ranking function to compare TrIFNs. The proposed ranking function is then used to solve fuzzy linear programming problem with data as TrIFNs. It is observed that the solution depends upon the attitude of the decision maker. An optimistic DM will yield a better solution than a pessimistic DM.

References

[1] D.F. Li, J.X. Nan and M.J. Zhang. A ranking method of triangular intuitionistic fuzzy numbers and application to decision making. *International Journal of Computational Intelligence Systems,* 3, No. 5, 522-530, 2010.

[2] Zadeh, L. A., "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets and Systems*, Vol. **1**, 1978, pp. 3-28.

[3] Bellman R.E. and Zadeh, L.A., "Decision-making in a fuzzy environment," *Management Science*, Vol. **l7**, 1970, pp. 141-164.

[4] Zimmermann, H.-J., "Description and optimization of fuzzy systems," *International Journal of General Systems*, Vol. **2**, 1976, pp. 209-215.

[5] Chopra, R., Saxena, R.R (2014). An approach to solve possibilistic linear programming problems. *Applied Mathematics*; **5:** No. 2

[6] Chopra, R., Saxena, R.R (2013). Model for fuzy multiple objective problems. *American Journal of Operations Research*; **3:** No.1

[7] R. Slowinski. A multicriteria fuzzy linear programming method for water supply system development planning. *Fuzzy Sets and Systems*, 19, 217-237, 1986.

[8] P. Czyzak, "Application of the FLIP method to farm structure optimization under uncertainty" in R. Slowinski and J. Teghem, eds., *Stochastic Versus Fuzzy Approaches to Multiobjective Mathematical Programming under Uncertainty*, Reidel, Dordrecht, 1990, pp. 263-278.

[9] J. L. Verdegay, "Application of fuzzy optimization in operational research," *Control and Cybernetics*, Vol. 13, 1984, pp. 229-239.

[10] J. F. C. Trappey, C. R. Liu, and T. C. Chang, "Fuzzy non-linear programming: theory and application in manufacturing," *International Journal of Production Research*, Vol. 26, 1988, pp. 957-985.

[11] R. Ostermark, "Fuzzy linear constraints in the capital asset pricing model," *Fuzzy Sets and Systems*, Vol. 30, 1989, pp. 93-102.

[12] Abbasbandy, S. Ranking of fuzzy numbers, some recent and new formulas. IFSA-EUSFLAT 2009, 642-646, 2009

[13] Chu, T.C. and Tsao, C.T. Ranking fuzzy numbers with an area between the centroid point and original point. *Computers Mathematics and Applications,* 43, 111-117, 2002.

[14] Jimenez, M. Ranking fuzzy numbers through the comparison of its expected intervals. *International Journal of Uncertainity, Fuzziness and Knowledge Based Systems,* 4, 379-388, 1996.

[15] Liu, X.W. and Han, S.L. Ranking fuzzy numbers with preference weighting function expectations. *Computers and Mathematics with Applications,* 49, 1731-1753, 2005.

[16] Maleki, H.R. Ranking functions and their applications to fuzzy linear programming. *Far East Journal of Mathematical Sciences,* 4, 283-301, 2002.

[17] Wang, X. and Kerre, E.E. Reasonable properties for the ordering of fuzzy quantities(i). *Fuzzy Sets and Systems,* 118, 375-385, 2001.

[18] Wang, X. and Kerre, E.E. Reasonable properties for the ordering of fuzzy quantities(ii). *Fuzzy Sets and Systems,* 118, 387-405, 2001.

[19] Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems,* 1986, 20, 87-96.

[20] D.F. Li. Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. *Micro Electronics Reliability.* 48, 1741, 2008.

[21] H.B. Mitchell. Ranking intuitionistic fuzzy numbers. *International Journal of Uncertainity, Fuzziness and knowledge based Systems*, 12, 377-386, 2004.

[22] A. Nagoorgani and K. Ponnalagu. A new approach on solving intuitionistic fuzzy linear programming problem. *Applied mathematical Sciences*, 6, No. 70, 3467-3474, 2012.

[23] D.F. Li, G.H. Chen and Z.G. Huang. Linear programming method for multiattribute decision making using IF sets. *Information Sciences,* 180, 1591-1609, 2010.