Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 12, Number 3 (2017), pp. 663-676
© Research India Publications
http://www.ripublication.com

A Note on Fuzziness in Inventory Management Problems

Dr. Jayjayanti Ray

Department of Mathematics, Gokhale Memorial Girls' College
(Calcutta University affiliation), Kolkata, INDIA.

From traditional view point, Science should strive for certainty in all its manifestations (precision, specificity, sharpness, consistency, etc.); hence, uncertainty (imprecision, non-specificity, vagueness, inconsistency, etc.) is regarded as unscientific. But according to the modern view, uncertainty which is unavoidable is considered essential to Science and has great utility. Impreciseness and uncertainty is a notable change among the various formal changes in Science and Mathematics in this century.

The modern concept of uncertainty evolved with the publication of a seminar paper by Zadeh, L.A. (1965), where he introduced a theory whose objects - *fuzzy sets*, are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a *degree*. Zadeh's paper challenged not only probability theory as the sole agent for uncertainty, but the very foundations upon which probability theory is based: Aristotelian two-valued logic. A fuzzy set can be defined mathematically by assigning to each possible individual in the Universe of discourse a value representing its grade of membership in the fuzzy set. Individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. Following Zadeh a membership grade allows finer detail, such that the transition from membership to non-membership is gradual rather than abrupt. Given a collection of objects U, a fuzzy set A in U is defined as a set of ordered pairs $A \equiv \{\langle x, \mu_A(x)\rangle \mid x \epsilon \text{ U}\}$ where $\mu_A(x)$ is called the membership function for the set of all objects x in U. The membership function relates to each x a membership grade $\mu_A(x)$, a real number in the closed interval $[0,1]$. So here we see that it is necessary to work with pairs $\langle x, \mu_A(x)\rangle$ whereas for classical sets a list of objects suffices, as their membership is understood. Thus the definition of a fuzzy set is the extension of the definition of a classical set, since membership values μ are permitted in the interval $0 \leq \mu \leq 1$, higher the value, the higher the membership; whereas in a classical set the membership values are restricted to $\mu \in \{0,1\}$. Since full membership or full non-membership in the fuzzy set can still be indicated by the values of 1 and 0, respectively, we can consider crisp set to be a restricted case of the more general concept of a fuzzy set, for which only these two grades of membership are allowed. Thus, a fuzzy set representing our subjective concept of sunny might assign a degree of membership of 1 to a cloud cover of 0 %, 0.8 to a cloud cover of 20 %, 0.4 to a cloud cover of 30 %, and 0 to a cloud cover of 75 %.

As it is possible to express most of Mathematics in the language of set theory, researchers are today looking at the consequences of 'fuzzifying' set theory, resulting in fuzzy logic, fuzzy numbers, fuzzy intervals, fuzzy arithmetic, fuzzy integrals, etc. With fuzzy logic based on fuzzy sets, a computer can process words from natural language, such as 'small', 'large', and 'approximately equal'.

❖ ***Basic concept of Fuzzy set theory***: There are two alternative ways to represent a membership function - continuous or discrete.

- A ***continuous fuzzy set*** A is defined by means of any of the following continuous membership functions $\mu_A(x)$:

  a) A *trapezoidal membership function:* It is a piece wise linear continuous function, controlled by four parameters $\{a, b, c, d\}$. Thus a trapezoidal fuzzy number A is thus expressed as $[a, b, c, d]$ and its membership function is defined as follows:

$$\mu_A(x)=\begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x \geq d \end{cases} \qquad \boxed{x \in R}$$

  b) A *triangular membership function:* It is also a piecewise linear function and is derived from the trapezoidal membership function. By setting $b = c$ above we have a triangular fuzzy number A defined as $[a, b, d]$ whose membership function is

$$\mu_A(x)=\begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{d-x}{d-b}, & b \leq x \leq d \\ 0, & x \geq d \end{cases} \qquad \boxed{x \in R}$$

c) *Smooth trapezoid and triangular membership functions* can be obtained by replacing the linear segments corresponding to the intervals $a \leq x \leq b$ and $c \leq x \leq d$ by a non-linear function.

➢ ***Discrete fuzzy sets*** are defined by means of discrete variable x_i $(i = 1,2, \ldots \ldots)$. Thus, a discrete fuzzy set A is defined by ordered pairs, $A = \{\langle x_1, \mu(x_1)\rangle, \langle x_2, \mu(x_2)\rangle, \ldots \ldots | x_i \in U, i = 1,2, \ldots .\}$.

➢ $\boldsymbol{\alpha}$ **− cut** operation **:** Given a fuzzy set A on X and any real number $\alpha \epsilon [0,1]$, then the $\alpha -$ cut , denoted by A_α is the crisp set $A_\alpha = \{x \in X\colon\ \mu_A(x) \geq \alpha\}$. The strong $\alpha -$ cut denoted by $A_{\alpha+}$ is the crisp set $A_{\alpha+} = \{x \in X\colon\ \mu_A(x) > \alpha\}$.

For example if A be a fuzzy set whose membership function is

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \end{cases}$$

To find $\alpha -$ cut of A, we first set $\alpha \epsilon [0,1]$ to both left and right reference functions of A, i.e. $\alpha = \frac{x-a}{b-a}$ *and* $\alpha = \frac{c-x}{c-b}$.

Expressing x in terms of α we have $x = (b-a)\alpha + a$ and $x = c - (c-b)\alpha$

which gives the $\alpha - cut$ of A as $\alpha_A = [(b-a)\alpha + a,\ c - (c-b)\alpha\,]$.

Various arithmetic operations of fuzzy numbers such as addition, subtraction, multiplication, division, etc. can be done using $\alpha -$ cut method. P. Dutta, H. Boruah and T. Ali (2011) have shown that this method is general and simple enough to deal with different types of fuzzy arithmetic including exponentiation, taking logarithm, extracting n^{th} root.

➢ The **support of a fuzzy set** A defined on X is a crisp set defined as $Supp(A) = \{x \in X, \mu_A(x) > 0\}$.

➢ The **height of a fuzzy set** A, denoted by $h(A)$ is the largest membership grade obtained by any element in the set, i.e., $h(A) = \underbrace{sup}_{x \in X}\ \mu_A(x)$.

➢ A **fuzzy number** is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

Fuzzy set theory is primarily concerned with how to quantitatively deal with imprecision and uncertainty, and offers the decision maker another tool in addition to the classical deterministic and probabilistic mathematical tools that are used in modelling real-world problems. Zimmermann (1996) discussed the concept of the fuzzy set theory and its applications. Kaufmann and Gupta (1991) explained the fuzzy arithmetic and their applications.

❖ **Study of fuzziness in Inventory management problems**

As almost every business must carry out some inventory for smooth and efficient running of its operation, inventory control plays an important role. The problem is to take decisions that how much should be stocked and when should be stocked for un-interrupted production. Harris, F. (1915) developed first inventory model. There are various types of uncertainties that cannot be appropriately treated by the usual probabilistic models. Now the question is how to define inventory optimization tasks in such environment and how to interpret optimal solutions. The fuzzy set theory was developed in the mid-1960s. Lotfi A. Zadeh (1965) introduced the concept of fuzzy set theory in inventory modelling. Later an extension principle was developed by R.E. Bellman and L.A. Zadeh (1970) in the field of decision-making problems in management sciences as well as OR sciences. Zadeh (1965, 1973) showed that for the new products and seasonal items it is better to use fuzzy numbers rather than probabilistic approaches. Jain, R. (1976) developed a fuzzy inventory model on decision making in the presence of fuzzy variables. Dubois, D. and Prade, H. (1978) defined some operations on fuzzy numbers. Kacpryzk, J. and Staniewski, P. (1982) developed an inventory model for long term inventory policy making through fuzzy decisions. Zimmerman, H.J. (1983) tried to use fuzzy sets in operational research. M. Gen, Y. Tsujimura, D. Zheng (1997) showed the applications of fuzzy st theory in inventory control problems.

Park, K.S. (1987) defines the fuzzy set theoretical interpretation of an EOQ problem. Vujosevic et al. (1996) developed an EOQ formula by assuming inventory cost as a fuzzy number. They considered an inventory model without backorder, and got fuzzy total cost. Park (1987) and Vujosevic et. al. (1996) developed the inventory models in fuzzy sense where ordering cost and holding cost are represented by fuzzy numbers. Park has represented costs as trapezoidal fuzzy numbers. Whereas Vujosevic et. al. represented ordering cost by triangular fuzzy number and holding cost by trapezoidal fuzzy number. Centroid of fuzzy total cost was taken as the estimate of fuzzy total cost. J.S. Yao and H.M. Lee (1996) developed another inventory model with fuzzy demand quantity and fuzzy production quantity. The same authors Yao and Lee (1998) developed an EOQ model by considering order quantity as fuzzy and allowing shortages. T.K. Roy and M. Maiti (1997) presented a fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Chang et. al. (1998) presented a fuzzy inventory model with backorder where the backorder quantity was fuzzified as the triangular fuzzy number. Yao, J.S. and Lee, H.M. (1999) developed a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number. Lee and Yao (1999a) proposed the inventory without backorder model in the fuzzy sense, where the order quantity is fuzzified as the triangular fuzzy number. Chang (1999) discussed the fuzzy production inventory model for fuzzify the product quantity as triangular fuzzy number. J.S. Yao, S.C. Chang and J.S. Su (2000) developed a fuzzy inventory model without backorder for fuzzy order quantity and fuzzy total demand quantity. Yao et al. (2000) assumed the order quantity and the total demand rate as triangular fuzzy numbers and obtained the fuzzy inventory model without shortages.

C. K. Kao and W. K. Hsu (2002) developed a single period inventory model with fuzzy demand. C. H. Hsieh (2002) developed an inventory model and gives an approach of optimization of fuzzy production. Yao and Chiang (2003) also considered the total cost of inventory without backorder. They fuzzified the total demand and cost of storing one unit per day into triangular fuzzy numbers and defuzzify by the centroid and the signed distance methods. Wu and Yao (2003) fuzzified the order quantity and shortage quantity into triangular fuzzy numbers in an inventory model with backorder and they obtained the membership function of the fuzzy cost and its centroid. Dutta et al. (2005) developed a model in presence of fuzzy random variable demand where the optimum is achieved using a graded mean integration representation. Chang et al. (2006) developed the mixture inventory model involving variable lead-time with backorders and lost sales. First they fuzzify the random lead-time demand to be a fuzzy random variable and then fuzzify the total demand to be the triangular fuzzy number and derive the fuzzy total cost. By the centroid method of defuzzification, they estimate the total cost in the fuzzy sense. S. K. De and A. Goswami (2006) through the investigation of an Economic Order Quantity (EOQ) model observed that the estimated inventory cost and starting time inventory cost are not necessarily the same. This arises as a result of the time gap in between the time of estimation and the starting time of the EOQ system, and a permissible delay in payment will affect the inventory total cost. Moreover, the political instability or uncertainty of a country (as well as the whole world) leads to a much more unstable situation in the present world economy. Thus, a change in inflation takes place, and the inflation rate is uncertain in nature. This study develops an EOQ model with a fuzzy inflation rate and fuzzy deterioration rate, and a delay in payment is also permissible. Lin (2008) developed the inventory problem for a periodic review model with variable lead-time and fuzzified the expected demand shortage and backorder rate using signed distance method to defuzzify.

Many of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables, and foodstuffs are subject to direct spoilage during storage period. The highly volatile liquids such as gasoline, alcohol, and turpentine undergo physical depletion over time through the process of evaporation. The electronic goods, radioactive substances, photographic film, and grain deteriorate through a gradual loss of potential or utility with passage of time. The deteriorated items repairing are a major problem in the supply chain of most of the business organizations. S. De and A. Goswami (2001) presented the EOQ model with fuzzy deterioration rate. J. Sujit D. Kumar, P. K. Kundu and A. Goswami (2007) developed an economic production quantity model with fuzzy demand and deterioration rate. An EOQ model for perishable items with fuzzy partial backlogging factor and fuzzy deterioration rate was developed by Halim et. al. (2008). In the paper of A. Roy, S. Kar, M. Maiti (2008) the inventory cost coefficients, storage space and budgetary cost are fuzzy and represented by fuzzy numbers, demands and rates of deterioration of the items being constant. They provide defuzzification techniques for two fuzzy inventory models using (i) extension principle and duality theory of non-linear programming and (ii) interval arithmetic. G.C. Mahata and A. Goswami (2009) has used fuzzy concepts to develop a fuzzy EOQ model with stock-dependent demand rate and non-linear holding cost by taking rate of deterioration to be a triangular fuzzy number.

A. Roy and G.P. Samanta (2009) have developed an inventory model considering that the cycle time is uncertain and described it by a triangular fuzzy number (symmetric). They discussed a fuzzy continuous review inventory model without backorder for deteriorating items. They used the signed distance method to fuzzify the total cost Gani and Maheswari (2010) developed an EOQ model with imperfect quality items with shortages where defective rate, demand, holding cost, ordering cost and shortage cost are taken as triangular fuzzy numbers. Graded mean integration method is used for defuzzification of the total profit. Liang-Yuh Ouyang, Jinn-Tsair Teng and Mei-Chuan Cheng (2010) explored and understood that in studies related to trade credit assumption that the interest rate is both fixed and predetermined is not true especially when interest rates fluctuate. They recast Chang *et al.*'s (2003) model by further fuzzifying the rate of interest charges, the rate of interest earned, and the deterioration rate into the triangular fuzzy number. They construct three different intervals to include the rate of interest charges, the rate of interest earned, and the deterioration rate, thus deriving the fuzzy total relevant inventory cost.

Ameli et al. (2011) developed a new inventory model to determine ordering policy for imperfect items with fuzzy defective percentage under fuzzy discounting and inflationary conditions. They used the signed distance method of defuzzification to estimate the value of total profit. Nezhad et al. (2011) developed a periodic review model and a continuous review inventory model with fuzzy setup cost, holding cost and shortage cost. Also they considered the lead-time demand and the lead-time p lus one period's demand as random variables. They use two methods in the name of signed distance and possibility mean value to defuzzify. Uthayakumar and Valliathal (2011) developed an economic production model for Weibull deteriorating items over an infinite horizon under fuzzy environment and considered some cost component as triangular fuzzy numbers and using the signed distance method to defuzzify the cost function.

P. K.De and A. Rawat (2011) developed a fuzzy inventory model without shortages by using triangular fuzzy number. C. K. Jaggi, S. Pareek, A. Sharma and Nidhi (2012) developed a fuzzy inventory model for deteriorating items with time varying demand and shortages. Sumana saha and Tripti Chakrabarty (2012) developed a fuzzy EOQ model with time varying demand and shortages. D. Dutta and Pawan Kumar (2012) considered a fuzzy inventory model without shortages using a trapezoidal fuzzy number. D. Dutta and Pawan Kumar (2013) considered an optimal replenishment policy for an inventory model without shortages by assuming fuzziness in demand, holding cost and ordering cost. Dipak Kumar Jana, Barun Das and Tapan Kumar Roy (2013) give a fuzzy generic algorithm approach for an inventory model for deteriorating items with backorders under fuzzy inflation and discounting over random planning horizon.

For solving various EOQ (Economic Order Quantity) models which arise in the real life situation, the values of the quantities such as demand rate, production rate, deterioration rate, etc. are not crisp but uncertain in nature. Hence these variables should be treated as fuzzy variables.

C. K.Jaggi, A.K. Bhunia, A. Sharma and Nidhi (2012), in their study, develop a crisp inventory model with constant deterioration, price dependent demand and time varying holding cost and partial backlogged shortages. Thereafter, to develop the corresponding fuzzy model, trapezoidal fuzzy numbers have been used to represent the uncertainty in all the parameters namely, demand, ordering cost, holding cost, purchase cost, deterioration rate, shortage cost and lost sale cost. From the numerical example, it is observed that the optimal profit of fuzzy model is lesser than that of crisp one. The reason behind this is due to uncertainty of several parameters. Hence they concluded that the average profit will be reduced when uncertainties are accounted in large manner. S. Pal, G. S. Mahapatra, G. P. Samanta (2014) developed an economic production model for single item with ramp type demand rate and deterioration rate of the item having two parameter Weibull's distribution. The effect of inflation is also considered when there is no shortage in the stock and the model is under finite time horizon. The inventory model is solved under crisp and fuzzy environment to evaluate the optimum solution of the model in different cases. B. Naserabadi, A. Mirzazadeh, and S. Nodoust (2014) developed an inventory model for items with uncertain deterioration rate, time-dependent demand rate with non increasing function, and allowable shortage under fuzzy inflationary situation. Recently H. Nagar and P. Surana (2015) developed the corresponding fuzzy inventory model for fuzzy deteriorating items with fuzzy demand rate under full backlogging. The average total inventory cost in fuzzy sense is derived. All inventory parameters including deterioration rate are fuzzified as the pentagonal fuzzy numbers. In another recent model developed by M.Maragatham, P.K.Lakshmidevi (2016) the holding cost, shortage cost, deterioration cost, purchasing cost and selling price are considered as trapezoidal fuzzy numbers. Last year N.K. Sahoo, B. S. Mohanty and P.K. Tripathy (2016) investigate the development of a fuzzy inventory model with time-varying demand, deterioration and salvage. The deterioration rate, demand, holding cost, unit cost and salvage value are taken as trapezoidal fuzzy numbers. Numerically comparing the crisp model with fuzzy model it is seen that if the uncertainties are accounted for in an appropriate manner, the time would decrease. In comparison with the crisp model, the fuzzy model is seen to be giving a relatively better optimal solution.

After the introduction of fuzzy set theory in 1965 by Zadeh, extensive research work has been done on *defuzzification* of fuzzy numbers. Among these, centroid method by C. Lee (1990), weighted average method H. by Heiiendoorn, C. Thomas (1993), graded mean value method by T.S. Liou, M.J.J. Wang (1992), nearest interval approximation method by P. Grzegorzewski (2002), graded mean integration value method by S.H. Chen, C.H. Hsieh (1999), etc., have drawn more attention. All these techniques replace the fuzzy parameters by their nearest crisp number / interval and the reduced crisp objective function is optimized. Study shows that among the various methods, the Signed Distance Method by J.S. Yao and J. Chiang (2003) is better for *defuzzification* . Syed and Aziz (2007), in their paper developed an inventory model without shortages, representing both the ordering and holding costs by fuzzy triangular number and calculating the optimal order quantity using Signed Distance Method and function principle for *defuzzification*. Umap (2010) formed a fuzzy EOQ model in which he used signed distance method and function principle method for defuzzification of total inventory costs as well as optimum order quantity. C.K. Jaggi, S. Pareek, A. Sharma, Nidhi (2012) present a fuzzy inventory model where for defuzzification, graded mean, signed distance and centroid methods are employed to evaluate the optimal time period of positive stock and total cycle length which minimizes the total cost. By given numerical example it has been tested that graded mean representation method gives minimum cost as compared to signed distance method and centroid method. Recently, H.P. UMAP(2014) considered a multi item EOQ model with stock dependent demand for deteriorating items in fuzzy environment. Inventory costs such as holding cost and setup cost have been represented by exponential membership function and profit, deteriorating rate and total investment constraint are represented by linear membership functions. The model has been solved by fuzzy non-linear programming (FNLP) method. In the fuzzy model developed by S. Kumar, U. S. Rajput (2015) by defuzzification using signed distance method and centroid method it has been observed that the total profit decreases as the optimal cycle time decreases and the profit given by the signed distance method is minimum as compared to the centroid method. In some of the models like Kasthuri et al. (2011), Kuhn-Tucker conditions for defuzzification can be used. Long back in the 20^{th} century, Roy and Maiti (1997, 1998) solved the classical EOQ problem with a fuzzy goal and fuzzy inventory costs using a fuzzy non-linear programming method where different types of membership functions for inventory parameters were specified. They examined the fuzzy EOQ problem with a demand-dependent unit price and an imprecise storage area using both fuzzy geometric and non-linear programming methods.

Research on the fuzzy sets has been growing steadily since the inception of the theory in the mid-1960s. The concepts and results pertaining to this theory is now quite impressive. Research on a broad variety of applications in inventory management has also been very active and has produced results that are perhaps even more impressive. To define inventory optimization tasks in such an unpredictable environment and to interpret optimal solutions, fuzzy set theory in inventory modelling gives an authenticity to the model formulated since fuzziness is the closest possible approach to reality.

REFERENCES:

[1] **Harris, F. (1915),** Operations and Cost. AW Shaw CO., Chicago

[2] **Zadeh, L.A. (1965)** Fuzzy Set. *Information Control*, **8**, 338-353.

[3] **R.E. Bellman and L.A. Zadeh (1970),** Decision making in a fuzzy environment, *Management Science*, 17(4):141-164.

[4] **L.A. Zadeh (1973),** Outline of a new Approach to the Analysis of Complex Systems and Decision Processes, *IEEE Transactions on Systems, Man and Cybernetics*, SMC-3:28-44.

[5] **Jain, R. (1976),** Decision Making in the Presence of Fuzzy Variables. *IIIE Transactions on Systems, Man and Cybernetics*, **17**, 698-703.

[6] **Dubois, D. and Prade, H. (1978),** Operations on Fuzzy Numbers. *International Journal of System Science*, vol. 9, no. 6, 613-626.

[7] **Kacpryzk, J. and Staniewski, P. (1982)**, Long Term Inventory Policy Making through Fuzzy Decision Making Methods. *Fuzzy Sets and System*, **8**, 117-132.

[8] **Zimmerman, H.J. (1983),** Using Fuzzy Sets in Operational Research. *European Journal of Operation Research*, **13**, 201-206.

[9] **Park, K.S. (1987),** Fuzzy Set Theoretical Interpretation of Economic Order Quantity. *IEEE Transactions on Systems, Man and Cybernetics*, SMC-17, 1082-1084.

[10] **A. Kaufmann and M.M. Gupta (1991),** Introduction to Fuzzy Arithmetic: Theory and Applications, *Van Nostrand Reinhold, New York*.

[11] **Hans J Zimmermann (1996),** "Fuzzy Set Theory and Its Applications," 3rd Ed. Dordrecht: Kluwer, Academic Publishers.

[12] **Vujosevic, M., Petrovic, D. , Petrovic, R. (1996)** EOQ Formula When Inventory Cost Is Fuzzy. *International Journal of Production Economics*, 45, 499-504.

[13] **Jing S Yao, Huey M Lee (1996),** "Fuzzy inventory with backorder for fuzzy order quantity", Information Sciences, vol. 93, pp. 283-319.

[14] **M. Gen, Y. Tsujimura, D. Zheng (1997),** "An application of fuzzy set theory to inventory control models", Computers and Industrial Engineering, vol. 33, pp. 553-556.

[15] **T. Roy and M. Maiti (1997)**, "A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity," European Journal of Operational Research, vol. 99, no. 2, pp. 425–432.

[16] **T. Roy and M. Maiti (1998),** "Multi-objective inventory models of deteriorating items with some constraints in a fuzzy environment," Computers & operations research, vol. 25, no. 12, pp. 1085–1095.

[17] **J.S. Yao and H.M. Lee (1998),** Economic Production Quantity for fuzzy demand quantity and fuzzy production quantity, *European journal of Operational Research*, 109:203-211.

[18] **San C Chang, Jing S Yao, Huey M Lee (1998),** "Economic reorder point for fuzzy backorder quantity", European Journal of Operational Research, vol. 109, pp. 183-202.

[19] **Yao, J.S. and Lee, H.M. (1999)** Fuzzy Inventory with or without Backorder for Fuzzy Order Quantity with Trapezoidal Fuzzy Number. *Fuzzy Sets and Systems*, 105, 311-337.

[20] **Yao, J.S. and Lee, H.M. (1999a)** Economic Order Quantity in Fuzzy Sense for Inventory without Backorder Model. *Fuzzy Sets and Systems*, **105**, 13-31.

[21] **S. Chang (1999),** "Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number", Fuzzy Set and Systems, vol. 107, pp.37-57.

[22] **S.H. Chen, C.H. Hsieh (1999)**, Graded mean integration representation of generalized fuzzy numbers, *Journal of Chinese Fuzzy Systems*, 5(2):1-7.

[23] **Jing S Yao, San C Chang, Jin S Su (2000),** "Fuzzy Inventory without backorder for fuzzy order quantity and fuzzy total demand quantity", Computer and Operations Research, vol. 27, pp. 935-962.

[24] **S. De and A. Goswami (2001),** A replenishment policy for items with finite production rate and fuzzy deterioration rate, *Opsearch*, 38:419-430.

[25] **Kao, C.K. and Hsu, W.K. (2002)** A Single Period Inventory Model with Fuzzy Demand. *Computers and Mathematicswith Applications*, 43, 841-848.

[26] **Hsieh, C.H. (2002)** Optimization of Fuzzy Production Inventory Models. *Information Sciences*, 146, 29-40.

[27] **P. Grzegorzewski (2002)**, Nearest interval approximation of a fuzzy number, *Fuzzy Sets and Systems*, 130: 321-330.

[28] **C. T. Chang, L. Y. Ouyang, and J. T. Teng (2003),** "An EOQ model for deteriorating items under supplier credits linked to ordering quantity" , Applied Mathematical Modelling, Vol. 27, pp. 983-996.

[29] **Kweimei Wu, Jing S Yao (2003),** "Fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity", European Journal of Operational Research, vol. 150, no. 2, pp. 320-352.

[30] **Jing S Yao, Jershan Chiang (2003),** "Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance", European journal of Operations research, vol. 148, pp. 401-409.

[31] **Pankaj Dutta, Dabjani Chakraborty, Akhil R Roy (2005),** "A single-period inventory model with fuzzy random variable demand", Mathematical and Computer Modelling, vol. 41 , no.8-9, pp. 915-922.

[32] **Hung C. Chang, Jing S Yao, Liang Y Ouyang (2006)**, "Fuzzy mixture inventory model involving fuzzy random variable lead-time demand and fuzzy total demand", European Journal of Operational Research, vol. 169, no. 1, pp. 65-80.

[33] **S. K. De and A. Goswami (2006)** An EOQ model with fuzzy inflation rate and fuzzy deterioration rate when a delay in payment is permissible, International Journal of Systems Science, Vol. 37 Issue 5, Pages 323 – 335. **Kumar, Sujit De, Kundu, P.K. and Goswami, A. (2007)** An Economic Production Quantity Inventory Model Involving Fuzzy Demand Rate and Fuzzy Deterioration Rate. *Journal of Applied Mathematics and Computing*, **12**, 251-260.

[34] **J.K. Syed and L.A. Aziz (2007)**, Fuzzy inventory without shortages using Signed distance method, *Applied Mathematics and Information Sciences-An International Journal*, 1(2):203-209.

[35] **Yu J Lin (2008)**, "A periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate", Computers & Industrial Engineering, vol. 54, no. 3, pp. 666-676.

[36] **K.A. Halim, B.C. Giri and K.S. Chaudhuri (2008)**, Fuzzy economic order quantity model for perishable items with stochastic demand, partial backlogging and fuzzy deterioration rate, *International journal of Operational Research*, 3:77-96.

[37] **A. Roy, S. Kar, M. Maiti (2008)** A deteriorating multi-item inventory model with fuzzy costs and resources based on two different defuzzification techniques, Applied Mathematical Modelling 32 , 208–223.

[38] **G.C. Mahata and A. Goswami (2009)**, Fuzzy EOQ models for Deteriorating items with Stock Dependent Demand and Nonlinear Holding Costs, *International Journal of Mathematical and Computer Science*, 5(2): 94-98.

[39] **Ajanta Roy, Guru P Samanta (2009),** "Fuzzy continuous review inventory model without backorder for deteriorating items", Electronic Journal of Applied Statistical Analysis, vol. 2, no.1, pp. 58-66.

[40] **Liang-Yuh Ouyang, Jinn-Tsair Teng and Mei-Chuan Cheng (2010)** A Fuzzy Inventory System with Deteriorating Items under Supplier Credits Linked to Ordering Quantity, Journal Of Information Science And Engineering, 26, 231-253.

[41] **Nagoor A Gani, S. Maheswari (2010),** "Economic order quantity for items with imperfect quality where shortages are backordered in fuzzy environment", Advances in Fuzzy Mathematics, vol. 5, no. 2, pp. 91-100.

[42] **Umap H.P (2010),** Fuzzy EOQ model for deteriorating items with two warehouses, Journal of Statistics and Mathematics, 1(2), 01-06.

[43] **P. Dutta, H. Boruah and T. Ali (2011)**, Fuzzy Arithmetic with and without using $\alpha -$ cut method: A Comparative Study, *International Journal of Latest Trends in Computing*, 2(1): 99-107.

[44] **M Ameli, A Mirzazadeh and M A Shirazi (2011),** "Economic order quantity model with imperfect items under fuzzy inflationary conditions", Trends in Applied Sciences Research, vol. 6, no.3, pp. 294-303.

[45] **Saheli S Nezhad, Shima M Nahavandi, Jamshid Nazemi (2011),** "Periodic and continuous inventory models in the presence of fuzzy costs", International Journal of Industrial Engineering Computations, vol. 2, pp. 167–178.

[46] **R Uthaykumar, M Valliathal (2011),** "Fuzzy economic production quantity model for Weibull deteriorating items with ramp type of demand", International Journal of Strategic Decision sciences, vol. 2, no. 3, pp. 55-90.

[47] **De, P.K. and Rawat, A. (2011)** A Fuzzy Inventory Model without Shortages Using Triangular Fuzzy Number. *Fuzzy Information and Engineering*, **3**, 59-68.

[48] **R.Kasthuri, P.Vasanthi, S.Ranganayaki and C.V.Seshaiah (2011),** Multi-Item inventory model involving three constraints: A Karush-Kuhn-Tucker conditions approach, American Journal of Operations Research, 1, 155-159.

[49] **C. K. Jaggi, A.K. Bhunia, A. Sharma and Nidhi (2012),** Fuzzy EOQ model for deteriorating items with price dependent demand and time-varying holding cost, AMO - Advanced Modeling and Optimization, Volume 14, Number 3.

[50] **Saha, S. and Chakrabarti, T. (2012)** Fuzzy EOQ Model with Time Dependent Demand and Deterioration with Shortages. *IOSR Journal of Mathematics*, **2**, 46-54.

[51] **Dutta, D. and Kumar, P. (2012)** Fuzzy Inventory Model without Shortage Using Trapezoidal Fuzzy Number with Sensitivity Analysis. *IOSR Journal of Mathematics*, 4, 32-37.

[52] **C.K. Jaggi, S. Pareek, A. Sharma, Nidhi (2012),** "Fuzzy inventory model for deteriorating items with time-varying demand and shortages", *American Journal of Operational Research*, Vol. 2(6), pp.81-92.

[53] **Dutta, D. and Kumar, P. (2013)** Optimal Ordering Policy for an Inventory Model for Deteriorating Items without Shortages by Considering Fuzziness in Demand Rate, Ordering Cost and Holding Cost. *International Journal of Advanced Innovation and Research*, **2**, 320-325.

[54] **Jana, D.K., Das, B. and Roy, T.K. (2013)** A Fuzzy Generic Algorithm Approach for an Inventory Model for Deteriorating Items with Backorders under Fuzzy Inflation and Discounting over Random Planning Horizon. *Advances in Operations Research*, 1-15.

[55] **S. Pal, G. S. Mahapatra, G. P. Samanta (2014)** An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment, International Journal of Production Economics, 2014, vol. 156, issue C, pages 159-166.

[56] **B. Naserabadi, A. Mirzazadeh, and S. Nodoust (2014)** A New Mathematical Inventory Model with Stochastic and Fuzzy Deterioration Rate under Inflation, Hindawi Publishing Corporation Chinese Journal of Engineering, Article ID 347857, 10 pages.

[57] **H.P. UMAP(2014)** Fuzzy Eoq Model for Deteriorating Items with Exponential Membership Function, American Journal of Applied Mathematics and Statistics, 2014, Vol. 2, No. 4, 203-206 .

[58] **S. Kumar, U. S. Rajput (2015)** Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging, Applied Mathematics, 6, 496-509.

[59] **H. Nagar and P. Surana (2015)** Fuzzy Inventory Model for Deteriorating Items with Fluctuating Demand and Using Inventory Parameters as Pentagonal Fuzzy Numbers, Journal of Computer and Mathematical Sciences, Vol. 6(2), 55-66.

[60] **M. Maragatham, P.K. Lakshmidevi (2016)** A Fuzzy Inventory Model For Non-Instantaneous Deteriorating Items Under Conditions Of Permissible Delay In Payments For N-Cycles, International Journal of Scientific Engineering and Technology, Volume No.5 Issue No.11, pp: 512-518.

[61] **N.K. Sahoo, B. S. Mohanty and P.K. Tripathy (2016)** Fuzzy inventory model with exponential demand and time-varying deterioration, Global Journal of Pure and Applied Mathematics, Volume 12, Number 3 (2016), pp. 2573–2589