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Some Properties of Upper and Lower α −Irresolute Intuitionistic Fuzzy Multifunctions

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Abstract

Recently Thakur and Bohre introduced the concept of upper and lower intuitionistic fuzzy (IF) α −irresolute Intuitionistic Fuzzy Multifunction (IFM). The aim of this paper is to obtain some properties of upper and lower IF α −irresolute IFM and to establish some theorems.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy multifunctions, lower Intuitionistic fuzzy α −irresolute and upper Intuitionistic fuzzy α −irresolute Intuitionistic fuzzy multifunctions.

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1. Introduction and Preliminaries

After the introduction of fuzzy sets by Zadeh [26] in 1965 and fuzzy topology by Chang [7] in 1967, several researches was conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set was introduced by Atanassov [2,3,4] as a generalization of fuzzy set. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [8] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In the 1999, the concept of intuitionistic fuzzy multifunctions was introduced by Ozbakir and Coker [18] from a topological space to an intuitionistic fuzzy topological space.

Throughout this paper $(X, \mathcal{T})$ and (Y, Γ) represents a topological space and an intuitionistic fuzzy topological space respectively. A subset A of a topological space $(X, \mathcal{T})$ is called α-open if A$\subset Int\left(Cl\left(Int(A)\right)\right)$. The complement of an α-open is called α-closed. Every open (resp. closed) set is α-open [16] (resp. α-closed[16]) but the converses may not be true. The family of all α-open (resp. α-closed) subsets of a topological space$(X, \mathcal{T})$ is denoted by αO(X) (resp. αC(X)). The intersection of all α-closed sets of X containing a set A of X is called the α-closure [13] of A. It is denoted by αCl(A). The union of all α-open sub sets of A of X is called the α-interior [13] of A .It is denoted by αInt(A) . A subset A of X is α-closed if and only if A $\supset Cl(Int(Cl(A)))$. A subset N of a topological space$(X, \mathcal{T})$ is called an α-neighborhood [13] of a point x (resp. a subset A) of X if there exists a α-open set O of X such that x $\in O \subset$N (resp. $A \subset O \subset N$). A mapping f from a topological space (X, $\mathcal{T}$) to another topological space (X*, $\mathcal{T}^*$) is said to be α-continuous [14, 17] (resp. α-irresolute [13]) if the inverse image of every open (resp. α −open) set of X* is α −open in X. Every continuous (resp. α-irresolute) mapping is α-continuous, but the converse may not be true [13,14]. The concepts of continuous and α-irresolute mappings are independent.

In 1983 Atanassov [2,3,4] introduced the concept of intuitionistic fuzzy sets.

Definition 1.1 [2, 3, 4]: Let Y be a nonempty fixed set. An intuitionistic fuzzy set $\tilde{A}$ in Y is an object having the form

$\tilde{A} = \{< y, \mu_{\tilde{A}}(y), \upsilon_{\tilde{A}}(y) > y \in Y \}$

where the functions $\mu_{\tilde{A}}$:Y→I and $\upsilon_{\tilde{A}}$:Y→I denotes the degree of membership (namely$\mu_{\tilde{A}}(y)$) and the degree of non membership (namely $\upsilon_{\tilde{A}}(y)$) of each element y∈Y to the set $\tilde{A}$ respectively, and $0 \leq \mu_{\tilde{A}}(y) + \upsilon_{\tilde{A}}(y) \leq 1$ for each y∈Y.

Definition 1.2 [2, 3, 4]: Let Y be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form $\tilde{A} = \{< y, \mu_{\tilde{A}}(y), \upsilon_{\tilde{A}}(y) > : y \in Y\}, \tilde{B} = \{< y, \mu_{\tilde{B}}(y), \upsilon_{\tilde{B}}(y) > : y \in Y\}$ and let
$\{\tilde{A}_\alpha : \alpha \in \Lambda\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:

1. $\tilde{A} \subseteq \tilde{B}$ if $\forall y \in Y, [\mu_{\tilde{A}}(y) \leq \mu_{\tilde{B}}(y) and \upsilon_{\tilde{A}}(y) \geq \upsilon_{\tilde{B}}(y)]$;
2. $\tilde{A} = \tilde{B}$ if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$;
3. $\tilde{A}^C = \{< y, \upsilon_{\tilde{A}}(y), \mu_{\tilde{A}}(y) > y \in Y\}$;
4. $\tilde{0} = \{< y, 0,1 >: y \in Y\}$ and $\tilde{1} = \{< y, 1,0 >: y \in Y\}$;
5. $\cap \tilde{A}_\alpha$=$\{< y, \wedge \mu_{\tilde{A}}(y), \vee \upsilon_{\tilde{A}}(y) >: y \in Y\}$;
6. $\cup \tilde{A}_\alpha = \{< y, \vee \mu_{\tilde{A}}(y), \wedge \upsilon_{\tilde{A}}(y) >: y \in Y\}$.

Definition 1.3 [9]: Two Intuitionistic Fuzzy Sets $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi coincident ($\tilde{A}q\tilde{B}$ for short) if $\exists$ $y \in Y$such that $\mu_{\tilde{A}}(y) > \upsilon_{\tilde{B}}(y)$ or $\upsilon_{\tilde{A}}(y) < \mu_{\tilde{B}}(y)$.

Lemma 1.1[9]: For any two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y,$\rceil(\tilde{A}q\tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c$.
In 1997 Coker [8] introduced the concept of intuitionistic fuzzy topology.

Definition 1.4[8]: An intuitionistic fuzzy topology on a non empty set Y is a family Γ of intuitionistic fuzzy sets in Y which satisfy the following axioms:

(O$_1$). $\tilde{0}, \tilde{1} \in \Gamma$
(O$_2$). $\tilde{A}_1 \cap \tilde{A}_2 \in \Gamma$ for any $\tilde{A}_1, \tilde{A}_2 \in \Gamma$
(O$_3$). $\cup \tilde{A}_\alpha$ for any arbitrary family $\{\tilde{A}_\alpha : \alpha \in \Lambda\} \in \Gamma$.

In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ, is known as an intuitionistic fuzzy open set in Y. The complement $\tilde{B}^c$of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 1.5 [8]: Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the interior and closure of $\tilde{A}$ are defined by:

cl($\tilde{A}$) = $\cap\{\tilde{K} : \tilde{K}$ is an intuitionistic fuzzy closed set in Y and $\tilde{A} \subseteq \tilde{K}\}$,
int($\tilde{A}$) = $\cup\{\tilde{G} : \tilde{G}$ is an intuitionistic fuzzy open set in Y and $\tilde{G} \subseteq \tilde{A}\}$.

In 1999 Ozbekir nad Coker [18] introduced the concept of intuitionistic fuzzy multifunction.

Definition 1.6 [18]: Let X and Y are two non empty sets. A function $F: (X, \mathcal{T}) \to (Y, \Gamma)$ is called intuitionistic fuzzy multifunction if F(x) is an intuitionistic fuzzy set in Y, $\forall$ x$\in$ X.

Definition 1.7 [18]: Let $F: (X, \mathcal{T}) \to (Y, \Gamma)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then F(A) = $\bigcup_{x \in A} F(x)$.

Definition 1.8[18]: Let $F: (X, \mathcal{T}) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:

$$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\} \text{ and } F^-(\tilde{A}) = \{x \in X : F(x) q \tilde{A}\}$$

Lemma 1.2[23]: Let $F: (X, \mathcal{T}) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction and $\tilde{A}, \tilde{B}$ be intuitionistic fuzzy sets in Y. Then:

1. $F^+(\tilde{1}) = F^-(\tilde{1}) = X$.
2. $F^+(\tilde{A}) \subseteq F^-(\tilde{A})$.
3. $[F^-(\tilde{A})]^c = F^+(\tilde{A}^c)$.
4. $[F^+(\tilde{A})]^c = F^-(\tilde{A}^c)$.
5. If $\tilde{A} \subseteq \tilde{B}$,then $F^+(\tilde{A}) \subseteq F^+(\tilde{B})$
6. If $\tilde{A} \subseteq \tilde{B}$,then $F^-(\tilde{A}) \subseteq F^-(\tilde{B})$.

Definition 1.9[10]:A subset $\tilde{A}$ of a topological space (Y, Γ) is called :

1. Intuitionistic fuzzy α-open if $\tilde{A} \subset Int\left(Cl\left(Int(\tilde{A})\right)\right)$.
2. Intuitionistic fuzzy α-closed if its complement is Intuitionistic fuzzy α-open.

Definition 1.10 [10]: Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the α-interior and α-closure of $\tilde{A}$ are defined by:

$\alpha Cl(\tilde{A}) = \cap\{\tilde{K} : \tilde{K}$ is an intuitionistic fuzzy α-closed set in Y and $\tilde{A} \subseteq \tilde{K}\}$,
$\alpha Int(\tilde{A}) = \cup\{\tilde{G} : \tilde{G}$ is an intuitionistic fuzzy α-open set in Y and $\tilde{G} \subseteq \tilde{A}\}$.

Recently Thakur and Bohre[24] introduced the concepts of Intuitionistic fuzzy lower α −irresolute and intuitionistic fuzzy upper α −irresolute intuitionistic fuzzy multifunctions.

Definition 1.11[24]: An Intuitionistic fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \Gamma)$ is said to be:

1. Intuitionistic fuzzy lower α −irresolute at a point $x_0 \in X$ if for any $\widetilde{W} \in IF\alpha O(Y)$, such that $F(x_0)q\widetilde{W}$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(x)q\widetilde{W}, \forall x \in U$.
2. Intuitionistic fuzzy lower α −irresolute if it has this property at each point of X.

Definition 1.12[24]: An Intuitionistic fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \Gamma)$ is said to be:

1. Intuitionistic fuzzy upper α −irresolute at a point $x_0 \in X$ if for any $\widetilde{W} \in IF\alpha O(Y)$, such that $F(x_0) \subset \widetilde{W}$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(U) \subset \widetilde{W}$
2. Intuitionistic fuzzy upper α −irresolute if it has this property at each point of X.

The object of this paper is to obtain some properties of Intuitionistic fuzzy lower α −irresolute and intuitionistic fuzzy upper α −irresolute intuitionistic fuzzy multifunctions.

A function F from a topological space $(X, \mathcal{T})$to IF topological space(Y, Γ) is called IFM if F(x) is an intuitionistic fuzzy set in $Y, \forall x \in X$.

2. Properties of Upper and Lower α −Irresolute Intuitionistic Fuzzy Multifunctions

Theorem 2.1: Let $F: (X, \mathcal{T}) \rightarrow (Y, \Gamma)$ be an intuitionistic fuzzy multifunction, Then the following statements are equivalent:

1. F is intuitionistic fuzzy lower α −irresolute.
2. $F^-(\tilde{G}) \in \alpha O(X), \forall \tilde{G} \in IF\alpha O(Y)$.

Proof: (a) ⟹ (b). Let $\tilde{G} \in IF\alpha O(Y)$ and $x_0 \in F^-(\tilde{G})$. Then $F(x_0)q\tilde{G}$. Therefore there exists a set $U \in \alpha O(X)$ containing x_0 such that $F(x)q\tilde{G}, \forall x \in U$. It follows that $x_0 \in U \subseteq F^-(\tilde{G})$. Hence, $F^-(\tilde{G})$ is the union of α −open sets of X is α −open in X.

(b) $\Longrightarrow$ (a). Let x_0 be arbitrarily chosen in X and $\widetilde{G}$ be any intuitionistic fuzzy α −open set of Y such that $F(x_0)q\tilde{G}$. Then $x_0 \in F^{-}(\tilde{G})$. Put $U = F^{-}(\tilde{G})$ then $U \in \alpha O(X)$ containing x_0 such that $F(x)q\tilde{G}, \forall x \in U$. Hence F is intuitionistic fuzzy lower α −irresolute.

Theorem 2.2: For an intuitionistic fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\Gamma)$ the following conditions are equivalent:

1. F is intuitionistic fuzzy upper irresolute.
2. $F^{+}(\tilde{G}) \in \alpha O(X), \forall \tilde{G} \in IF\alpha O(Y)$.

Proof: (a) $\Longrightarrow$ (b). Let $\tilde{G} \in IF\alpha O(Y)$ and $x_0 \in F^{+}(\tilde{G})$. Then $F(x_0) \subset \tilde{G}$. Therefore there exists a set $U \in \alpha O(X)$ containing x_0 such that $F(x) \subset \tilde{G}$. It follows that $x_0 \in U \subseteq F^{+}(\tilde{G})$. Hence, $F^{+}(\tilde{G})$ is the union of α −open sets of X is α −open in X.

(b) $\Longrightarrow$ (a). Let x_0 be arbitrarily chosen in X and $\widetilde{G}$ be any intuitionistic fuzzy α −open set of Y such that $F(x_0) \subset \tilde{G}$. Then $x_0 \in F^{+}(\tilde{G})$. Put $U = F^{+}(\tilde{G})$ then $U \in \alpha O(X)$ containing x_0 such that $F(U) \subset \tilde{G}$. Hence F is intuitionistic fuzzy upper α −irresolute.

Definition 2.1: An intuitionistic fuzzy set $\tilde{V}$ of an intuitionistic fuzzy topological space (Y,Γ) is called a semi $\alpha - q$ −neighbourhood of an intuitionistic fuzzy set $\tilde{A}$ of Y if ∃ an $\widetilde{U} \in IF\alpha O(Y)$ such that $\tilde{A}q\widetilde{U} \subset \tilde{V}$.

Theorem 2.3: If $F:(X,\mathcal{T}) \to (Y,\Gamma)$ be an intuitionistic fuzzy multifunction, then $\left(\alpha Cl(F)\right)^{-}(\tilde{V}) = F^{-}(\tilde{V})$ for every $\tilde{V} \in IF\alpha O(Y)$.

Proof: Let $\tilde{V} \in IF\alpha O(Y)$ and $x \in \left(\alpha Cl(F)\right)^{-}(\tilde{V})$. Then $\tilde{V}q(\alpha Cl(F(x)))$. It follows that $\tilde{V}q(\alpha Cl(F))(x)$. Since $\tilde{V} \in IF\alpha O(Y)$, we have $\tilde{V}q(F(x))$ and hence $x \in F^{-}(\tilde{V})$. Thus $\left(\alpha Cl(F)\right)^{-}(\tilde{V}) \subset F^{-}(\tilde{V})$. Conversely, if $x \in F^{-}(\tilde{V})$. Then $F(x)q\tilde{V}$ and so $\alpha Cl(F(x))q\tilde{V}$, because $F(x) \subset \alpha Cl(F(x))$. Hence $x \in \left(\alpha Cl(F)\right)^{-}(\tilde{V})$ and $F^{-}(\tilde{V}) \subset \left(\alpha Cl(F)\right)^{-}(\tilde{V})$ Consequently $\left(\alpha Cl(F)\right)^{-}(\tilde{V}) = F^{-}(\tilde{V})$.

Corollary 2.1: If F: $(X,\mathcal{T}) \to (Y,\theta)$ be a fuzzy multifunction, then $\left(\alpha Cl(F)\right)^{-}(V) = F^{-}(V)$ for every $V \in F\alpha O(Y)$.

Corollary 2.2: If F: $(X,\mathcal{T}) \to (Y,\mathcal{T}^{*})$ be a multifunction, then $\left(\alpha Cl(F)\right)^{-}(V) = F^{-}(V)$ for every $V \in \alpha O(Y)$.

Theorem 2.4: An intuitionistic Fuzzy multifunction $F:(X,T) \to (Y,\Gamma)$ is intuitionistic fuzzy lower α −irresolute if and only if $\alpha ClF:(X,T) \to (Y,\Gamma)$ is intuitionistic fuzzy lower α −irresolute.

Proof: ***Necessity.*** Suppose that F is intuitionistic fuzzy lower α −irresolute. Let $x \in X$ and $\tilde{V} \in IF\alpha O(Y)$ such that $\tilde{V}q\big(\alpha Cl(F)\big)(x)$. By theorem 2.3, we have $x \in \big(\alpha Cl(F)\big)^{-}(\tilde{V}) = F^{-}(\tilde{V})$ and hence $\tilde{V}q\big(F(x)\big)$. Since F is intuitionistic fuzzy lower α −irresolute there exits $U \in \alpha O(X)$ such that $\tilde{V}q\big(F(u)\big)$ for each $u \in U$. Therefore $\tilde{V}q\big(\alpha Cl(F)\big)(u)$ for each $u \in U$ and thus αClF is intuitionistic fuzzy lower α −irresolute.

Sufficiency: Suppose αClF is intuitionistic fuzzy lower α −irresolute. Let $x \in X$ and $\tilde{V} \in IF\alpha O(Y)$ such that $\tilde{V}q\big(F(x)\big)$. By theorem 2.3 we have $x \in \big(\alpha Cl(F)\big)^{-}(\tilde{V}) = F^{-}(\tilde{V})$ and hence $\tilde{V}q\big(\alpha Cl(F)\big)(x)$. Since αClF is intuitionistic fuzzy lower α −irresolute there exits $U \in \alpha O(X)$ such that $\tilde{V}q(\alpha Cl(F))(u)$ for each $u \in U$. Therefore $u \in \big(\alpha Cl(F)\big)^{-}(\tilde{V}) = F^{-}(\tilde{V})$ for each $u \in U$ and it shows that $\tilde{V}q\big(F(u)\big)$ for each $u \in U$. Hence F is intuitionistic fuzzy lower α −irresolute.

Corollary 2.3: A Fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \theta)$ is fuzzy lower α −irresolute if and only if $\alpha ClF: (X, \mathcal{T}) \rightarrow (Y, \theta)$ is fuzzy lower α −irresolute.

Corollary 2.4: A multifunction F: $(X, \mathcal{T}) \rightarrow (Y, \mathcal{T}^*)$ is lower α −irresolute if and only if αClF: $(X, T) \rightarrow (Y, T^*)$ is lower α −irresolute.

Definition 2.2: A Family $\{U_\omega : \omega \in \Lambda\}$ of open sets in X is said to be open cover of X if $\cup \{U_\omega : \omega \in \Lambda\} = X$, and a finite subfamily of open cover of X which is also open cover of X is called a finite sub cover of $\{U_\omega : \omega \in \Lambda\}$.

Theorem 2.5: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. An Intuitionistic fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \Gamma)$ is intuitionistic fuzzy upper α −irresolute if and only if the restriction $F|U_\omega: U_\omega \rightarrow Y$ is intuitionistic fuzzy upper α −irresolute for each $\omega \in \Lambda$.

Proof: ***Necessity.*** Suppose that F is intuitionistic fuzzy upper α −irresolute. Let $\omega \in \Lambda$ and $x \in U_\omega$. Let $\tilde{V} \in IF\alpha O(Y)$ such that $\left(F|U_\omega\right)(x) \subset \tilde{V}$. Since F is intuitionistic fuzzy upper α −irresolute and $F(x) = \left(F|U_\omega\right)(x)$, there exists $G \in \alpha O(X)$ containing x such that $F(G) \subset \tilde{V}$. Put $U = G \cap U_\omega$, then $x \in U \in \alpha O(X)$ and $\left(F|U_\omega\right)(U) = F(U) \subset \tilde{V}$. Therefore, it follows that $\left(F|U_\omega\right)$ is intuitionistic fuzzy upper α −irresolute.

Sufficiency. Let $x \in X$ and $\tilde{V} \in IF\alpha O(Y)$ such that $F(x) \subset \tilde{V}$. Since $\{U_\omega : \omega \in \Lambda\}$ be an open cover of X, there exists $\omega \in \Lambda$ such that $x \in U_\omega$. Now $F|U_\omega: U_\omega \rightarrow Y$ is intuitionistic fuzzy upper α −irresolute and $F(x) = \left(F|U_\omega\right)(x)$, there exists $U \in$

$\alpha O(U_\omega)$ containing x such that $\left(F|U_\omega\right)(U) \subset \tilde{V}$. Since each $U_\omega$is open in X $U \in \alpha O(X)$ containing x and $F(U) \subset \tilde{V}$. Hence F is intuitionistic fuzzy upper α −irresolute.

Corollary 2.5: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. A Fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \theta)$ is fuzzy upper α −irresolute if and only if the restriction $F|U_\omega : U_\omega \rightarrow Y$ is fuzzy upper α −irresolute for each $\omega \in \Lambda$.

Corollary 2.6: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. A multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \mathcal{T}^*)$ is upper α −irresolute if and only if the restriction $F|U_\omega : U_\omega \rightarrow Y$ is upper α −irresolute for each $\omega \in \Lambda$.

Theorem 2.6: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. An Intuitionistic fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \Gamma)$ is intuitionistic fuzzy lower α −irresolute if and only if the restriction $F|U_\omega : U_\omega \rightarrow Y$ is intuitionistic fuzzy lower α −irresolute for each $\omega \in \Lambda$.

Proof: ***Necessity.*** Suppose that F is intuitionistic fuzzy lower α −irresolute. Let $\omega \in \Lambda$ and $x \in U_\omega$. Let $\tilde{V} \in IF\alpha O(Y)$ such that $\left(F|U_\omega\right)(x) q \tilde{V}$. We have $F(x) = \left(F|U_\omega\right)(x)$ and hence $F(x) q \tilde{V}$. Since F is intuitionistic fuzzy upper α −irresolute, there exists $G \in \alpha O(X)$ containing x such that $F(g) q \tilde{V}$ for each $g \in G$. Put $U = G \cap U_\omega$. Then $U \in \alpha O(X)$ and $\left(F|U_\omega\right)(U) = F(U) \subset \tilde{V}$. Hence $\left(F|U_\omega\right)$ is intuitionistic fuzzy lower α −irresolute.

Sufficiency. Let $x \in X$ and $\tilde{V} \in IF\alpha O(Y)$ such that $F(x) q \tilde{V}$. Since $\{U_\omega : \omega \in \Lambda\}$ be an open cover of X, there exists $\omega \in \Lambda$ such that $x \in U_\omega$. Since $F(x) = \left(F|U_\omega\right)(x)$, we have $\left(F|U_\omega\right) q \tilde{V}$. Since $F|U_\omega : U_\omega \rightarrow Y$ is intuitionistic fuzzy lower α −irresolute and $F(x) = \left(F|U_\omega\right)(x)$, there exists $U \in \alpha O(U_\omega)$ containing x such that $\left(F|U_\omega\right)(u) q \tilde{V}$ for each for each $u \in U$. Since each$U_\omega$is open in X therefore $U \in \alpha O(X)$ containing x and $F(U) q \tilde{V}$. Therefore F is intuitionistic fuzzy lower α −irresolute.

Corollary 2.7: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. A fuzzy multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \theta)$ is fuzzy lower α −irresolute if and only if the restriction $F|U_\omega : U_\omega \rightarrow Y$ is fuzzy lower α −irresolute for each $\omega \in \Lambda$.

Corollary 2.8: Let $\{U_\omega : \omega \in \Lambda\}$ be an open cover of a topological space $(X, \mathcal{T})$. A multifunction $F: (X, \mathcal{T}) \rightarrow (Y, \mathcal{T}^*)$ is lower α −irresolute if and only if the restriction $F|U_\omega: U_\omega \rightarrow Y$ is lower α −irresolute for each $\omega \in \Lambda$.

Conclusion
The present paper obtained some properties of intuitionistic fuzzy upper and lower α −irresolute intuitionistic fuzzy multifunctions.

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