Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 7, Number 1 (2012), pp. 43-49

http://www.ripublication.com/afm.htm

On Fuzzy Contra g^* p-Continuous Functions

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Abstract

In this paper we introduce and study the new class of functions called fuzzy contra g^*p-continuous and almost fuzzy contra g^*p-continuous mappings on fuzzy topological spaces. We investigate some of their properties. Also we provide the relation between fuzzy contra g^*p-continuous mappings and fuzzy almost contra g^*p-continuous mappings.

Keywords: Fuzzy topology, fuzzy generalized closed set, fuzzy g^*p-closed set, fuzzy contra pre continuous function, fuzzy g^*p-continuous function, fuzzy almost contra continuous functions.

Introduction

The fuzzy pre-open and fuzzy pre-continuous mappings were introduced and generalized by Bin Shahana [4]. N. Levine [8] introduced the concepts of generalized closed sets in general topology in the year 1970. T. Fukutake, R.K. Saraf, M. Caldas and S. Mishra [6] introduced the notation of generalized pre-closed sets in fuzzy topological space. S. S. Benchalli and G. P. Siddapur [3] introduced and investigate g^*p-continuous maps in fuzzy topological spaces. In 2011, P.G. Patil T.D. Rayanagoudar and Mahesh K. Bhat [11] introduced and studied the concepts of contra g^*p-continuous and almost contra g^*p-continuous mappings in general topological spaces.

In this paper we introduce and study the new class of mappings called fuzzy contra g^*p-continuous and fuzzy almost contra g^*p-continuous functions in fuzzy topological spaces. Also we define the relation between of fuzzy contra g^*p -continuous and fuzzy almost contra g^*p-continuous spaces and study some of their properties.

Preliminaries

Let X be a non empty set. A collection τ of fuzzy sets in X is called a fuzzy topology on X if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of τ and τ is closed with respect to any union and finite intersection. The members of τ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The **closure** of a fuzzy set λ (denoted by (λ)) is the intersection of all fuzzy closed which contains λ. The **interior** of a fuzzy set λ (denoted by $int(\lambda)$) is the union of all fuzzy open subsets of λ. A fuzzy set λ in X is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

1. semi-open fuzzy set [1] if $\lambda \leq cl\big(int(\lambda)\big)$ and semi-closed fuzzy set if $int\big(cl(\lambda)\big) \leq \lambda$.
2. pre-open fuzzy set [4] if $\lambda \leq int\big(cl(\lambda)\big)$ and pre-closed fuzzy set if $cl\big(int(\lambda)\big) \leq \lambda$.
3. semi-preopen fuzzy set [12] (=β set) if $\lambda \leq cl(int\big(cl(\lambda)\big))$ and semi-preclosed fuzzy set if $int\left(cl\big(int(\lambda)\big)\right) \leq \lambda$.
4. regular open fuzzy set [1] if$\lambda = int(cl(\lambda))$ and regular closed fuzzy set if $\lambda = cl\big(int(\lambda)\big)$.

The pre-closure (resp. semi-closure, semi-preopen) of a fuzzy set λ in fuzzy topological space (X, τ) is intersection of all pre-closed (resp. semi-closed, semi-preclosed) fuzzy sets in X containing λ and is denoted by $pcl(\lambda)$ (resp. $scl(\lambda), spcl(\lambda)$).

Definition 2.2: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

1. generalized closed fuzzy set (g-closed) fuzzy set [2] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in(X, τ).
2. generalized pre-closed fuzzy set (gp-closed) fuzzy set [6] if $pcl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is open fuzzy set in(X, τ).
3. g^*- closed fuzzy set (g^*-closed) fuzzy set [8] if $cl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in(X, τ).
4. g^*-preclosed fuzzy set (g^*p-closed) fuzzy set [3] if $pcl(\lambda) \leq \eta$ whenever $\lambda \leq \eta$ and η is g-open fuzzy set in(X, τ).

The complement of g-closed (resp. gp-closed, g^*-closed and g^*p-closed) fuzzy sets are called fuzzy g-open (resp. gp-open, g^*-open and g^*p-open) sets in fuzzy topological spaces.

Definition 2.3: A fuzzy topological space (X, τ) is called T_p^* -space [6] if every g^*p-closed fuzzy set is a closed fuzzy set in X.

Definition 2.4: A function f from a fuzzy topological space (X, τ) to fuzzy topological space (Y, σ) is called:

1. fuzzy-contra continuous if $f^{-1}(\lambda)$ is fuzzy closed in X for every fuzzy open set λ of Y [5].
2. fuzzy contra precontinuous if $f^{-1}(\lambda)$ is fuzzy preclosed in X for every fuzzy open set λ of Y [7].
3. fuzzy g-continuous if $f^{-1}(\lambda)$ is fuzzy g-closed in X for every fuzzy closed set λ of Y [2].
4. fuzzy gp- continuous if $f^{-1}(\lambda)$ is fuzzy gp-closed in X for every fuzzy closed set λ of Y [6].
5. fuzzy g^*- continuous if $f^{-1}(\lambda)$ is fuzzy g^*-open in X for every fuzzy open set λ of Y [8].
6. fuzzy g^*p-continuous if $f^{-1}(\lambda)$ is fuzzy g^*p -open in X for every fuzzy open set λ of Y [3].
7. fuzzy almost continuous if $f^{-1}(\lambda)$ is fuzzy open in X for every fuzzy regular open set λ of Y [1].

Fuzzy Contra g^*p-Continuous Functions

Definition 3.1. A function $f: X \to Y$ is called **fuzzy contra g^*p-continuous** if $f^{-1}(\lambda)$ is fuzzy g^*p-closed set in X for every open set λ in Y.

Theorem 3.2. Every fuzzy contra continuous function is fuzzy contra g^*p-continuous function. .

Proof: It follows from the fact that every fuzzy closed set is g^*p-closed set.[]
The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ and μ, λ be fuzzy sets in X and Y, defined as $\mu(x_1) = 0.2$, $\mu(x_2) = 0.4$, $\mu(x_3) = 0.6$, $\lambda(y_1) = 0.3$, and $\lambda(y_2) = 0.5$. Let $\tau = \{0, \mu, 1\}$ and $\tau' = \{0, \lambda, 1\}$ be fuzzy topologies on sets X and Y respectively. We see that map $f : X \to Y$ defined as $f(x_1) = y_1, f(x_2) = y_1$ and $f(x_3) = y_2$. Then f is fuzzy contra g^*p-continuous but not fuzzy contra continuous.

Theorem 3.4. Every fuzzy contra pre-continuous mapping is fuzzy contra g^*p-continuous function.

Proof. Straight forward and follows from the definitions.
The converse of the above theorem need not be true as seen from Example 3.2.

Theorem 3.5. If a function $f: (X, \tau) \to (Y, \sigma)$ is fuzzy contra g^*p –continuous and (X, τ) is fuzzy T_p^*-space, than f is fuzzy contra continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is g^*p-closed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. Thus f is fuzzy contra continuous function.

Theorem 3.6. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy contra precontinuous and (X,τ) is fuzzy T_p^*-space, than f is fuzzy contra g^*p-continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is preclosed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is g^*p-closed fuzzy set in X. Thus f is fuzzy contra g^*p-continuous function.

Theorem 3.7. If a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy contra g^*p –continuous and (X,τ) is fuzzy T_p^*-space, than f is fuzzy contra pre-continuous.

Proof. Let λ be open fuzzy set in Y. Then $f^{-1}(\lambda)$ is g^*p-closed fuzzy set in X. Since X is fuzzy T_p^* -space. $f^{-1}(\lambda)$ is closed fuzzy set in X. And every closed fuzzy set is preclosed fuzzy set. Thus f is fuzzy contra pre-continuous function.

Theorem 3.8. Let (X,τ) and (Y,σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \rightarrow Y$.

1. f is fuzzy contra g^*p- continuous.
2. $f^{-1}(\lambda)$ is g^*p-open fuzzy set in X for each closed fuzzy set λ in Y.
3. for each $x \in X$ and each closed fuzzy set λ in Y containing $f(x)$. there exist a g^*p-open fuzzy set η in X containing x such that $f(\eta) \leq \lambda$.
4. for each $x \in X$ and open fuzzy set μ in Y non-containing $f(x)$, there exists a g^*p-closed fuzzy set ϑ in X non-containing x such that $f^{-1}(\mu) \leq \vartheta$.

Proof. (1)$\Rightarrow$ (2). Let λ be a closed fuzzy set in (Y,σ). Then 1-λ is fuzzy open. By (1), $f^{-1}(1-\lambda) = 1 - f^{-1}(\lambda)$ is g^*p-closed fuzzy set in X. So $f^{-1}(\lambda)$ is g^*p –open fuzzy set in X.
(2) $\Rightarrow$ (1). proof as above.
(2) $\Rightarrow$ (3). Let λ be any closed fuzzy set in Y containing $f(x)$. By (2). $f^{-1}(\lambda)$ is g^*p-open fuzzy set in (X,τ) and $x \in f^{-1}(\lambda)$. Take $\eta = f^{-1}(\lambda)$. Then $f(\eta) \leq \lambda$.
(3) $\Rightarrow$ (2). Let λ be a closed fuzzy set in Y and $x \in f^{-1}(\lambda)$. From (3), there exists a g^*p-open fuzzy set η in X containing x such that $\eta \leq f^{-1}(\lambda)$. We have $f^{-1}(\lambda) = \cup_{x \in f^{-1}(\lambda)} \eta$. Thus $f^{-1}(\lambda)$ is g^*p-open fuzzy set in (X,τ).
(3) $\Rightarrow$ (4). Let μ be any open fuzzy set in Y non-containing $f(x)$. Then 1-μ is a closed fuzzy set containing $f(x)$. By (3) there exists a g^*p-open fuzzy set η in X containing x such that $f(\eta) \leq 1-\mu$. Hence $\eta \leq f^{-1}(1-\mu) \leq 1 - f^{-1}(\mu)$ and then $f^{-1}(\mu) \leq 1-\eta$. Take $\vartheta = 1-\eta$. We obtain that ϑ is a g^*p-closed fuzzy set in X non-containing x.

The converse can be shown easily.

Definition 3.9. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy Contra g^*p – irresolute** if $f^{-1}(\lambda)$ is g^*p-closed fuzzy set in X for every g^*p-open fuzzy set λ in Y.

Theorem 3.10. A function $f:(X,\tau) \to (Y,\sigma)$ is fuzzy contra g^*p-continuous if and only if $f^{-1}(\lambda)$ is g^*p-open fuzzy set in X for every g^*p-closed fuzzy set λ in Y.

Theorem 3.11. Every fuzzy contra g^*p-irresolute mapping is fuzzy contra g^*p-continuous.

Proof. Let $f: X \to Y$ is fuzzy contra g^*p-irresolute function. Let λ be a fuzzy open set in Y. Then λ is g^*p-open fuzzy set in Y. Since f is fuzzy contra g^*p-irresolute. $f^{-1}(\lambda)$ is g^*p- fuzzy closed set in X. Hence f is fuzzy contra g^*p-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.12. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$. Let μ be fuzzy set in X and ϑ be a fuzzy set in Y defined as $\mu(x_1) = 0.2$, $\mu(x_2) = 0.4$, $\vartheta(y_1) = 0.1$ and $\vartheta(y_2) = 0.5$. Let $\tau = \{0, \mu, 1\}$ and $\tau' = \{0, \vartheta, 1\}$ be the fuzzy topologies on sets X and Y respectively. The map $f: X \to Y$ $f: X \to Y$ defined as $f(x_i) = y_i$, $i = 1,2$ is fuzzy contra g^*p - continuous function. The fuzzy set γ in Y defined as $y_1) = 0.3$, $\gamma(y_2) = 0.3$ is g^*p –open fuzzy set in Y but $f^{-1}(\gamma)$ is not g^*p –closed fuzzy set in X. Hence the map $f: X \to Y$ is fuzzy contra g^*p-continuous but not fuzzy contra g^*p –irresolute function.

Theorem 3.13. Let $f: X \to Y$, $g: Y \to Z$ be two functions then
$gof: X \to Z$ is fuzzy contra g^*p-continuous, if f is fuzzy contra g^*p-continuous and g are fuzzy continuous.
$gof: X \to Z$ is fuzzy contra g^*p-continuous if f is fuzzy contra g^*p-irresolute and g is fuzzy g^*p-continuous.

Fuzzy Almost Contra g^*p-Continuous Function

Definition 4.1. A function $f:(X,\tau) \to (Y,\sigma)$ is called **Fuzzy almost contra g^*p –Continuous** if $f^{-1}(\lambda)$ is fuzzy g^*p-closed set in X for every regular open set λ in Y.

Theorem 4.2. Every fuzzy contra g^*p-continuous function is fuzzy almost contra g^*p-continuous.

Proof. Since every regular fuzzy open set is open fuzzy set, such that every fuzzy contra g^*p-continuous mappings is fuzzy almost contra g^*p-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.3: Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ λ, and μ be a fuzzy set in X and Y defined as $\lambda(x_1) = 0.2$, $\lambda(x_2) = 0.4$, $\mu(y_1) = 0.3$, $\mu(y_2) = 0.5$. Let $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \mu, 1\}$ be fuzzy topologies on sets X and Y respectively. The map $f: (X, \tau) \to (Y, \tau')$ defined as $f(x_i) = y_i, i = 1,2$ is fuzzy almost contra g^*p-continuous map but not fuzzy contra g^*p-continuous.

Definition 4.4. A function $f: X \to Y$ is said to be fuzzy regular set connected [] if $f^{-1}(\lambda)$ is fuzzy clopen in X for every fuzzy regular open set λ of Y.

Theorem 4.5. If a function $f: X \to Y$ is fuzzy almost contra g^*p-continuous and almost continuous, then f is fuzzy regular set connected.

Proof. Let λ be a fuzzy regular open set in (Y, σ). Since f is fuzzy almost contra g^*p-continuous and fuzzy almost continuous, $f^{-1}(\lambda)$ is fuzzy g^*p-closed and open. Hence $f^{-1}(\lambda)$ is fuzzy clopen. Therefore f is fuzzy regular set connected.

Definition 4.6. A fuzzy topological spaces (X, τ) is called fuzzy g^*p-connected if X cannot be written as the disjoint union of two non-empty fuzzy g^*p-open sets.

Theorem 4.7. Let (X, τ) and (Y, σ) be two fuzzy topological spaces. The following statement are equivalent for a function $f: X \to Y$.

1. f is fuzzy almost contra g^*p- continuous.
2. $f^{-1}(\lambda)$ is fuzzy g^*p-open set in X for every regular closed set λ in Y.
3. for each $x \in X$ and each fuzzy regular closed set λ in Y containing $f(x)$. there exist a fuzzy g^*p-open set η in X containing x such that $f(\eta) \leq \lambda$.
4. for each $x \in X$ and fuzzy regular open set μ in Y non-containing $f(x)$, there exists a fuzzy g^*p-closed set ϑ in X non-containing x such that $f^{-1}(\mu) \leq \vartheta$.

Proof. As theorem 3.8.

Theorem 4.8: Let X, Y and Z be fuzzy topological spaces and let $f: X \to Y$ and $g: Y \to Z$ be maps. If f is fuzzy contra g^*p-continuous and g is fuzzy almost continuous then $gof: X \to Z$ is fuzzy almost contra g^*p-continuous.

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