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A Fuzzy Critical Path Analysis by Ranking Method

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Abstract

In this paper an algorithm is presented to perform critical path analysis in a fuzzy environment. The trapezoidal fuzzy numbers, given by decision makers or characterized by historical data, are utilized to assess the activity times in a project network. And a fuzzy critical path analysis is proposed for project network problem. So, the grade of membership that the project can be completed within a prefixed period can be computed. By using this algorithm, the ambiguities involved in the assessment activity times in a project network can be effectively improved and thus a more convincing and effective project management decision-making can be obtained.

Keywords: Fuzzy Set Theory, Trapezoidal Fuzzy Numbers, Fuzzy Critical Path Analysis, Project Network.

Introduction

To maximize resource utilization and minimize overall cost, project management has always been an important issue for government agencies and industrial organizations. The network techniques used to tackle project analysis are Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) [19]. With CPM a deterministic assessment for activity time was used [7, 16], while with PERT random time assessments were employed [11, 14, 18]. Chen et al. [7] incorporated time-window constraint and time schedule constraint into the traditional activity network. And a linear time algorithm is developed for finding the critical path in an activity network with these time constraints. Levner and Kats [11] proposed an algorithm for finding critical paths in PERT networks with variable arc length depending on a parameter. This algorithm can be applied to solve cyclic robotic scheduling problem in O(mn2) time (where m and n stand the number of arcs and nodes in the network, respectively). All the methods stated above are based on the concept of accurate measure and crisp evaluation or random process.

In a PERT project network, the activity times are assumed to be a beta distribution and three time estimates (optimistic time estimate, most likely time estimate and pessimistic time estimate) are used to estimate the means and variances of the activity times. However, goodness-of-fit test for the activity times not only the process is troublesome but also the result may show that the assumption of beta distribution is not held.

When decision-makers engage in evaluating activity times, they tend to, in fact, give assessments based on their wisdom, professional knowledge, experience and available information. In reality, due to the availability and uncertainty of information as well the variation of management scenario, it is often difficult to obtain the exact activity time estimates. Apart from these, linguistic terms [21] such as "approximately between 12 and 18 hours", "around 28 hours", are frequently used to convey their estimations. Thus, the conventional approaches, both deterministic and random process, tend to be less effective in conveying the imprecision or vagueness nature of the linguistic assessment [4, 15, 17]. The fuzzy set theory can play a significant role in this kind of decision-making environment. Nasution [15] proposed a fuzzy critical path method by considering the interactive fuzzy subtraction and by observing that only the nonnegative part of the fuzzy numbers can have physical interpretation. Chanas and Zielinski [4] proposed a method to make critical path analysis in the network with fuzzy activity times (interval activity times, fuzzy numbers of L-R type) by directly applying the extension principle [20] to the classical criticality notion treated as a function of activity duration time in the network. And two methods of calculation of the path degree of criticality are presented. Slyeptsov and Tyshchuk [17] presented an efficient method of computation of fuzzy time windows for late start and finish times of operations in the problem of fuzzy network. Fuzzy set theory was used to tackle problems in which a source of vagueness is involved. Linguistic terms can be properly represented by the approximate reasoning of fuzzy set theory [21]. To effectively deal with the ambiguities involved in the process of linguistic estimate times, the trapezoidal fuzzy numbers are used to characterize fuzzy measures of linguistic values. Besides, to reduce the complexity of model development and computations of solving problem, and incorporate the decision-maker's risk attitude into the problem of fuzzy network, an algorithm which is intuitive and easy implementation,is thus proposed to perform critical path analysis in a fuzzy environment.

The Representation of Fuzzy Activity Time

In this paper, the fuzzy activity time, denoted by FET_{ij}, of activity A_{ij} in a project network is represented by trapezoidal fuzzy number $FET_{ij} = (c_{ij}, a_{ij}, b_{ij}, d_{ij})$ [1, 17], where $c_{ij,}$ d_{ij} are minimum and maximum values of assessing activity time for $A_{ji,}$ whereas a_{ij} and b_{ij} are the first quartile and third quartile of activity time for Aij. If there is only one set of four historical data, then c_{ij}, a_{ij}, b_{ij}, d_{ij} can be sorted from minimum to maximum.For example, if the current four historical data of activity are 6, 9, 3, and 8, the trapezoidal fuzzy number of evaluation value is (3, 6, 8, 9). Conversely, if one has no further information with respect to activity A_{ij}, the fuzzy activity time $FET_{ij} = (c_{ij}, a_{ij}, b_{ij}, d_{ij})$ can be evaluated subjectively by the decision-maker based on his/her knowledge, experience and subjective judgment. Applying the extension principle [19], the extended algebraic operations of any two fuzzy activity times and can be expressed as:

Addition : $\oplus$
$FET_1 \oplus FET_2 = (c_1, a_1, b_1, d_1) \oplus (c_2, a_2, b_2, d_2)$
$= (c_1 + c_2, a_1 + a_2, b_1 + b_2, d_1 + d_2)$

Subtraction : Θ
$FET_1 \Theta FET_2 = (c_1, a_1, b_1, d_1) \Theta (c_2, a_2, b_2, d_2)$
$= (c_1 - d_2, a_1 - b_2, b_1 - a_2, d_1 - c_2)$

The Rank of Trapezoidal Fuzzy Number

In fuzzy critical path analysis, ranking methods are essential. Many methods of ranking fuzzy numbers have been proposed (e.g., Bortolan and Degani [1], Buckley [2],Campos and Gonzalez [3], Chen [6], Gonzalez [9], Kim and Park [10], Liou and Wang [13]). However, certain shortcomings of some of the methods have been reported in Bortolan and Degani [1], Chen [6] and Kim and Park [10]. For ease of implementation, powerfulness in problem solving and considering the decision-maker' risk attitude, combining the ranking method proposed by Liang and Wang [12] and the concept of evaluating decision-maker's risk attitude presented by Chang and Chen [5], a useful ranking method for decision-making problem characterized by trapezoidal fuzzy numbers is developed. Let $FET_{ij} = (c_{ij}, a_{ij}, b_{ij}, d_{ij})$ be the fuzzy activity time of activity A_{ij}. The decision maker's risk attitude index β can be obtained by

$$\beta = \left[\sum_i \sum_j \frac{a_{ij} - b_{ij}}{(a_{ij} - c_{ij}) + (d_{ij} - b_{ij})} \right] \Big/ t \qquad (1)$$

where A_{ij} and t denotes the set of all activities and the number of activities in network.

For a fuzzy number A_i with membership function $f_{A_i}(x)$ we define

$m_i = \min\{x / f_{A_i}(x) = 1\} + \max\{x / f_{A_i}(x) = 1\}$

now, we rank the fuzzy number A_i and A_J according to the following rules-

$A_i > A_j \Leftrightarrow R(A_i) > R(A_j)$ or $R(A_i) = R(A_j)$ and $m_i > m_j$

then the ranking value $R(A_i)$ of the trapezoidal fuzzy number A_i can be obtained as follow.

$$R(A_i)=\beta\left[(d_i-x_i)/(x_2-x_1-b_i+d_i)\right]$$
$$+(1-\beta)\left[1-(x_2-c_i)/(x_2-x_1+a_i-c_i)\right] \quad (2)$$

Where
$x_1=\min(c_1,c_2,...c_n)$ and $x_2=\max(d_{1,}d_2,...d_n)$

By using Eq. (2) and taking the β value calculated by Eq. (1), one can easily calculate the ranking values of the n trapezoidal fuzzy numbers. Based on the ranking rules described above, the ranking of the n trapezoidal fuzzy numbers can then be effectively determined.

Fuzzy Critical Path Analysis

Notation

N: The set of all nodes in a project network
A_{ij} : The activity between nodes i and j
FET_{ij} : The fuzzy activity time of A_{ij}
FES_j : The earliest fuzzy time of node j
ELF_j : The latest fuzzy time of node j
FTS_{ij} : The total slack fuzzy time of A_{ij}
S(j): The set of all successor activities of node j
NS(j): The set of all nodes connected to all successor activities of node j, i.e.,
NS(j) = {k | $A_{jk} \in$ S(j), k$\in$ N}
F(j): The set of all predecessor activities of node j
NP(j): The set of all nodes connected to all predecessor activities of node j, i.e.,
NP(j) = {i | Aij $\in$ F(j), i $\in$ N}
Pi: The i-th path
P: The set of all paths in a project network.
FCPM(P_k): The fuzzy completion time of path Pk in a project network.

The important properties and theorem in FCPM

In here, the important properties and theorem used in the fuzzy CPM [14] are briefly introduced. Set the initial node to zero for starting, i.e., FES1 = (0, 0, 0, 0). Then, the following properties are true.

Property 1: $FES_j = \max\{FES_i \oplus FET_{ij} | i \in NP(j), j \neq 1, j \in N\}$

Property 2: $FLF_j = \min\{FLF_k \ \Theta\ FET_{jk} | k \in NS(j), j \neq n, j \in N\}$

Property 3: $FTS_{ij} = FLF_j \ \Theta\ (FES_i \oplus FET_{ij}),\ 1 \leq i < j \leq n;\ i, j \in N$

Property 4: $\sum_{\substack{1\le i<j\le n \\ i,j\in P_k}} FTS_{ij}, P_K \in P$

Definition 1: Assume that there exist a path P_C in a project network such that $FCPM(P_C) = \min\{FCPM(P_i)/P_i \in P\}$ then the path P_C is a fuzzy critical path.

Theorem 4.1: Assume that the fuzzy activity times of all activities in a project network are trapezoidal fuzzy numbers, then there exists fuzzy critical path in the project network.

Proof. Set FES_1 = (0, 0, 0, 0) to indicate that the project starts at time zero, then FES_j of the j-th node can be calculated by the sum of FES_i and FET_{ij}, where $i \in NP(j)$ and j = 2, 3, . . . , n. Then there must uniquely exist a maximal FES_j, such that,

$FES_j = \max\{FES_i \oplus FET_{ij} | i \in NP(j), j \neq 1, j \in N\}$

Furthermore, let $FLF_n = FES_n$ to indicate the latest and earliest start of the last node of the project are the same, then FLF_j of the j-th node can be calculated by subtracting FET_{jk} from FLF_k, where $k \in NS(j)$, and j = n − 1, n − 2, . . . , 2, 1.

Similarly, there uniquely exists a minimal FLF_j, such that,

$FLF_j = \min\{FLF_k \ \Theta\ FET_{jk} | k \in NS(j), j \neq n, j \in N\}$

and so is FTS_{ij}. It is also true for any $P_k \in P$ such $FCPM(P_k)$ is unique. And at least there exists a path P_C such that

$FCPM(P_C) = \min\{FCPM(P_i)/P_i \in P\}$ therefore, we complete the proof.

Fuzzy critical path analysis

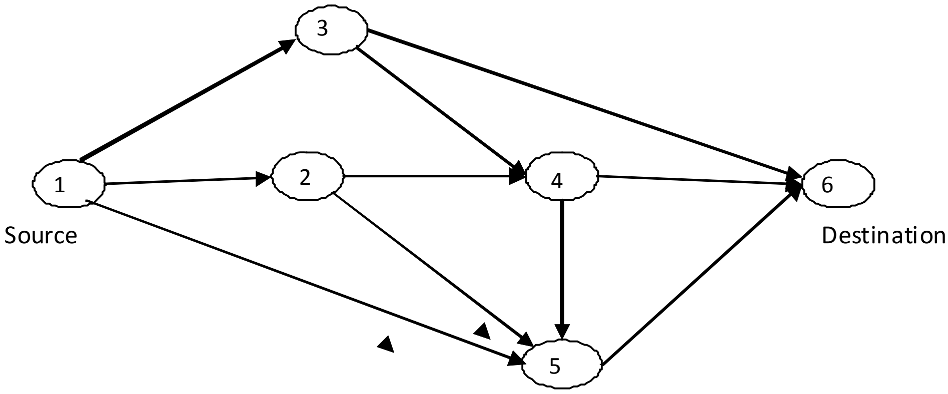
In this section, a fuzzy critical path analysis algorithm is developed to find a critical path of a project network in a fuzzy environment. The description of the algorithm is presented in the following.

Fuzzy critical path analysis algorithm:

1. Identify activities in a project.
2. Establish precedence relationships of all activities.
3. Estimate the fuzzy activity time with respect to each activity.
4. Construct the project network.
5. Let FES_1 = (0, 0, 0, 0) and calculate FES_j, j = 2, 3, . . . , n, by using property 1.
6. Let $FLF_n = FES_n$ and calculate FLF_j, j = n−1, n − 2, . . . , 2, 1, by using property 2.
7. Calculate FTS_{ij} with respect to each activity in a project network by using property 3.
8. Find all the possible paths and calculate $FCPM(P_k)$ by using property 4.
9. Find the fuzzy critical path by using definition 1. and theorem 1.
10. Find the grade of membership that the project can be completed at scheduled time.

Numerical Example

In this section, a hypothetical project problem is presented to demonstrate the computational process of fuzzy critical path analysis proposed above. Suppose there is a project network, as Figure 1, with the set of node N = {1, 2, 3, 4}, the fuzzy activity time for each activity as shown in Table 1.



Activity A_{ij}	Fuzzy activity time FET_{ij}
A_{12}	(2,2,3,4)
A_{13}	(2,3,3,6)
A_{15}	(2,3,4,5)
A_{24}	(2,2,4,5)
A_{25}	(2,4,5,8)
A_{34}	(1,1,2,2)
A_{36}	(7,8,11,15)
A_{45}	(2,3,3,5)
A_{46}	(3,3,4,6)
A_{56}	(1,1,1,2)

By using equation (5), the decision maker's risk attitude index β is equal to 0.602. The computational procedures are as follows:

Step 1. Set $FES_1 = (0, 0, 0, 0)$ and calculate FES_j , j = 2, 3, 4, 5,6 by using property 1.

$FES_2 = FES_1 \oplus FET_{12} = (0,0,0,0) + (2,2,3,4) = (2,2,3,4)$

$FES_3 = FES_1 \oplus FET_{13} = (0,0,0,0) + (2,3,3,6) = (2,3,3,6)$

$FES_4 = \max\{FES_2 \oplus FET_{24}, FES_3 \oplus FET_{34}\}$

$= \max\{(4,4,7,9), (3,4,5,8)\}$

by equation (1) and taking β = 0.602, the ranking value of (4,4,7,9) and (3,4,5,8) can be obtained: $x_1 = 3$, $x_2 = 9$,

$R((4,4,7,9)) = 0.602[(9-3)/(9-3-7+9)] + 0.398[1 - (9-4)/(9-3+4-4)] = 0.51783,$

$R((3,4,5,8)) = 0.602[(8-3)/(9-3-5+8)] + 0.398[1 - (9-3)/(9-3+4-3)] = 0.39130,$

Since
$R((4,4,7,9)) > R((3,4,5,8))$, $FES_4 = (4,4,7,9)$.

$FES_5 = \max\{FES_1 \oplus FET_{15}, FES_2 \oplus FET_{25}, FES_4 \oplus FET_{45}\}$
$= \max\{(2,3,4,5), (4,6,8,12),(6,7,10,14)\}$

by equation (1) and taking $\beta = 0.602$, $FES_5 = (6,7,10,14)$.
$FES_6 = \max\{FES_3 \oplus FET_{36}, FES_4 \oplus FET_{46}, FES_5 \oplus FET_{56}\}$
$= \max\{(9,11,14,21), (7,7,11,15),(7,8,11,16)\}$

by equation (1) and taking $\beta = 0.602$, $FES_6 = (9,11,14,21)$.

Step 2. Set $FLF_6 = (9,11,14,21)$ and calculate FLF_j , j = 5,4,3,2,1 by using property 2.
$FLF_5 = FLF_6 \ominus FET_{56} = (9,11,14,21) \ominus (1,1,1,2) = (7,10,13,20)$,
$FLF_4 = \min\{FLF_5 \ominus FET_{45}, FLF_6 \ominus FET_{46}\} = \min\{(2,7,10,18), (3,7,11,18)\}$,

by equation (1) and taking $\beta = 0.602$, $FLF_4 = (2,7,10,18)$.
$FLF_3 = \min\{FLF_6 \ominus FET_{36}, FLF_4 \ominus FET_{34}\} = \min\{(-6,0,6,14), (0,5,9,17)\}$,

by equation (5) and taking $\beta = 0.602$, $FLF_3= (-6,0,6,14)$.
$FLF_2 = \min\{FLF_4 \ominus FET_{24}, FLF_5 \ominus FET_{25}\} = \min\{(-3,3,8,16), (-1,5,9,18)\}$,

by equation (1) and taking $\beta = 0.602$, $FLF_2= (-3,3,8,16)$.
$FLF_1 = \min\{FLF_2 \ominus FET_{12}, FLF_3 \ominus FET_{13} ,FLF_5 \ominus FET_{15} \}$
$= \min\{(-7,0,6,14), (-12,- 3,3,12),(2,6,10,18)\}$,

by equation (1) and taking $\beta = 0.602$, $FLF_1 = (-12,-3,3,12)$.

Step 3. Calculate FTS_{ij} with respect to each activity by property 3.
$FTS_{12} = FLF_2 \ominus (FES_1 \oplus FET_{12})$
$=(-3,3,8,16) \ominus ((0, 0, 0, 0) \oplus (2,2,3,4)) =(-7,0,6,14)$
$FTS_{13} = FLF_3 \ominus (FES_1 \oplus FET_{13})$
$=(-6,0,6,14) \ominus ((0, 0, 0, 0) \oplus (2,3,3,6)) =(-12,-3,3,12)$
$FTS_{15} = FLF_5 \ominus (FES_1 \oplus FET_{15})$
$=(7,10,13,20) \ominus ((0, 0, 0, 0) \oplus (2,3,4,5)) =(2,6,10,18)$
$FTS_{24} = FLF_4 \ominus (FES_2 \oplus FET_{24})$
$=(2,7,10,18) \ominus ((2,2,3,4) \oplus (2,2,4,5)) =(-7,0,6,14)$
$FTS_{25} = FLF_5 \ominus (FES_2 \oplus FET_{25})$
$=(7,10,13,20) \ominus ((2,2,3,4) \oplus (2,4,5,8)) =(-5,2,7,16)$
$FTS_{34} = FLF_4 \ominus (FES_3 \oplus FET_{34})$
$=(2,7,10,18) \ominus ((2,3,3,6) \oplus (1,1,2,2)) =(-6,2,6,15)$
$FTS_{36} = FLF_6 \ominus (FES_3 \oplus FET_{36})$
$=(9,11,14,21) \ominus ((2,3,3,6) \oplus (7,8,11,15)) =(-12,-3,3,12)$
$FTS_{45} = FLF_5 \ominus (FES_4 \oplus FET_{45})$

=(7,10,13,20) ϴ ((4,4,7,9) ⊕ (2,3,3,5)) =(-7,0,6,14)
FTS_{46} = FLF_6 ϴ (FES_4 ⊕ FET_{46})
=(9,11,14,21) ϴ ((4,4,7,9) ⊕ (3,3,4,6)) =(-6,0,7,14)
FTS_{56} = FLF_6 ϴ (FES_5 ⊕ FET_{56})
=(9,11,14,21) ϴ ((7,10,13,20) ⊕ (1,1,1,2)) =(-7,0,6,14)

Step 4. Find all the possible paths and calculate FCPM(P_k) by using property 4.
P = {(1, 3, 6), (1, 3, 4, 6), (1, 2, 4,6),(1,2,5,6),(1,5,6),(1,2,4,5,6)}

Let path P_1 = (1, 3,6), then FCPM(P_1) = FTS_{13} ⊕ FTS_{36}
= (−12,-3,3,12) ⊕ (−12,-3,3,12) = (−24,-6,6,24)

Let path P_2 = (1, 3,4,6), then FCPM(P_2) = FTS_{13} ⊕ FTS_{34} ⊕ FTS_{46}
= (−12,-3,3,12) ⊕ (−6,2,6,15) ⊕ (−6,0,7,14)= (−24,-1,16,41)

Let path P_3 = (1, 2,4,6), then FCPM(P_3) = FTS_{12} ⊕ FTS_{24} ⊕ FTS_{46}
= (−7,0,6,14) ⊕ (−7,0,6,14) ⊕ (−6,0,7,14)= (−20,0,19,42)

Let path P_4= (1, 2,5,6), then FCPM(P_4) = FTS_{12} ⊕ FTS_{25} ⊕ FTS_{56}
= (−7,0,6,14) ⊕ (−5,2,7,16) ⊕ (−7,0,6,14)= (−19,2,19,44)

Let path P_5 = (1,5,6), then FCPM(P_5) = FTS_{15} ⊕ FTS_{56}
= (2,6,10,18) ⊕ (−7,0,6,14) = (−5,6,16,32)

Let path P_6 = (1, 2,4,5,6), then FCPM(P_6) = FTS_{12} ⊕ FTS_{24} ⊕ FTS_{45} ⊕ FTS_{56}
= (−7,0,6,14) ⊕ (−7,0,6,14) ⊕ (−7,0,6,14) ⊕ (−7,0,6,14)= (−28,0,24,56)

Step 5. Find the fuzzy critical path by equation (5) and taking $\beta = 0.602$, the ranking value of FCPM(P_i), i = 1, 2, 3,4,5,6 can be obtained:
R(FCPM(P_1)) = 0.4305
R(FCPM(P_2)) = 0.5133
R(FCPM(P_3)) = 0.5326
R(FCPM(P_4)) = 0.5158
R(FCPM(P_5)) = 0.5036
R(FCPM(P_6)) = 0.5456

Since R(FCPM(P_1)) < R(FCPM(P_5)) < R(FCPM(P_2)) < R(FCPM(P_4)), < R(FCPM(P_6)) the fuzzy critical path is P_1

Conclusion

In project evaluation analysis, it is very often that information available for making decision is vague and uncertain. Therefore, it is rather difficult to obtain exact activity assessment data. In addition, goodness-of-fit test that the activity times follow a beta distribution is not only hard to carry out but also the conclusion may show that the assumption of beta distribution is not held. Hence the conventional precisionbased/ random-oriented project analysis tends to be less effective in conveying available information in such an imprecise and fuzzy decision environment. This paper proposes an algorithm to tackle the problem in fuzzy project decision analysis. The method can take account the rating attitude (optimistic/pessimistic) of the decision makers. Furthermore, the proposed algorithm can be computerized. Thus by conducting fuzzy or non-fuzzy activity time assessments, the decision-makers can obtain the fuzzy critical path automatically.

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