Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 7, Number 1 (2012), pp. 1-9

http://www.ripublication.com/afm.htm

t-Intuitionistic Fuzzy Quotient Group

P.K. Sharma

Post Graduate Department of Mathematics,
D.A.V. College, Jalandhar City, Punjab, India
E-mail: pksharma@davjalandhar.com

Abstract

In this paper, the notion of t-intuitionistic fuzzy cosets of an intuitionistic fuzzy normal subgroup and t-intuitionistic fuzzy quotient group are defined and discussed. A homomorphism from a given group onto the set of all t-intuitionistic fuzzy quotient group is established. Some related results has been derived.

Keywords: Intuitionistic fuzzy (IFS), Intuitionistic fuzzy subgroup (IFSG), (α, β)–Cut set, t-intuitionistic fuzzy coset, t-intuitionistic fuzzy quotient group.

Mathematics Subject Classification: 03F55.

Introduction
The notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] as a generalization of Zadeh's fuzzy sets. IFS makes description of the objective world become more realistic, practical and accurate, making it very promising. Instead of using fuzzy approach, past researchers have studied IFSs to be applied in variety of area such as decision making[10], medical diagnostics[9] and pattern recognition[4] and seem to be more popular than fuzzy sets in recent years. Author has already introduced the notion of t-intuitionistic fuzzy cosets of an intuitionistic fuzzy module and t-intuitionistic fuzzy quotient module in [8] In this paper, we introduce the notion of t-intuitionistic fuzzy cosets of an intuitionistic fuzzy normal subgroup and t-intuitionistic fuzzy quotient group and discuss some of their properties.

Preliminaries
We first recall some definition for the sake of completeness of the topic under study.

Definition (2.1)[1] Let X be a fixed non-empty set. An intuitionistic fuzzy set (IFS) A of X is an object of the following form A = {< x, μ_A (x), ν_A (x) >: x $\in$X}, where μ_A: X $\rightarrow$ [0, 1] and ν_A: X $\rightarrow$ [0, 1] define the degree of membership and degree of non-membership of the element x $\in$X respectively and for any x $\in$X, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Remark (2.2) (i): When $\mu_A(x) + \nu_A(x) = 1$, i.e. when $\nu_A(x) = 1-\mu_A(x) = \mu_A^c(x)$. Then A is called fuzzy set. **(ii)** We use the notation A = (μ_A, ν_A) to denote the IFS A of X.

Definition (2.3)[5]: Let G be a group. An intuitionistic fuzzy subset (IFS) A = (μ_A, ν_A) of G is called intuitionistic fuzzy subgroup (IFSG) of G if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\mu_A(x^{-1}) = \mu_A(x)$
(iii) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$
(iv) $\nu_A(x^{-1}) = \nu_A(x)$, for all x, y $\in$G

Or Equivalently A is IFSG of G if and only if
$\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$

Definition (2.4)[5] An IFSG A = (μ_A, ν_A) of a group G is said to be intuitionistic fuzzy normal subgroup of G (In short IFNSG) of G if

(i) $\mu_A(xy) = \mu_A(yx)$
(ii) $\nu_A(xy) = \nu_A(yx)$, for all x, y $\in$G

Or Equivalently A is an IFNSG of a group G is normal if and only if
$\mu_A(y^{-1} x y) = \mu_A(x)$ and $\nu_A(y^{-1} x y) = \nu_A(x)$, for all x, y $\in$G

Definition (2.5)[5]: Let A be intuitionistic fuzzy set of a universe set X. Then (α, β)-cut of A is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS A is given by

$C_{\alpha,\beta}(A) = \{ x: x \in X \text{ such that } \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta \}$,

where $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

Theorem (2.6) [5,7]: If A is IFS of a group G. Then A is IFSG (IFNSG) of G if and only if $C_{\alpha,\beta}(A)$ is a subgroup (normal) of group G, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

t-Intuitionistic fuzzy quotient group

Definition (3.1) Let A be an IFSNG of a group G. Let t $\in$ [0,1]. For any x$\in$G

Define an IFS A_x^t of G called t – Intuitionistic fuzzy coset of A in G as follows

$A^t{}_x(g) = (\mu_{A^t{}_x}(g)\ ,\nu_{A^t{}_x}(g)\ )$, where
$\mu_{A^t{}_x}(g) = \min\{\mu_A(gx^{-1}),\ t\}$ and $\nu_{A^t{}_x}(g) = \max\{\nu_A(gx^{-1}),\ 1\text{-}\ t\}$ for all x, g $\in$ G.

Proposition (3.2) Let S be the set of all t – intuitionistic fuzzy cosets of an IFNSG A in G. i.e. S = { $A^t{}_x$: x$\in$G }. Then the binary operations $\otimes$ defined on the set S as follows:

$$A^t{}_x \otimes A^t{}_y = A^t{}_{xy} \quad , \text{ for all } x , y \in G$$

is a well defined operation.

Proof. Let $A^t{}_x = A^t{}_{x'}$ and $A^t{}_y = A^t{}_{y'}$, for some x , y , x$'$, y$'$ $\in$ G

Let g$\in$G be any element, then

$[A^t{}_x \otimes A^t{}_y](g) = (A^t{}_{xy})(g) = (\mu_{A^t{}_{xy}}(g)\ ,\nu_{A^t{}_{xy}}(g))$

Now $\mu_{A^t{}_{xy}}(g) = \min\{\mu_A g(xy)^{-1},\ t\} = \min\{\mu_A(gy^{-1})x^{-1},\ t\} = \mu_{A^t{}_x}(gy^{-1}) = \mu_{A^t{}_{x'}}(gy^{-1})$

$= \min\{\mu_A(gy^{-1})x'^{-1},\ t\} = \min\{\mu_A(x'^{-1}g)y^{-1},\ t\} = \mu_{A^t{}_y}(x'^{-1}g) = \mu_{A^t{}_{y'}}(x'^{-1}g)$

$= \min\{\mu_A(x'^{-1}g)y'^{-1},\ t\} = \min\{\mu_A y'^{-1}(x'^{-1}g),\ t\} = \min\{\mu_A(y'^{-1}x'^{-1})g,\ t\}$

$= \min\{\mu_A(x'y')^{-1}g,\ t\} = \min\{\mu_A g(x'y')^{-1},\ t\}$

$= \mu_{A^t{}_{x'y'}}(g)$

Similarly, we can show that $\nu_{A^t{}_{xy}}(g) = \nu_{A^t{}_{x'y'}}(g)$, $\forall$ g $\in$ G.

Therefore $\otimes$ is well defined operation on S.

Lemma (3.3): If A is IFNSG of a group G. Then

$A^t{}_x = A^t{}_{x'} \iff$ Nx = Nx$'$, for x , x$'$ $\in$ G , wherer N = $C_{t,1-t}(A)$

Proof. Let $A^t{}_x = A^t{}_{x'}$ for x , x$'$ $\in$ G , then $A^t{}_x(x') = A^t{}_{x'}(x')$

$\left(\min\{\mu_A(x'x^{-1}),\ t\},\ \max\{\nu_A(x'x^{-1})\ ,1\text{ - }t\}\right) = \left(\min\{\mu_A(x'x'^{-1}),\ t\}, \max\{\nu_A(x'x'^{-1})\ ,1\text{ - }t\}\right)$

$= \left(\min\{\mu_A(e),\ t\}, \max\{\nu_A(e)\ ,1\text{ - }t\}\right)$

$= (t\ ,1\text{- }t)$

$\Rightarrow \min\{\mu_A(x'x^{-1}),\ t\} = t$ and $\max\{\nu_A(x'x^{-1})\ ,1\text{ - }t\} = 1 - t$

$\Rightarrow \mu_A(x'x^{-1}) \geq t$ and $\nu_A(x'x^{-1}) \leq 1\text{ - t}$ and so $x'x^{-1} \in C_{t,1-t}(A) = N$

$\Rightarrow$ N x = N x$'$ ....(*)

Now, we show that if N x = N x$'$, then $A^t{}_x = A^t{}_{x'}$ Let for some y$\in$G, $A^t{}_x$ (y) $\neq$ $A^t{}_{x'}$(y)

i.e. $\left(\min\{\mu_A(xy^{-1}),\ t\}, \max\{\nu_A(xy^{-1})\ ,1\text{ - }t\}\right) \neq \left(\min\{\mu_A(x'y^{-1}),\ t\}, \max\{\nu_A(x'y^{-1})\ ,1\text{ - }t\}\right)$

Suppose $\mu_A(x y^{-1}) < t$ and $\mu_A(x' y^{-1}) \geq t$
Therefore $\nu_A(x y^{-1}) \leq 1\text{-}t$ and $\nu_A(x' y^{-1}) \leq 1\text{-}t$
$\Rightarrow x' y^{-1} \in C_{t, 1\text{-}t}(A) = N$
$\Rightarrow N x' y^{-1} = N \Rightarrow Nx y^{-1} = N$ (Using (*))
$\Rightarrow x y^{-1} \in N$ and so $\mu_A(x y^{-1}) \geq t$, a contradiction

Similarly, if $\mu_A(x y^{-1}) \geq t$ and $\mu_A(x' y^{-1}) < t$ also leads to contradiction
Therefore either $\mu_A(x y^{-1}) \geq t$ and $\mu_A(x' y^{-1}) \geq t$ i.e. $\nu_A(x y^{-1}) \leq 1\text{-}t$ and $\nu_A(x' y^{-1}) \leq 1\text{-}t$ or $\mu_A(x y^{-1}) < t$ and $\mu_A(x' y^{-1}) < t$ i.e. $\nu_A(x y^{-1}) \leq 1\text{-}t$ and $\nu_A(x' y^{-1}) \leq 1\text{-}t$

In the first case

Min $\{ \mu_A(x y^{-1}), t \} = t$ and max $\{ \nu_A(x y^{-1}), 1\text{-}t \} = 1\text{-}t$
And so $A^t{}_x(y) = (t,\ 1-t)$ and also
Min $\{ \mu_A(x' y^{-1}), t \} = t$ and max $\{ \nu_A(x' y^{-1}), 1\text{-}t \} = 1\text{-}t$
And so $A^t{}_{x'}(y) = (t,\ 1-t)$. Thus $A^t{}_x(y) = A^t{}_{x'}(y)$, for all $y \in G$
Therefore $A^t{}_x = A^t{}_{x'}$

In the second case

Min $\{ \mu_A(x y^{-1}), t \} = \mu_A(x y^{-1}) < t$ and max $\{ \nu_A(x y^{-1}), 1\text{-}t \} = 1\text{-}t$
And also
Min $\{ \mu_A(x'y^{-1}), t \} = \mu_A(x' y) < t$ and max $\{ \nu_A(x' y^{-1}), 1\text{-}t \} = 1\text{-}t$
Now since $N x = N x'$, therefore let $x = n x'$, where $n \in N$
So that $\mu_A(n) \geq t$ and $\nu_A(n) \leq 1 - t$

$$\begin{aligned} A^t{}_{x'}(y) &= \left(\min\{\mu_A(yx'^{-1}),\ t\}, \max\{\nu_A(yx'^{-1}),1\text{ - }t\}\right) \\ &= \left(\mu_A(yx^{-1}n),\ 1\text{- }t\right) \\ &= \left(\mu_A(nyx^{-1}),\ 1\text{- }t\right) \\ &\geq \left(\mu_A(n) \wedge \mu_A(yx^{-1}),\ 1\text{- }t\right) \\ &= \left(\mu_A(yx^{-1}),\ 1\text{- }t\right) \\ &= \left(\min\{\mu_A(yx^{-1}),\ t\}, \max\{\nu_A(yx^{-1}),1\text{ - }t\}\right) \\ &= A^t{}_x(y) \end{aligned}$$

Thus $A^t{}_{x'}(y) \geq A^t{}_x(y)$, for all $y \in G$

Similarly $A^t{}_x(y) = \left(\min\{\mu_A(yx^{-1}),\ t\}, \max\{\nu_A(yx^{-1}), 1 - t\}\right)$

$$= \left(\mu_A(yx^{-1}),\ 1 - t\right)$$
$$= \left(\mu_A(yx'^{-1}n^{-1}),\ 1 - t\right)$$
$$\geq \left(\mu_A(n) \wedge \mu_A(yx'^{-1}),\ 1 - t\right)$$
$$= \left(\mu_A(yx'^{-1}),\ 1 - t\right)$$
$$= \left(\min\{\mu_A(yx'^{-1}),\ t\}, \max\{\nu_A(yx'^{-1}), 1 - t\}\right)$$
$$= A^t{}_{x'}(y)$$

Thus $A^t{}_x(y) \geq A^t{}_{x'}(y)$, for all $y \in G$

Therefore $A^t{}_x = A^t{}_{x'}$

Hence the result proved.

Proposition (3.4) The set S of all t-Intuitionistic fuzzy cosets of an IFNSG A of a group G, form a group under the well-defined operations ⊗.

Proof. It is easy to check that the identity element of S is $A^t{}_e$, where e is the identity element of group G, and the inverse of an element $A^t{}_x$ is $A^t{}_{x^{-1}}$.

Proposition (3.5) The IFS B of S defined by $B(A^t{}_a) = \left(\mu_B(A^t{}_a),\ \nu_B(A^t{}_a)\right)$ *Where* $\mu_B(A^t{}_a) = \underset{A^t{}_x = A^t{}_a}{Sup}\{\mu_A(x) : x \in G\}$ and $\nu_B(A^t{}_a) = \underset{A^t{}_x = A^t{}_a}{Inf}\{\nu_A(x) : x \in G\}$ is a IFSG of S, called t-Intuitionistic fuzzy quotient group.

Proof. Let a, b $\in$G and let $B(A^t{}_a) = (\theta_1,\ \theta_2)$ and $B(A^t{}_b) = (\phi_1,\ \phi_2)$, *where*

$$\theta_1 = \underset{A^t{}_x = A^t{}_a}{Sup}\{\mu_A(x) : x \in G\},\ \theta_2 = \underset{A^t{}_x = A^t{}_a}{Inf}\{\nu_A(x) : x \in G\} \text{ and}$$
$$\phi_1 = \underset{A^t{}_x = A^t{}_b}{Sup}\{\mu_A(x) : x \in G\},\ \phi_2 = \underset{A^t{}_x = A^t{}_b}{Inf}\{\nu_A(x) : x \in G\}$$

$\exists$ x, y $\in$ G such that $\theta_1 - \in < \mu_A(x)$, Nx = N*a* and $\phi_1 - \in < \mu_A(y)$, Ny = Nb

Therefore Nxy = N*a*b $\Rightarrow A^t{}_{xy} = A^t{}_{ab} \Rightarrow A^t{}_x \otimes A^t{}_y = A^t{}_a \otimes A^t{}_b$

So that $\mu_A(xy) \leq \mu_B(A^t{}_x \otimes A^t{}_y) = \mu_B(A^t{}_a \otimes A^t{}_b)$

Now $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) = \mu_A(x)[say] > \theta_1 - \in$

$\therefore\ \theta_1 - \in\ <\ \mu_A(xy) \leq \mu_B(A^t{}_a \otimes A^t{}_b)$, $\forall \in > 0$

so that $\theta_1 \leq \mu_B(A^t{}_a \otimes A^t{}_b)$

Now two cases arises

Case(*i*) When $\theta_1 \geq \phi_1$, then $\theta_1 - \in \ \geq \ \phi_1$ - $\in$

ϕ_1 - $\in \ \leq \ \theta_1 - \in \ \leq \mu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b)$, $\forall \ \in \ > 0$

so that $\phi_1 \ \leq \ \mu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b)$

Therefore $\mu_B(\mathrm{A}^t{}_a) \wedge \mu_B(\mathrm{A}^t{}_b) = \ \theta_1 \wedge \phi_1 \ = \phi_1 \ \leq \ \mu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b)$

Case(*ii*) When $\theta_1 < \phi_1$, then

$\mu_B(\mathrm{A}^t{}_a) \wedge \mu_B(\mathrm{A}^t{}_b) = \ \theta_1 \wedge \phi_1 \ = \theta_1 \ \leq \ \mu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b)$

*Thu*s in *any case we get* $\mu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b) \geq \ \mu_B(\mathrm{A}^t{}_a) \wedge \mu_B(\mathrm{A}^t{}_b)$

Similarly, we can show that $\nu_B(\mathrm{A}^t{}_a \otimes \mathrm{A}^t{}_b) \leq \ \nu_B(\mathrm{A}^t{}_a) \vee \nu_B(\mathrm{A}^t{}_b)$

Next to show that $\mathrm{B}(\mathrm{A}^t{}_a) = \mathrm{B}((\mathrm{A}^t{}_a)^{-1}) = \mathrm{B}(\mathrm{A}^t{}_{a^{-1}})$

Since $\mathrm{A}^t{}_x = \mathrm{A}^t{}_a \Rightarrow$ Nx = N*a* and $\mathrm{A}^t{}_y = \mathrm{A}^t{}_{a^{-1}} \Rightarrow$ Ny = $\mathrm{N}a^{-1} = (\mathrm{N}a)^{-1} = (\mathrm{Nx})^{-1} = \mathrm{Nx}^{-1}$

yx $\in$ N $\Rightarrow$ Nyx = N $\Rightarrow$ $\mathrm{A}^t{}_{yx} = \mathrm{A}^t{}_e \Rightarrow \mathrm{A}^t{}_y \otimes \mathrm{A}^t{}_x = \mathrm{A}^t{}_e$ so $\mathrm{A}^t{}_y = \mathrm{A}^t{}_{x^{-1}}$

and we know that $\mu_A(x^{-1}) = \mu_A(x)$. Thus $\mu_B(\mathrm{A}^t{}_a) = \ \mu_B(\mathrm{A}^t{}_{a^{-1}})$.

Similarly, we van show that $\nu_B(\mathrm{A}^t{}_a) = \ \nu_B(\mathrm{A}^t{}_{a^{-1}})$. Whence $\mathrm{B}(\mathrm{A}^t{}_a) = \mathrm{B}(\mathrm{A}^t{}_{a^{-1}})$.

Proposition (3.6) A mapping *f*: G → S, where G is a group and S is the set of all t-intuitionistic fuzzy cosets of the IFNSG A of G defined by *f* (x) = $A^t{}_x$, is an onto homomorphism with ker *f* = N (= $C_{t,1-t}$ (A)), where t $\in$[0,1])

Proof. Clearly *f* is an onto homomorphism

Let x $\in$ ker *f*, then *f* (x) = identity element of S = $A^t{}_e$

Therefore $A^t{}_x = A^t{}_e$ so N x = N e = N $\Rightarrow$ x $\in$ N

$\Rightarrow$ ker $f \subseteq$ N (1)

Conversely, let x $\in$ N $\Rightarrow$ Nx = N so that

N $\mathrm{xg}^{-1} = \mathrm{Ng}^{-1}$ $\forall$ g $\in$ G. If possible let x $\notin$ ker*f* i.e. $A^t{}_x \neq A^t{}_e$ therefore there exists g $\in$ G such that $A^t{}_x(g) \neq A^t{}_e(g)$

Suppose $\mu_A(\mathrm{xg}^{-1}) < \mathrm{t}$ and $\mu_A(\mathrm{g}^{-1}) \geq \mathrm{t}$, i.e. $\nu_A(\mathrm{xg}^{-1}) \leq 1\text{-}\mathrm{t}$ and $\nu_A(\mathrm{g}^{-1}) \leq 1\text{-}\mathrm{t}$

Therefore $\mu_A(\mathrm{g}^{-1}) \geq \mathrm{t}$ and $\nu_A(\mathrm{g}^{-1}) \leq 1\text{-}\mathrm{t}$ $\Rightarrow$ $\mathrm{g}^{-1} \in \mathrm{N}$ so $\mathrm{Ng}^{-1} = \mathrm{N}$

i.e. $\mathrm{Nxg}^{-1} = \mathrm{N}$ $\Rightarrow$ $\mathrm{xg}^{-1} \in \mathrm{N}$ and so $\mu_A(\mathrm{xg}^{-1}) \geq t$, a contradiction

Similarly, $\mu_A(\mathrm{xg}^{-1}) \geq t$ and $\mu_A(\mathrm{g}^{-1}) < t$ is not possible .

$\therefore$ either $\mu_A(\mathrm{xg}^{-1}) \geq t$, $\mu_A(\mathrm{g}^{-1}) \geq \mathrm{t}$ i.e. $\nu_A(\mathrm{xg}^{-1}) \leq 1\text{-}\mathrm{t}$ and $\nu_A(\mathrm{g}^{-1}) \leq 1\text{-}\mathrm{t}$

or $\mu_A(\mathrm{xg}^{-1}) < \mathrm{t}$, $\mu_A(\mathrm{g}^{-1}) < t$ i.e. $\nu_A(\mathrm{xg}^{-1}) \leq 1\text{-}\mathrm{t}$ and $\nu_A(\mathrm{g}^{-1}) \leq 1\text{-}\mathrm{t}$

In the first case

Min { μ_A (xg^{-1}), t } = t and Max { ν_A (xg^{-1}), 1-t } = 1-t and so $A^t{}_x(g) = (t, 1-t)$ similarly we get $A^t{}_e(g) = (t, 1-t)$. Thus $A^t{}_x(g) = A^t{}_e(g)$

In the second case

Min { μ_A (x g^{-1}), t } = μ_A (xg^{-1}) < t and Max { ν_A (xg^{-1}), 1-t } = 1-t

$A^t{}_x(g) = (\min\{\mu_A(\mathrm{x\,g}^{-1}), t\}, \max\{\nu_A(\mathrm{x\,g}^{-1}), 1-\mathrm{t}\})$

$= (\mu_A(\mathrm{x\,g}^{-1}), 1\text{-t})$

$\geq (\mu_A(\mathrm{x}) \wedge \mu_A(\mathrm{g}), 1\text{-t})$

$= (\mu_A(\mathrm{g}), 1\text{-t})$ $[\because \mathrm{x} \in \mathrm{N} \therefore \mu_A(\mathrm{x}) \geq \mathrm{t}$ and $\mu_A(\mathrm{g}) = \mu_A(\mathrm{g}^{-1}) < t]$

$= (\min\{\mu_A(\mathrm{eg}^{-1}), t\}, \max\{\nu_A(\mathrm{eg}^{-1}), 1-\mathrm{t}\})$

$= A^t{}_e(g)$

$= (\mu_A(\mathrm{g}^{-1}), 1\text{-t})$

$= (\mu_A(\mathrm{xg}^{-1}\mathrm{x}^{-1}), 1\text{-t})$ [As A is IFNSG of G so $\mu_A(\mathrm{xg}^{-1}\mathrm{x}^{-1}) = \mu_A(\mathrm{g}^{-1})$]

$\geq (\mu_A(\mathrm{x\,g}^{-1}) \wedge \mu_A(\mathrm{x}), 1\text{-t})$

$= (\mu_A(\mathrm{x\,g}^{-1}), 1-t)$

$= (\min\{\mu_A(\mathrm{x\,g}^{-1}), t\}, \max\{\nu_A(\mathrm{x\,g}^{-1}), 1-\mathrm{t}\})$

$= A^t{}_x(g)$

i.e. $f(x) = \textit{Identity element of}$ S and so x $\in$ ker f

$\therefore$ N $\subseteq$ ker f so ker f = N.

Proposition (3.7) If f: G → S, is an onto homomorphism, then f (A) = B, where A is IFS of G and B is IFS of S.

Proof. *Let* $A^t{}_a \in$ S be any element of S , where $a \in G$ such that $f(a) = A^t{}_a$

Let A be IFS of G , then

$$f(A)(A^t{}_a) = \begin{cases} \left(Sup\{\mu_A(x) : \mathrm{x} \in f^{-1}(A^t{}_a)\},\ \mathrm{Inf}\{\nu_A(x) : \mathrm{x} \in f^{-1}(A^t{}_a)\} \right) \\ (\mu_A(e)\ ,\ \nu_A(e)\ ) \end{cases}$$

$$= \begin{cases} \left(Sup\{\mu_A(x) : A^t{}_x = A^t{}_a\},\ \mathrm{Inf}\{\nu_A(x) : A^t{}_x = A^t{}_a\} \right) \\ (\mu_A(e)\ ,\ \nu_A(e)\ ) \end{cases}$$

$= \mathrm{B}(A^t{}_a)$

Hence f (A) = B

Theorem (3.8): Let A be a IFNSG of G and B be a IFSG of S, then

$$\mathrm{C}_{t,1-t}(B) = \{A^t{}_e\}$$

Proof.

Now $B(A^t{}_e) = \left(\mu_B(A^t{}_e)\ ,\ \nu_B(A^t{}_e)\right)$, where

$\mu_B(A^t{}_e) = Sup\{\mu_A(x) :\ Nx = N\}$

$= Sup\{\mu_A(x) : x \in N\}$

$\geq \mu_A(n)$, for all $n \in N = C_{t,1-t}(A)$

$\geq t$

Similarly , we can show that $\nu_B(A^t{}_e) \leq 1-t$. Thus $A^t{}_e \in C_{t,1-t}(B)$

Let $A^t{}_a \in C_{t,1-t}(B) \Rightarrow \mu_B(A^t{}_a) \geq t$ and $\nu_B(A^t{}_a) \leq 1-t$

Let $\theta_1 = \mu_B(A^t{}_a) = Sup\{\mu_A(x) :\ Nx = Na\}$ and

$\theta_2 = \nu_B(A^t{}_a) = Inf\{\nu_A(x) :\ Nx = Na\}$

Therefore $\theta_1 \geq t$ and $\theta_2 \leq 1-t$. Let $\varepsilon > 0$ be given, ∃'s $x, y \in G$ such that
$Nx = Na$ so that $x a^{-1} = n_1 \in N$ and $\mu_A(x) > \theta_1 - \varepsilon \geq t-\varepsilon$ and
$Ny = Na$ so that $y a^{-1} = n_2 \in N$ and $\nu_A(y) < \theta_2 + \varepsilon \leq (1-t) + \varepsilon$

$$\mu_A(a) = \mu_A(an_1n^{-1}{}_1) \geq \mu_A(an_1) \wedge \mu_A(n_1) = \begin{cases} \geq t & if\ \mu_A(an_1) \geq t \\ = \mu_A(an_1) & if\ \mu_A(an_1) < t \end{cases}$$ and

$$\nu_A(a) = \nu_A(an_2n^{-1}{}_2) \leq \nu_A(an_2) \vee \nu_A(n_2) = \begin{cases} \leq 1-t & if\ \nu_A(an_2) \leq 1-t \\ = \nu_A(an_2) & if\ \nu_A(an_2) > 1-t \end{cases}$$

Thus in any case $\mu_A(a) > t-\varepsilon$ and $\nu_A(a) < (1-t) + \varepsilon$, for all $\varepsilon > 0$
$\Rightarrow \mu_A(a) \geq t$ and $\nu_A(a) \leq (1-t)$ implies that $a \in C_{t,1-t}(A)$
$\Rightarrow Na = N$ so $A^t{}_a = A^t{}_e$ Hence $C_{t,1-t}(B) = \{A^t{}_e\}$.

References

[1] K.T. Atanassov, "Intutionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, no. 1, 1986, pp. 87–96

[2] K. T. Atanassov, "New operations defined over the intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 61, no. 2, 1994, pp. 137–142

[3] L. Atanassov, "On Intuitionistic fuzzy versions of L. Zadeh's extension principle ", Notes on intuitionistic Fuzzy Sets, (13) (3), 2006, 33-36

[4] Dengfeng Li, Cheng Chutian, " New similarity measure of IFS and application to pattern recognition", J. pattern recognition letter, Vol. 23, 2002, 221-225

[5] Sharma, P.K., " (α, β)-Cut of Intuitionistic fuzzy groups" International Mathematics Forum, Vol. 6, 2011, no. 53, 2605-2614

[6] Sharma, P.K., "Homomorphism of Intuitionistic fuzzy groups " International Mathematics Forum, Vol. 6, 2011, no. 64, 3169-3178

[7] Sharma, P.K., " On the direct product of Intuitionistic fuzzy groups", International Mathematical Forum, Vol. 7, 2012, no. 11, 523-530

[8] Sharma, P.K., " t-Intuitionistic fuzzy Quotient modules" International Journal of Fuzzy Mathematics & System (IJFMS), Vol., 2, No. 1, 2012, 37-42

[9] Supriya K.De, R.Biswas A. R. Roy, " An application of Intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and System, 117 (2001), 209-213

[10] Szmidt E and Kacprzyk J. " Intuitionistic fuzzy set in group decision making, Notes IFS 2 (1), 1996, 11-14

[11] Zadeh,L.A., " Fuzzy Sets", Information and Control, 8, (1965), 338-353