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Common Fixed Point Theorems for Hybrid Pairs of OWC Mappings Satisfying Generalized Contractive Condition of Integral Type in Symmetric Spaces.

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Abstract

In this paper, we study the existence and uniqueness of common fixed point theorems for hybrid pairs of owc mappings satisfying a generalized contractive condition of integral type in symmetric spaces. These results unifies, extends and complements many results existing in literature, also contain every theorem on hybrid and multivalued self mappings of metric space.

Keywords: Common fixed points, weakly compatible mappings, occasionally weakly compatible mappings (owc) and symmetric spaces.

AMS subject classifications: 47H10, 54H25

Introduction and preliminaries

Generalization of the Banach contraction mapping principle is one of pivotal results of analysis and has been a heavily investigated branch of research. It is widely considered as the source of metric fixed point theory and the significance lies in its vast applicability in a number of branches of mathematics.

In particular, establishment of fixed point theorems for a mapping satisfying contractive condition with no requirement of continuity at each point was firstly initiated by Kannan [2] in 1968. After that, there flows a flood of papers and several author studied fixed point theorems for a pair of mappings. The notion of weakly commuting mappings was introduced by Sessa [3] that weakened the concept of commutativity of two mappings. Jungck [4] generalized the concept of weakly commuting mappings by adding the notion of compatible mappings. This concept was further improved by Jungck and Rhoades [5] with notion of weakly compatible mappings. AL-Thagafi and Shahzad [6] gave a definition which is proper generalization of nontrivial weakly compatible maps which have coincidence points. Recently, Jungck and Rhoades [7] studied fixed point results for occasionally weakly compatible (owc). Abbas and Rhoades [10, 11] obtained common fixed point theorems for hybrid pairs of single valued and multivalued owc maps defined on symmetric space. For other relaxed fixed point theorems in symmetric space and their applications, one may refer to [12-17].

In 2002 Branciari [1] analyzed the existence of fixed point for mapping f defined on complete metric space (X, d) satisfying a contractive condition of integral type. (see the following theorem).

Theorem 1.1 Let (X, d) be a complete metric space, $\alpha \in (0, 1)$ and $f : X \to X$ be a mapping such that for each $x, y \in X$, $\int_0^{d(fx,fy)} \phi(t)\,dt \le \int_0^{d(x,y)} \phi(t)\,dt$, where $\phi : [0, +\infty) \to [0, +\infty)$ is Lebesgue-integrable mapping which is summable (i.e., with finite integral) on each compact subset of $[0, +\infty)$, nonnegative, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \phi(t) > 0$; then f has a unique fixed point $a \in X$ such that for each $x \in X$, $\lim_{n\to\infty} f^n x = a$.

The aim of this paper is to obtain fixed point theorems for maps involving hybrid pairs of single valued and multivalued owc maps satisfying a generalized contractive condition of integral type in the frame work of symmetric space.

Lemma 1.2 (Lebesgue Dominated Convergence Theorem) If a sequence $\{f_n\}$ of Lebesgue measurable functions converges almost everywhere to f and if there exist an integrable function ≥ 0 such that $|f_n(x)| \le g(x)$ for every n, then $| \quad |$

Definition 1.2 A symmetric on X is mapping $d : X \times X \to [0, \infty)$ such that

$d(x, y) = 0$ iff $x = y$,

$d(x, y) = d(y, x)$

A set X together with a symmetric d is called symmetric space.

We will use the following notations, throughout this paper, where (X, d) is a symmetric space, and $A \subseteq X$, $d(x, A) = \inf\{d(x, a): a \in A\}$, and B(X) is the class of all nonempty, bounded subset of X. The diameter of A, $B \in B(X)$ is denoted and defined by

$$\delta(A, B) = \sup\{d(a, b): a \in A, b \in B\}.$$

Clearly, $\delta(A, B) = \delta(B, A)$. For $\delta(\{a\}, B)$ and $\delta(\{a\}, \{b\})$, we write $\delta(a, B)$ and $d(a, b)$, respectively. We appeal to the fact that $\delta(A, B) = 0$ if and only if $A = B = \{x\}$ for $A, B \in B(X)$.

Recall that $x \in X$ is called coincident point (resp., common fixed point) of $f : X \to X$ and $T : X \to B(X)$ if $fx \in Tx$ (resp., $x = fx \in Tx$)

Definition 1.3 Maps $f : X \to X$ and $T : X \to B(X)$ are said to be compatible if $fTx \in B(X)$ for each $x \in X$ and $\delta(fTx_n, Tfx_n) \to 0$, whenever $\{x_n\}$ is sequence in X such that $Tx_n \to \{t\}$ ($\delta(Tx_n, t) \to 0$) and $fx_n \to t$ for some $t \in X$ [23].

Definition 1.4 Maps $f : X \to X$ and $T : X \to B(X)$ are said to be weakly compatible if $fTx = Tfx$ whenever $fx \in Tx$.

Definition 1.5 Maps $f : X \to X$ and $T : X \to B(X)$ are said to be occasionally weakly compatible (owc) if and only if there exist some point $x \in X$ such that $fx \in Tx$ and $fTx \subseteq Tfx$.

Assume that $F : [0, \infty) \to R$ satisfies the following.

i. $F(0) = 0$and $F(t) > 0$ for each $t \in (0, \infty)$.

ii. F is nondecreasing on $[0, \infty)$.

Define, $\Gamma[0, \infty) = \{F$: F satisfies (i)-(ii) above$\}$.

Let $\psi : [0, \infty) \to R$ satisfies the following

iii. $\psi(t) < 0$ for each $t \in (0, \infty)$.

iv. ψ is nondecreasing on $[0, \infty)$.

Define, $\Psi[0, \infty) = \{\psi : \psi$ satisfies (iii)-(iv) above$\}$.

For some example of mappings F which satisfy (i)-(ii), one may refer to [8].

Common Fixed Point Theorems

Now, we prove the following main theorems.

Theorem 2.1 Let f, g be self mapping of a symmetric space (X, d) and T, S be mappings from X into B(X) such that the pairs {f, T} and {g, S} are occasionally weakly compatible, satisfying

$$\int_0^{F(\delta(Tx,Sy))} \phi(s)ds \le \int_0^{\psi(F(M(x,y)))} \phi(s)ds \tag{2.1}$$

for each $x, y \in X$, for which $fx \neq gy$, where

$$M(x, y) = \max\{d(fx, gy), d(fx, Tx), d(gy, Sy), \delta(fx, Sy), \delta(gy, Tx)\} \tag{2.2}$$

Then f, g, T and S have a unique common fixed point.

Proof: Since the pairs {f, T} and {g, S} are occasionally weakly compatible, therefore by definition, there exist x, y in X such that $fx \in Tx$, $gy \in Sy$, $fTx \subseteq Tfx$ and $gSy \subseteq Sgy$. Also $d(f^2x, g^2y) \le \delta(Tfx, Sgy)$.

Using (2.2), we have

$$M(fx, gy) = \max\{d(f^2x, g^2y), d(f^2x, Tfx), d(g^2y, Sgy), \delta(f^2x, Sgy), \delta(g^2y, Tfx)\}$$
$$\le \delta(Tfx, Sgy) \tag{2.3}$$

We first show that $fx = gy$, for otherwise, by (2.1), we have

$$\int_0^{F(\delta(Tfx,Sgy))} \phi(s)ds \le \int_0^{\psi(F(M(fx,gy)))} \phi(s)ds$$

$$\le \int_0^{\psi(F(\delta(Tfx,Sgy)))} \phi(s)ds$$

$$< \int_0^{(F(\delta(Tfx,Sgy)))} \phi(s)ds \qquad (2.4)$$

It leads to a contradiction and hence, $gy = fx$. Obviously, $d(fx, g^2y) \le \delta(Tx, Sfx)$. Next, we claim that $x = fx$, if not then consider by (2.2)

$$M(x, fx) = \max\{d(fx, g^2y), d(fx, Tx), d(g^2y, Sgy), \delta(gy, Sgy), \delta(g^2y, Tx)\}$$

$$\le \delta(Tx, Sfx) \qquad (2.5)$$

Using (2.5), we have by (2.1),

$$\int_0^{F(\delta(Tx,Sfx))} \phi(s)ds \le \int_0^{\psi(F(M(x,fx)))} \phi(s)ds$$

$$\le \int_0^{\psi(F(\delta(Tx,Sfx)))} \phi(s)ds$$

$$< \int_0^{(F(\delta(Tx,Sfx)))} \phi(s)ds \qquad (2.6)$$

Which is again a contradiction and the claim follows. On the same account, we can prove $y = gy$.

Thus f, g, T and S have a common fixed point and uniqueness follows easily from (2.1).

Corollary 2.2 The theorem (2.1), will remain proved if the contractive condition (2.2) is replaced by any of following

i. $M(x, y) = h \max\{d(fx, gy), d(fx, Tx), d(gy, Sy), \frac{1}{2}[\delta(fx, Sy) + \delta(gy, Tx)]\}$, where $0 \le h < 1$.

ii. $M(x, y) = \alpha\, d(fx, gy) + \beta\{d(fx, Tx) + d(gy, Sy)\} + \gamma \max\{ d(fx, gy), \delta(fx, Sy), \delta(gy, Tx)\}$, where $\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = 0$.

iii. $M(x, y) = (\alpha + \beta + \gamma) \max\{d(fx, gy), d(fx, Tx), d(gy, Sy), \delta(fx, Sy), \delta(gy, Tx)\}$.

iv. $M(x, y) = \max\{d(fx, fy), \frac{1}{2}[d(fx, Tx) + d(fy, Ty)], \frac{1}{2}[\delta(fx, Ty) + \delta(fy, Tx)]\}$, where we have assumed $f = g$ and $T = S$. As all above cases are special cases of condition (2.2), result follows from theorem (2.1).

Theorem 2.3 Let f, g be self mapping of a symmetric space (X, d) and T, S be mappings from X into B(X) such that the pairs {f, T} and {g, S} are occasionally weakly compatible, satisfying

$$\int_0^{F((\delta(Tx,Sy))^p)} \phi(s)ds \le \int_0^{\psi(F(M_p(x,y)))} \phi(s)ds \qquad (2.7)$$

for each $x, y \in X$, for which $fx \neq gy$, where

$$M_p(x,y) = \alpha(\delta(Tx, gy))^p$$
$$+ (1 - \alpha) \max \{(d(fx, Tx))^p, (d(gy, Sy))^p, (d(fx, Tx))^{p/2} (d(gy, Tx))^{p/2},$$
$$(\delta(gy,Tx))^{p/2} (\delta(fx,Sy))^{p/2}\}, \tag{2.8}$$

where $0 < a \leq 1$, and $p \geq 1$, then f, g, T and S have a unique common fixed point.

Proof: The result follows immediately on the same pattern as in theorem (2.1).

Define $G = \{ g : R^5 \to R^5\}$ such that

(g_1) g is nondecreasing in the 4th and 5th variables,

(g_2) if $u \in R^+$ is such that

$u \leq g(u, 0, 0, u, u)$ or $u \leq g(0, u, 0, u, u)$ or $u \leq g(0, 0, u, u, u)$,

then $u = 0$.

Theorem 2.4 Let f, g be self mapping of a symmetric space (X, d) and T, S be mappings from X into B(X) such that the pairs {f, T} and {g, S} are occasionally weakly compatible, satisfying

$$\int_0^{F(\delta(Tx,Sy))} \phi(s)ds \leq \int_0^{P(x,y)} \phi(s)ds \tag{2.9}$$

for each $x,y \in X$, for which $fx \neq gy$, where

$$P(x, y) = g\{F(d(fx, gy)), F(d(fx, Tx)), F(d(gy, Sy)), F(\delta(fx, Sy)),$$
$$F(\delta(gy, Tx))\} \tag{2.10}$$

Then f, g, T and S have a unique common fixed point.

Proof: Since the pairs {f, T} and {g, S} are occasionally weakly compatible, therefore by definition, there exist x, y in X such that $fx \in Tx$, $gy \in Sy$, $fTx \subseteq Tfx$ and $gSy \subseteq Sgy$. Also $d(fx, gy) \leq \delta(Tx, Sy)$. First, we show that $gy = fx$. Suppose not, then by (2.10), we have

$$P(x, y) = g\{F(d(fx, gy)), 0, 0, F(\delta(fx, Sy)), F(\delta(gy, Tx))\}$$
$$\leq g\{F(\delta(Tx, Sy)), 0, 0, F(\delta(Tx, Sy)), F(\delta(Sy, Tx))\}.$$

By (g_2), we get $P(x, y) = 0$, with this, (2.9) implies that $\int_0^{F(\delta(Tx,Sy))} \phi(s)ds \leq 0$, implies that $\delta(Tx, Sy) = 0$, further implies that $d(fx, gy) = 0$.

Which is a contradiction, hence claim follows i.e., $fx = gy$. Also $d(fx, f^2x) \leq \delta(Tfx, Sy)$. Next we claim that, $fx = f^2x$ if not, then by (2.10), we have

$$P(fx, y) = g\{F(d(f^2x, gy)), 0, 0, F(\delta(f^2x, Sy)), F(\delta(gy, Tfx))\}$$
$$\leq g\{F(\delta(Tfx, Sy)), 0, 0, F(\delta(Tfx, Sy)), F(\delta(Sy, Tfx))\}.$$

By (g_1) and (g_2), we have, $P(x, y) \leq 0$, using this (2.9) gives $\int_0^{F(\delta(Tfx,Sy))} \phi(s)ds \leq$

0, implies that $\delta(Tfx, Sy) = 0$, further implies that $d(f^2x, fx) = 0$,

Which is a contradiction, hence our claim follows i.e., $fx = f^2x$, in the similar fashion, we can prove $gy = gy^2$. Hence fx is common fixed point of f, g, T and S and uniqueness follows easily from (2.1).

A control function Φ: $R^+ \to R^+$ is continuous, monotonically increasing function that satisfies $\Phi(2t) \leq 2\Phi(t)$ and $\Phi(0) = 0$ if and only if $t = 0$.

Let ψ: $R^+ \to R^+$ be such that $\psi(t) < t$, for each $t > 0$.

Theorem 2.5 Let f, g be self mapping of a symmetric space (X, d) and T, S be mappings from X into B(X) such that the pairs {f, T} and {g, S} are occasionally weakly compatible, satisfying

$$\int_0^{F(\Phi(\delta(Tx,Sy)))} \phi(s)ds \leq \int_0^{\psi(F(M_\Phi(x,y)))} \phi(s)ds \tag{2.11}$$

for each $x, y \in X$, for which $fx \neq gy$, where

$$M_\Phi(x, y) = \max\{F(\Phi(fx, gy)), F(\Phi(fx, Tx)), F(\Phi(gy, Sy)), \tfrac{1}{2}[\Phi(\delta(fx, Sy)) + \Phi(\delta(gy, Tx))]\} \tag{2.12}$$

Then f, g, T and S have a unique common fixed point.

Proof: Since the pairs {f, T} and {g, S} are occasionally weakly compatible, therefore by definition, there exist x, y in X such that $fx \in Tx$, $gy \in Sy$, $fTx \subseteq Tfx$ and $gSy \subseteq Sgy$. Also $d(fx, gy) \leq \delta(Tx, Sy)$.

$$M_\Phi(x, y) = \max\{F(\Phi(fx, gy)), 0, 0, \tfrac{1}{2}[\Phi(2\delta(Tx, Sy))]\}$$
$$\leq \Phi(\delta(Tx, Sy)) \ (2.13)$$

First, we show that gy = fx. Suppose not, and then by (2.11) and (2.13), we have

$$\int_0^{F(\Phi(\delta(Tx,Sy)))} \phi(s)ds \leq \int_0^{\psi(F(M_\Phi(x,y)))} \phi(s)ds$$
$$\leq \int_0^{\psi(F(\Phi(\delta(Tx,Sy))))} \phi(s)ds$$
$$< \int_0^{F(\Phi(\delta(Tx,Sy)))} \phi(s)ds$$

It leads to contradiction, therefore $\delta(Tx, Sy) = 0$, which further implies that $d(fx, gy) = 0$ i.e., $fx = gy$. Hence the claim follows. Also, $d(fx, f^2x) \leq \delta(Tfx, Sy)$. Next we claim that, $fx = f^2x$ if not, then by (2.12), we obtain

$$M_\Phi(fx, y) = \max\{\Phi(d(f^2x, gy)), 0, 0, \tfrac{1}{2}[\Phi(2\delta(Tfx, Sy))]\}$$
$$\leq \Phi(\delta(Tfx, Sy)) \ (2.14)$$

Next we claim that, $fx = f^2x$ if not, then by (2.12), we have

$$\int_0^{F(\Phi(\delta(Tfx,Sy)))} \phi(s)ds \leq \int_0^{\psi(F(M_\Phi(fx,y)))} \phi(s)ds$$
$$\leq \int_0^{\psi(F(\Phi(\delta(Tfx,Sy))))} \phi(s)ds$$
$$< \int_0^{F(\Phi(\delta(Tfx,Sy)))} \phi(s)ds$$

Again, we approaches to contradiction, therefore δ(Tfx, Sy) = 0, further implies that $d(f^2x, fx) = 0$, hence our claim follows i.e., $fx = f^2x$, in the similar fashion, we can prove $gy = gy^2$. Hence fx is common fixed point of f, g, T and S. Uniqueness follows easily from (2.11).

Set G = {ψ: [0, ∞) → [0, ∞) ; ψ is a continuous and decreasing mapping with ψ (t) iff t =0}

Theorem 2.6 Let f, g be self mapping of a symmetric space (X, d) and T, S be mappings from X into B(X) such that the pairs {f, T} and {g, S} are occasionally weakly compatible, satisfying

$$\int_0^{\psi(\delta(Tx,Sy))} \phi(s)ds \leq \int_0^{\psi(d(fx,g y))-\Phi(d(fx,gy))} \phi(s)ds \qquad (2.15)$$

for every x, y ∈ X, for which right hand side of (2.15) is not equal to 0, where ψ, Φ∈ G, then f, g, T and S have a unique common fixed point.

Proof: Since the pairs {f, T} and {g, S} are occasionally weakly compatible, therefore by definition, there exist x, y in X such that fx ∈ Tx, gy ∈ Sy, fTx ⊆ Tfx and gSy ⊆ Sgy. Also d(fx, gy) ≤ δ(Tx, Sy). First, we show that gy = fx. Suppose not, then by (2.15), we have

$$\int_0^{\psi(\delta(Tx,Sy))} \phi(s)ds \leq \int_0^{\psi(d(fx,gy))-\Phi(d(fx,gy))} \phi(s)ds$$
$$\leq \int_0^{\psi(d(Tx,Sy))-\Phi(d(Tx,Sy))} \phi(s)ds$$

This leads to contradiction. Therefore fx = gy. Hence the claim follows. Again, $d(f^2x, fx) \leq \delta(Tfx, Sy)$, now we claim that $f^2x = fx$, if not, the condition (2.15) implies that

$$\int_0^{\psi(\delta(Tfx,Sy))} \phi(s)ds \leq \int_0^{\psi(d(f^2x,gy))-\Phi(d(f^2x,gy))} \phi(s)ds$$
$$= \int_0^{\psi(d(f^2x,fx))-\Phi(d(f^2x,fx))} \phi(s)ds$$
$$\leq \int_0^{\psi(d(Tfx,Sy))-\Phi(d(Tfx,Sy))} \phi(s)ds$$

which is a contradiction, and hence the claim follows. On the similar account, it can be proved that $gy = g^2y$. Hence fx is common fixed point of f, g, T and S. Uniqueness follows easily from (2.15).

Example 2.7 Let x = {a, b, c}. Define $d : X \times X \to [0, \infty)$ by

$d(a, a) = d(b, b) = d(c, c) =0$, $d(a, b) = d(b, a) = b$,

$d(a, c) = d(c, a) = e$, (with $e > c + a$) $d(b, c) = d(c, b) = a$,

Obviously, d is symmetric but not metric on X.

We define $T, S : X \to B(X)$ by

$T(a) = \{a, c\}$, $T(b) = \{a, b, c\}$, $T(c) = \{ a, c\}$,

$S(a) = \{a, b\}$, $S(b) = \{a, c\}$, $S(c) = \{ b, c\}$,

And $f, g : X \to X$ as follows:

$f(a) = a$, $f(b) = c$, $f(c) = a$,

$g(a) = a$, $g(b) = a$, $g(c) = b$,

Now, it is obvious $f(a) \in T(a)$ but $fT(a) \neq Tf(a)$, and $f(c) \in T(c)$ but $fT(c) \neq Tf(c)$, therefore {f, T} is not weakly compatible. On the other hand, $f(b) \in T(b)$ but $fT(b) = Tf(b)$. Hence {f, T} is occasionally weakly compatible. Also, $g(a) \in S(a)$ but $gS(a) \neq Sg(a)$, and $g(c) \in S(c)$ but $gS(c) \neq Sf(c)$, therefore {g, S} is not weakly compatible. On the other hand, $g(b) \in S(b)$ but $gS(b) = Sg(b)$. Hence {g, S} is occasionally weakly compatible. As $f(a) = g(a) \in T(a)$ and $f(a) = g(a) \in S(a)$, so a is a unique common fixed point of f, g, T and S.

Remark 2.8

As integral contractive condition are indeed generalizations of corresponding contractive condition. Every contractive condition of integral type automatically induces the corresponding conractive condition, not including integral, by setting $\phi(s) = 1$over R^+.

Weakly compatible are owc but converse is not true, as in above example.

The Class of symmetric spaces is more general than that of metric spaces. Therefore the following results can be seen as special cases of results of our paper.

i. ([19, Theorem 1] and [20, theorem 1]) are special case of theorem 2.5.
ii. [21, Theorem 1], [22, Theorem 2.1], [23, Theorem 4.1] and [24, Theorem 2] are special case of corollary 2.2 part (i). Moreover, [25, Theorem 2] and [26, Theorem 1] also become special case of corollary 2.2 part (i).
iii. ([27, Theorem 2], [28, Theorem 1], [29, Theorem 1& 2]) is a special case of Theorem 2.1.
iv. [30, Theorem 3.1] becomes a special case of corollary 2.2 part (iii).

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