Advances in Fuzzy Mathematics
ISSN 0973-533X Volume 5, Number 2 (2010), pp. 147–156

http://www.ripublication.com/afm.htm

# Some Non Linear Arithmetic Operations on Triangular Fuzzy Numbers $(m, \alpha, \beta)$

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## Abstract

In this paper some non linear arithmetic operations on triangular fuzzy numbers $(m, \alpha, \beta)$ are discussed. Direct mathematical expressions to evaluate exponential, square root, logarithms, inverse exponential etc. of positive fuzzy numbers of type $(m, \alpha, \beta)$ are obtained using the basic analytical principles of algebraic mathematics and Taylor series expansion. The devised formulas are applied in various numerical examples that have an implementation scope in several engineering and real life systems.

**Keywords:** fuzzy numbers; taylor series.
**2000 AMS Subject Classification:** 03E72; 11D61; 41A58.

## 1. Introduction

Fuzzy numbers are one way to describe the vagueness and lack of precision of data. They are developed based on the theory of fuzzy sets which Zadeh [12] introduced in 1965. Algebraic equations involving fuzzy numbers are an important application of fuzzy set theory. They can be used for expert system reasoning, fuzzy process modeling and control, and so forth.

Previous studies on fuzzy numbers have shown that there are no opposite and reverse fuzzy numbers in the sense of group structure [3,9,11]. Mathematically, for any fuzzy number $A$, $A + (-A) \neq 0$ and $A \times ({}^{1}/_{A}) \neq 1$.

Fuzzy systems including fuzzy set theory and fuzzy logic have many successful applications. Sophisticated fuzzy set theoretic methods have been applied to various areas ranging from fuzzy topological spaces to quantum optics, medicine and so on [7,10].

Even though many operations that can be carried out using ordinary numbers can also be carried with fuzzy numbers, but many properties change as we pass from one family to the other. Although this is important, it should not prevent us from using the concepts and theory of fuzzy numbers. Some of the interesting arithmetic work on fuzzy numbers can be found in [3,4,5].

But Most of the recent research work on special behavior of fuzzy numbers like factorial, exponential, logarithmic and trigonometric is limited and much of the work is concentrated on linear fuzzy system of equations and their applications [1,2]. A good and a detailed representation of fuzzy numbers in the domain of combinatorics and their related arithmetic behavior is described by Kauffman and Gupta [6] but the method of explanation is based upon the $\alpha$-cut approach and a clear view of its application on triangular fuzzy numbers is not adequately comprehendible.

In this paper a specific attention is given to positive fuzzy numbers of type $(m, \alpha, \beta)$ and the related mathematical expressions for evaluation of logarithm, exponential, inverse exponential, $n^{th}$ positive root etc. are derived using simple analytical mathematics [8]. For example in order to derive the exponential of a positive fuzzy number $(m, \alpha, \beta)$, taylor series expansion for the exponent is used along with simple mathematical multiplication of two positive triangular fuzzy numbers given by Dubois and Prade [4,5].

The rest of this paper is organized as follows. In section 2, some of the preliminary concepts on fuzzy numbers are reviewed. In section 3, the proposed non linear arithmetic functions on positive fuzzy numbers of type $(m, \alpha, \beta)$, such as exponential, logarithm etc. are described. In section 4, the proposed mathematical expressions are investigated with the help of numerical examples that belong to the domain of general and applied mathematics. In section 5, the paper is concluded.

## 2. Preliminary concepts

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed [2,3,4,5].

**Definition 2.1.** Let $X$ denote a universal set. Then the fuzzy subset $\tilde{A}$ of $X$ is defined by its membership function

$$\mu_{\tilde{A}} : X \rightarrow [0,1],$$

which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$, to each element $x \in X$, where the value of $\mu_{\tilde{A}}(x)$ at $x$ shows the grade of membership of $x$ in $\tilde{A}$.

**Definition 2.2.** A fuzzy number is a convex normalized fuzzy set of the real line $R^1$ whose membership function is piecewise continuous.

**Definition 2.3.** A fuzzy number $M$ is called positive (negative), denoted by $M > 0$ $(M < 0)$, if its membership function $\mu_M(x)$ satisfies $\mu_M(x) = 0, \forall x \leq 0$ $(\forall x \geq 0)$.

Similarly, a fuzzy number $M$ is called non positive (non negative), denoted by $M \leq 0$ ($M \geq 0$), if its membership function $\mu_M(x)$ satisfies $\mu_M(x) = 0, \forall x > 0$ ($\forall x < 0$).

**Definition 2.4.** A fuzzy number $M$ is said to be a $LR$ fuzzy number if

$$\mu_M(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), x \leq m, \alpha > 0, \\ R\left(\dfrac{x-m}{\beta}\right), x \geq m, \beta > 0 \end{cases}$$

Where $m$ is the mean value of $M$ and $\alpha, \beta$ are left and right spreads, respectively and a function $L(.)$ the left shape function satisfying:

1. $L(x) = L(-x)$
2. $L(0) = 1 \; and \; L(1) = 0$
3. $L(x) \; is \; non \; increasing \; on \; [0, \infty)$

Naturally, a right shape function $R(.)$ is similarly defined as $L(.)$. Using its mean value and left and right spreads and shape functions, such a $LR$ fuzzy number $M$ is symbolically written as $M = (m, \alpha, \beta)_{LR}$. Clearly $M$ is non negative, if and only if $m - \alpha \geq 0$.

**Definition 2.5.** Two $LR$ fuzzy number $M = (m, \alpha, \beta)_{LR}$ and $N = (n, \gamma, \delta)_{LR}$ are said to be equal if and only if

$$m = n, \alpha = \gamma, \beta = \delta.$$

**Definition 2.6.** For two $LR$ fuzzy number $M = (m, \alpha, \beta)_{LR}$ and $N = (n, \gamma, \delta)_{LR}$ the formula for extended addition, extended opposite, extended subtraction and extended multiplication are summarized as follows:

*Addition*

$$(m, \alpha, \beta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (m + n, \alpha + \gamma, \beta + \delta)_{LR}$$

*Opposite*

$$M = -(m, \alpha, \beta)_{LR} = (-m, \beta, \alpha)_{RL}$$

*Subtraction*

Let $M = (m, \alpha, \beta)_{LR}$ and $N = (n, \gamma, \delta)_{RL}$ be two $LR$ and $RL$ fuzzy numbers, respectively.

$$M \ominus N = (m, \alpha, \beta)_{LR} \ominus (n, \gamma, \delta)_{RL} = (m - n, \alpha + \delta, \beta + \gamma)_{LR}$$

*Multiplication*

The approximate formula for the extended multiplication of two symmetric $LR$ fuzzy numbers can be summarized as follows:

If $M > 0$ and $N > 0$ then

$$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} \cong (mn, m\gamma + n\alpha - \alpha\gamma, m\delta + n\beta + \beta\delta)_{LR}$$

*Scalar multiplication*

$$\lambda \otimes (m,\alpha,\beta)_{LR} = \begin{cases} (\lambda m, \lambda\alpha, \lambda\beta)_{LR}, \lambda \geq 0 \\ (\lambda m, -\lambda\beta, -\lambda\alpha)_{RL}, \lambda < 0 \end{cases}$$

*Inverse*

$$1/_{(m,\alpha,\beta)_{LR}} = \left(1/_m, 1/_m - 1/_{m+\beta}, 1/_{m-\alpha} - 1/_m\right)_{RL}$$

## 3. Non linear arithmetic operations

### 3.1. Modulus

The modulus of a positive triangular fuzzy number $\widetilde{M} = (m,\alpha,\beta)$ is proposed as follows:

$$|\widetilde{M}| = |(m,\alpha,\beta)| = \begin{cases} (m,\alpha,\beta)\ \widetilde{M} \geq 0 \\ (-m,\beta,\alpha)\ \widetilde{M} < 0 \end{cases}$$

### 3.2. Square root

The square root of a positive triangular fuzzy number $\widetilde{M} = (m,\alpha,\beta) > 0$ is obtained as follows:

$$\sqrt{\widetilde{M}} = \sqrt{(m,\alpha,\beta)} = (x,y,z) \Rightarrow (x,y,z)^2 = (m,\alpha,\beta)$$

on applying the multiplication rule from definition

If $\tilde{A} > 0$ and $\tilde{B} > 0$ then

$$(a,\alpha,\beta)_{LR} \otimes (b,\gamma,\delta)_{LR} \cong (ab, a\gamma + b\alpha - \alpha\gamma, a\delta + b\beta + \beta\delta)_{LR}$$

we get

$$\sqrt{(m,\alpha,\beta)} = \left(\sqrt{m}, \sqrt{m} - \sqrt{m-\alpha}, \sqrt{m+\beta} - \sqrt{m}\right) \tag{1}$$

which is again a positive $LR$ type fuzzy number

### 3.3 A general recursive formula for $(\mathbf{m},\boldsymbol{\alpha},\boldsymbol{\beta})^{\mathbf{n}}$

Using the multiplication rule for two positive fuzzy numbers:

If $M > 0$ and $N > 0$ then

$$(m,\alpha,\beta)_{LR} \otimes (n,\gamma,\delta)_{LR} \cong (mn, m\gamma + n\alpha - \alpha\gamma, m\delta + n\beta + \beta\delta)_{LR}$$

we can have a general formula for the $n^{th}(n > 0)$ power of $(m,\alpha,\beta)$

$$(m,\alpha,\beta)^n = (m^n, f(m,\alpha,n), g(m,\beta,n))$$

where $f$ and $g$ are polynomial functions as follows

$$f(m,\alpha,n) = a_{1n}m^{n-1}\alpha + a_{2n}m^{n-2}\alpha^2 + a_{3n}m^{n-3}\alpha^3 + \cdots + a_{nn}\alpha^n$$

$$g(m,\beta,n) = b_{1n}m^{n-1}\beta + b_{2n}m^{n-2}\beta^2 + b_{3n}m^{n-3}\beta^3 + \cdots + b_{nn}\beta^n$$

and

$$a_{1n} = n,\ a_{nn} = (-1)^{n-1}$$

$$a_{in} = (-1)^{i-1}\{|a_{i,n-1}| + |a_{i-1,n-1}|\}, 1 < i < n$$

$$b_{in} = |a_{i,n}|\ 1 \leq i \leq n$$

A general recursive formula for $(-(\mathrm{m},\alpha,\beta))^{\mathrm{n}}$ :

A general recursive formula for $-\tilde{x} = (-(m,\alpha,\beta))^n$ is obtained as follows:

$$(-(m,\alpha,\beta))^n = \begin{cases} (m,\alpha,\beta)^n \text{ n} \in \text{even} \\ -(m,\alpha,\beta)^n \text{ n} \in \text{odd} \end{cases}$$

It is to be noted that $-(m,\alpha,\beta)^n = (-m^n, g(m,\beta,n), f(m,\alpha,n))$
where $f$ and $g$ are polynomial functions as follows

$$f(m,\alpha,n) = a_{1n}m^{n-1}\alpha + a_{2n}m^{n-2}\alpha^2 + a_{3n}m^{n-3}\alpha^3 + \cdots + a_{nn}\alpha^n$$

$$g(m,\beta,n) = b_{1n}m^{n-1}\beta + b_{2n}m^{n-2}\beta^2 + b_{3n}m^{n-3}\beta^3 + \cdots + b_{nn}\beta^n$$

and

$$a_{1n} = n,\ a_{nn} = (-1)^{n-1}$$

$$a_{in} = (-1)^{i-1}\{|a_{i,n-1}| + |a_{i-1,n-1}|\}, 0 < i < n$$

$$b_{in} = |a_{i,n}|\ 0 \le i \le n$$

However multiplication with $-1$ changes the number from $LR$ type triangular fuzzy number to $RL$ type triangular fuzzy number.

### 3.4 Exponential of a non negative triangular fuzzy number

The exponential of a triangular fuzzy number has been calculated in [6] using the $\alpha$-cut approach but in this paper we intend to formulate the exponential of a fuzzy number using the taylor series expansion method. Since exponential of a real number is defined as follows, we will extend the same formula for a triangular fuzzy number

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, -\infty < x < \infty$$

For $\tilde{x} = (m,\alpha,\beta)$, we have

$$e^{\tilde{x}} = \tilde{1} + \frac{\tilde{x}}{1!} + \frac{\tilde{x}^2}{2!} + \frac{\tilde{x}^3}{3!} + \cdots, \tilde{x} \ge 0$$

or

$$e^{(m,\alpha,\beta)} = \tilde{1} + \frac{(m,\alpha,\beta)}{1!} + \frac{(m,\alpha,\beta)^2}{2!} + \frac{(m,\alpha,\beta)^3}{3!} + \cdots, \tilde{x} \ge 0$$

$$(m,\alpha,\beta)^2 = (m^2, 2m\alpha - \alpha^2, 2m\beta + \beta^2)$$
$$(m,\alpha,\beta)^3 = (m^3, 3m^2\alpha - 3m\alpha + \alpha^3, 3m^2\beta + 3m\beta + \beta^3)$$

Similarly using the general recursive formula for $(m,\alpha,\beta)^n$ we have:

$$e^{(m,\alpha,\beta)} = \tilde{1} + \frac{(m,\alpha,\beta)}{1!} + \frac{(m,\alpha,\beta)^2}{2!} + \frac{(m,\alpha,\beta)^3}{3!} + \cdots, \tilde{x} \ge 0$$
$$e^{(m,\alpha,\beta)} = (1,0,0) + \sum_{i=1}^{\infty} \frac{(m,\alpha,\beta)^i}{i!}$$

$$\Rightarrow \left(1+\frac{m}{1!}+\frac{m^2}{2!}+\frac{m^3}{3!}\dots\right), \left(\alpha\left(\frac{1}{1!}+\frac{2m}{2!}+\frac{3m^2}{3!}+\cdots\right) - \alpha^2\left(\frac{1}{2!}+\frac{3m}{3!}+\cdots\right)\dots\right), \left(\beta\left(\frac{1}{1!}+\frac{2m}{2!}+\frac{3m^2}{3!}+\cdots\right) + \beta^2\left(\frac{1}{2!}+\frac{3m}{3!}+\cdots\right)+\cdots\right)$$

$$\Rightarrow (e^m, e^m - e^{m-\alpha}, e^{m+\beta} - e^m) \tag{2}$$

Using the above equation the following set of corollaries can be easily verified.

**Corollary 3.1.** $e^{\tilde{x}}.e^{\tilde{y}} = e^{\tilde{x}+\tilde{y}}$ if f $\tilde{x},\tilde{y} \geq 0$
**Corollary 3.2.** $(e^{\tilde{x}})^a = e^{a\tilde{x}}$ if f $\tilde{x} \geq 0$ and $a \in R^+$
**Corollary 3.3.** $\frac{e^{\tilde{x}}}{e^{\tilde{y}}} = e^{\tilde{x}-\tilde{y}}$ if f $\tilde{x} \geq \tilde{y} \geq 0$

### 3.5 Inverse Exponential of a non negative triangular fuzzy number $(\mathbf{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})_{\mathbf{LR}}$

Using Def 2.6

$$e^{-(m,\alpha,\beta)} = \frac{1}{e^{(m,\alpha,\beta)}} = (\frac{1}{e^m}, \frac{1}{e^m} - \frac{1}{e^{m+\beta}}, \frac{1}{e^{m-\alpha}} - \frac{1}{e^m})_{RL} \tag{3}$$

### 3.6 Logarithm of a positive triangular fuzzy number

$$\log_e(m,\alpha,\beta) = (x,y,z) \Rightarrow e^{(x,y,z)} = (m,\alpha,\beta)$$
$$\Rightarrow x = \log_e m, y = \log_e \frac{m}{m-\alpha}, z = \log_e \frac{m+\beta}{m}$$

$$\log_e(m,\alpha,\beta) = \left(\log_e m, \log_e \frac{m}{m-\alpha}, \log_e \frac{m+\beta}{m}\right) \tag{4}$$

Using the above equation the following set of corollaries can be easily verified.
**Corollary 3.4.** $\text{Log}_e \tilde{x} + \log_e \tilde{y} = \log_e \tilde{x}\tilde{y} \; if \; \tilde{x},\tilde{y} > 0$
**Corollary 3.5.** $\log_e \tilde{x} - \log_e \tilde{y} = \log_e \frac{\tilde{x}}{\tilde{y}} \; if \; \tilde{x} \geq \tilde{y} > 0$
**Corollary 3.6.** $\log_e \tilde{x}^a = a \log_e \tilde{x} \; if \; \tilde{x} > 0, a \in I^+$

### 3.7 Positive solutions of $(\mathbf{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})^{\mathbf{1/n}}$

If the spreads are small in comparison to the mean i.e. $\alpha, \beta \ll m$, Then the multiplication rule between two positive fuzzy numbers can be reduced to

If $M > 0$ and $N > 0$ then

$$(m,\alpha,\beta)_{LR} \otimes (n,\gamma,\delta)_{LR} \cong (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$$

Using this multiplication rule we can evaluate the $n^{th}$ $(n > 0)$ positive root of a fuzzy number as:

$$(m,\alpha,\beta)^{1/n} \simeq \left(m^{1/n}, \frac{\alpha m^{1/n}}{nm}, \frac{\beta m^{1/n}}{nm}\right) \tag{5}$$

**Proposition 3.1.** $(m,\alpha,\beta)^{1/2} \simeq \left(m^{1/2}, \frac{\alpha m^{-1/2}}{2}, \frac{\beta m^{-1/2}}{2}\right)$

*Proof*

For small spreads $\sqrt{(m,\alpha,\beta)} = \left(\sqrt{m}, \sqrt{m} - \sqrt{m-\alpha}, \sqrt{m+\beta} - \sqrt{m}\right)$ reduces to

$$(m,\alpha,\beta)^{1/2} = \left(m^{1/2}, m^{1/2} - (m-\alpha)^{1/2}, (m+\beta)^{1/2} - m^{1/2}\right)$$

From binomial expansion [8]

$$(1+x)^n = 1 + nx\,, x \ll 1$$

Hence for small spreads $\alpha, \beta \ll m$

$$m^{1/2} - (m-\alpha)^{1/2} = m^{1/2}\left(1 - \left(1 - \frac{\alpha}{2m}\right)\right) = \frac{\alpha m^{-1/2}}{2}$$

$$(m+\beta)^{1/2} - m^{1/2} = m^{1/2}\left(\left(1 + \frac{\beta}{2m}\right) - 1\right) = \frac{\beta m^{-1/2}}{2}$$

Thus,

$$(m,\alpha,\beta)^{1/2} \simeq \left(m^{1/2}, \frac{\alpha m^{-1/2}}{2}, \frac{\beta m^{-1/2}}{2}\right)$$

### 3.8 Other arithmetic operators

1. $\widetilde{M}^{\widetilde{N}}$ if $\widetilde{M} > 0, \widetilde{N} \geq 0$

Let $\widetilde{M} = (m,\alpha,\beta), \widetilde{N} = (n,\gamma,\delta)$

$$(m,\alpha,\beta)^{(n,\gamma,\delta)} = e^{(n,\gamma,\delta)\ln(m,\alpha,\beta)} = e^{(n,\gamma,\delta)(\log_e m, \log_e \frac{m}{m-\alpha}, \log_e \frac{m+\beta}{m})}$$

on further solving we get:

$$(m,\alpha,\beta)^{(n,\gamma,\delta)} = \left(m^n, m^n - (m-\alpha)^{(n-\gamma)}, (m+\beta)^{(n+\delta)} - m^n\right) \tag{6}$$

2. $a^{\tilde{x}}$ where $a \geq 1$ $and$ $\tilde{x} \geq 0$

Let $\tilde{x} = (m,\alpha,\beta)$

$$a^{\tilde{x}} = e^{\tilde{x}\ln a} = (a^m, a^m - a^{m-\alpha}, a^{m+\beta} - a^m)$$

## 4. Numerical examples

**Example 4.1.** Suppose you deposit \$603 in a bank account that whose continuous interest rate is a fuzzy number (0.6,0.01,0.01) p.a. because of market fluctuations. How much money will you have 7 years later assuming that interest paid is compounded continuously i.e. $A(t) = A(0)e^{\tilde{r}t}$

Solution Since $A(0) = \$603, t = 7, r = (0.6,0.01,0.01)$, we have from (2)

$$A(7) = 603.\,e^{7(0.6,0.01,0.01)} = \$(40211.86,2717.3,2914.7)$$

**Example 4.2.** A hard boiled egg at $(98,1,2)$℃ is put in a sink of $(18,1,0)$℃. If the system obeys Newton's law of cooling i.e. $T = T_s + (T_0 - T_s)e^{-kt}$, evaluate the temperature of the egg after 2 minutes assuming the rate constant $k = (\ln 2\,, 0.05,0)\ per\ min.$

Solution Assuming the variables to be fuzzy we can evaluate the final temperature as follows

$$\tilde{T} = (18,1,0) + \big((98,1,2) - (18,1,0)\big)e^{-(2\ln 2,0.1,0)}$$
$$\tilde{T} = (18,1,0) + (80,1,3)(0.25,0,0.026)$$
$$\tilde{T} = (38,1,2.92)℃$$

**Example 4.3.** Find all the positive solution of equation $e^{\tilde{x}} = \sqrt{(4,3,5)}$

Solution Using Eq. 1 $\sqrt{(4,3,5)} = (2,1.1)$

Hence using Eq. 4

$$\tilde{x} = \ln(2,1,1) = (\ln 2\,, \ln 2\,, \ln\frac{3}{2})$$

**Example 4.4.** Compute the value of $(3,2,2)^{(2,1,2)}$

Solution Using Eq. 6 we get $(3,2,2)^{(2,1,2)} = (3^2, 3^2 - 1^1, 5^4 - 3^2) = (9,8,616)$

**Example 4.5.** A ball is thrown from a height of $(16,3,1)$ meters under gravity($g = 9.8\ ms^{-2}$). Compute the approximate time when the ball will hit the ground.

Solution Using laws of motion we may write:

$$\tilde{S} = \frac{1}{2}g\tilde{t}^2$$
$$\tilde{t} = \sqrt{\frac{2\tilde{S}}{g}} = 0.45(4,0.4,0.12) = (1.8,0.18,0.05)\ \text{Seconds}$$

**Example 4.6.** Solve the following equation for positive fuzzy solution:

$$\sqrt{\tilde{x} + \sqrt{\tilde{x} + \sqrt{\tilde{x} + \cdots\infty}}} = (4,3,1)$$

Solution Let $\tilde{x} = (m, \alpha, \beta)$. The equation can be rewritten as

$$\sqrt{(m,\alpha,\beta) + (4,3,1)} = (4,3,1)$$
$$\sqrt{(m + 4, \alpha + 3, \beta + 1)} = (4,3,1)$$

Using Eq. 1 we get,

$$\sqrt{m+4} = 4,$$
$$\sqrt{m+4} - \sqrt{m-\alpha+1} = 3$$
$$\sqrt{m+\beta+5} - \sqrt{m+4} = 1$$

on solving we get,

$$\tilde{x} = (m,\alpha,\beta) = (12,12,8)$$

**Example 4.7.** Find all the solutions of the equation $\sqrt{|\tilde{x}|} = (5,2,0)$

Solution The equation can be simply written as $|\tilde{x}| = (5,2,0)^2 = (25,16,0)$
Hence the equation has two solutions $\tilde{x}_1 = (25,16,0)$ and $\tilde{x}_2 = (-25,0,16)$.

**Example 4.8.** Solve $\sqrt{\tilde{x} + \sqrt{\tilde{x} + \sqrt{\tilde{x} + \cdots\infty}}}$ where $\tilde{x} = (6,5,6)$.

Solution The given equation can be re written as $\sqrt{\tilde{x}+\tilde{y}} = \tilde{y}$
Let $\tilde{y} = (p,q,r)$, therefore $\sqrt{(6+p, 5+q, 6+r)} = (p,q,r)$
or

$$\sqrt{6+p} = p \Rightarrow p = 3$$
$$\sqrt{6+p} - \sqrt{1+p-q} = q \Rightarrow \frac{5-\sqrt{5}}{2}$$
$$\sqrt{15+r} - 3 = r \Rightarrow r = 1$$

Hence the $LR$ type positive fuzzy solution is $\left(3, \frac{5-\sqrt{5}}{2}, 1\right)$.

**Example 4.9.** Evaluate the positive fuzzy solution of $(27,2,1)^{1/3}$

Solution Using Eq. 5, we get

$$(27,2,1)^{1/3} \simeq (3, \frac{2}{27}, \frac{1}{27})$$

## 5. Conclusion

In this paper several new mathematical expressions for positive fuzzy numbers of type $(m,\alpha,\beta)$ are devised. The proposed expressions are direct and do not need the computation of $\alpha$-cut of the fuzzy number on which the operation is desired. The expressions have been devised using traditional algebraic mathematics and are easy to comprehend. Various numerical examples are also solved to demonstrate the use of contrived expressions. Similar expressions for such type of fuzzy numbers in the field of trigonometry and geometry are left as future works.

## References

[1] Buckley J.J. and Qu Y.,1991, Solving systems of linear fuzzy equations, Fuzzy Sets and Systems, 43, pp. 33-43.

[2] Dehghan M., Hashemi B., 2006, Solution of the fully fuzzy linear system using the decomposition procedure, Applied Mathematics and Computation 182, pp. 1568-1580.

[3] Dubois D. and Prade H.,1979, Fuzzy real algebra: Some results, Fuzzy Sets Sys. 2, pp. 327-349.

[4] Dubois D. and Prade H., 1978, Operations on fuzzy numbers, International journal of Systems Science, pp. 613-626.

[5] Dubois D. and Prade H., 1980, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York.

[6] Kauffmann A., Gupta M. M., 1991, Introduction to Fuzzy Arithmetic: Theory and Applications, New York: Van Nostrand Reinhold.

[7] Kearfott R.B. and Kreinovich V., 1996, Applications of Interval Computations, Kluwer Academic Publishers, Dordrecht/Boston/London.

[8] Lipschutz S., 2005, Shaum's Outline of Theory and Problems of Linear Algebra, 3rd edition, McGraw Hill Book Company, New York.

[9] Mizumoto M. and Tanaka K.,1979, Some properties of fuzzy numbers, in Advances in Fuzzy Set Theory and Applications (M. M. Gupta, R. K. Ragade, and R. R. Yager, Eds.), North-Holland, Amsterdam, pp. 153-164.

[10] Nguyen H.T. and Kreinovich V.,1996, Nested intervals and sets: concepts, relations to fuzzy sets, and applications ,Applications of Interval Computations, R.B. Kearfott et al., eds., Kluwer Academic Publishers, Netherlands, pp. 254–290.

[11] Yager R., 1979, On solving fuzzy mathematical relationships, Information Control 41, pp. 29-55.

[12] Zadeh L.A., 1965, Fuzzy sets, Information and Control, 8, pp. 338–353.