Advances in Fuzzy Mathematics
ISSN 0973-533X Volume 5, Number 2 (2010), pp. 111–119

http://www.ripublication.com/afm.htm

Goal Programming and Fuzzy Goal Programming Techniques in the Bank Investment Plans under the Scenario of Maximizing Profit and Minimizing Risk Factor: A Case Study

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Abstract

In this paper we have shown that the risk and profit objectives can not be taken as constraint together in multi-objective linear programming problem and also in goal programming problem, but they are taken as constraint together in fuzzy environment. That is why the fuzzy goal programming technique is being followed to solve the bank investment allocation problem where the under- and over-deviational variables are introduced to each of the membership goals by assigning the highest degree (unity) as the aspiration levels of each objective goal. Also we have shown that in bank investment problem liquidity constraint can never be taken as fuzzy if the highest value of aspiration levels of each constraint goal is 1 and also the aspiration level for the capital adequacy objective can not be set smaller than 90.

Keywords: Fuzzy sets; Goal programming; Fuzzy Goal programming

Introduction

Commercial banks perform primarily different functions. Commercial banks, by their nature and functions, are multi product institutions. They naturally want to maximize their profits at the same time minimize the risk criteria. This conflicting situation leads naturally to think for multi objective optimization in the investment plans of the commercial banks.

In this paper we consider the optimal investment plan of a fictitious bank called **'Bank AXN'** and solve it by goal programming and fuzzy goal programming technique.

In 1955 the roots of goal programming lie in the journal (management science) by Charnes, Cooper and Ferguson [2]. The basic theory of GP is given by Ignizio[3]. Fuzzy sets are mathematical concept proposed by prof. L.A. Zadeh [7] in 1965. Bellman and Zadeh [1] proposed that a fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the goals/objectives and constraints. The use of fuzzy set theory in goal programming was first considered by Narasimhan[6]. In 1978, Zimmerman, H.J. [8] extended his fuzzy linear programming approach to the multi objective linear programming problem. In the recent past, R. H. Mohamod [5] studied some fuzzy programming models by using the concept of conventional goal programming. In this paper, we construct fuzzy goal programming problem where the membership functions(of the objectives) are considered as fuzzy goals by assigning the highest degree (unity) as the aspiration level and by introducing under- and over-deviational variables to each of them. In the achievement function, the under-deviation and over-deviation variables of the goals are minimized. For our discussion of the optimal investment plans of bank we take the constraints, objectives and capital of a fictitious bank to be called **'Bank AXN'** as under. These assumptions are realistic in the scenario of the contemporary commercial banks operating in the country (India). Let the **'Bank AXN'** has a capital of Rs. 530 crores(1 crore = 10 million), demand deposits (checking account) of Rs.150,000 crores and time deposit (saving accounts and certificates of deposit) of Rs. 450,000 crores. Table 1 displays the categories among which the **'Bank AXN'** must divide its capital and deposited funds. Rates of return are also provided for each category together with other information related to risk.

Table 1: 'Bank AXN's Investment Opportunities.

Investment Category, *j*	Return Rate (%)	Liquid Part (%)	Required Capital (%)	Risk Asset ?
i) Cash	0.0	100.0	0	No
ii) short term investment	4.0	99.5	.5	No
iii)Government: 1 to 5 years	3.5	96.0	5	No
iv)Government: 5 to 10 years	7.0	90.0	6	No
v)Instalment loans	11.5	0.0	16	Yes
vi)Cash credit	12.0	0.0	18	Yes
vii)Commercial loans	10.5	0.0	10	Yes

We model **'Bank AXN's** investment decisions with a decision variable for each category of investment in table 1: x_j = amount invested in category j, $j = 1, 2, \ldots\ldots, 7$

We further assume the following conditions which are inconformity with international standard: i) At least 47% of demand deposit and 36% of time deposit should be kept as liquid part. ii) At least 14% of demand deposit and 4%of time deposit should be kept as cash reserve. iii) At least 5% of sum of capital and deposits should be invested in all the categories (diversification). iv)At least 4% of sum of capital and deposits should be given as commercial loans[4]. We have three objectives such as profit, capital-adequacy (the ratio of required capital for bank solvency to actual capital) and risk asset ratio (sum of the investments having risk / capital). A low risk asset ratio indicates a financially secure institution and a low value of capital –adequacy ratio indicates minimum risk.

Multi Objective Linear Programming Model of Bank 'Bank AXN's Investment Problem:

Max $(.040\, x_2 + .035\, x_3 + .070\, x_4 + .115\, x_5 + .120\, x_6 + .105\, x_7)$ (profit)

Min $(1/530)\,[.005\, x_2 + .050\, x_3 + .060\, x_4 + .160\, x_5 + .180\, x_6 + .100\, x_7]$ (capital-adequacy)

Min $(1/530)\,[x_5 + x_6 + x_7]$ (risk-asset)

Subject to $x_1 + \ldots\ldots + x_7 = 530 + 150{,}000 + 450{,}000$ (invest all)

$x_1 + .995\, x_2 + .960\, x_3 + .900\, x_4 \geq .47 \times 150{,}000 + .36 \times 450{,}000$ (liquidity)

Where $x_1 \geq .14 \times 150{,}000 + .04 \times 450{,}000$ (cash reserve)

$x_j \geq .05\,(530 + 150{,}000 + 450{,}000)$ for all $j = 2,\ldots,7$ (diversification)

$x_7 \geq .40\,(530 + 150{,}000 + 450{,}000)$ (commercial)

We rank the three objectives in the order: (1) risk-asset (2) profit objective (3) capital-adequacy objective. Now by solving the above multi objective linear programming problem using Pre-emptive or Lexicographic method in the order mentioned we get the optimal solutions as: $x_1^* = 39000$, $x_2^* = 30026.5$, $x_3^* = 30026.5$, $x_4^* = 201210.8$, $x_5^* = 30026.5$, $x_6^* = 30027.7$, $x_7^* = 240212$ with optimal **risk-asset = 566.54** and **profit = 48615.38**. The problem with capital-adequacy as objectivegives no feasible solution when we take the other two objectives (i.e. profit and risk-asset) as constraints with risk $\leq$ 566.54 and profit $\geq$ 48615.38. This means that risk asset objective $\leq$ 566.54 and profit objective $\geq$ 48615.38 is not tenable at the same time, some deviations are to be accepted. Let us now consider the goal programming modelling of the above problem. For the discussion ahead, we consider goal programming model with three aspiration levels for profit, risk and capital adequacy objective. The aspiration level for the third one is obtained by substituting the solution mentioned earlier in capital adequacy objective and it comes out to be 90.48. Now we set $g_1 = 560$, $g_2 = 48700$, $g_3 = 90$. Here we minimize the unwanted deviational variables.

Goal Linear Program Model of the 'Bank AXN's Investment Problem:

$$\text{MIN}\,(d_1^+ + d_2^- + d_3^+)$$

$$\text{Subject to}\quad \frac{1}{530}(x_5 + x_6 + x_7) + d_1^- - d_1^+ = 560$$

$$.040\,x_2 + .035\,x_3 + .070\,x_4 + .115\,x_5 + .120\,x_6 + .105\,x_7 + d_2^- - d_2^+ = 48700$$

$$\frac{1}{530}(.005\,x_2 + .050\,x_3 + .060\,x_4 + .160\,x_5 + .180\,x_6 + .100\,x_7) + d_3^- - d_3^+ = 90$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 600530$$

$$x_1 + .995\,x_2 + .96\,x_3 + .9\,x_4 \geqq 232500$$

$$\text{Where,}\quad x_1 \geqq .14 \times 150{,}000 + .04 \times 450{,}000$$

$$x_j \geqq .05\,(530 + 150{,}000 + 450{,}000)\ \text{for all } j = 2,\dots,6$$

$$x_7 \geqq .40\,(530 + 150{,}000 + 450{,}000)$$

$$d_k^-,\ d_k^+ \geqq 0,\ d_k^- \cdot d_k^+ = 0,\ k = 1,2,3$$

Solving the above G.P problem by the software LINGO (ver. 11) we get the optimal solutions as $x_1^* = 39000$, $x_2^* = 30026.5$, $x_3^* = 30026.5$, $x_4^* = 199518.3$, $x_5^* = 30026.5$, $x_6^* = 31720.20$,

$x_7^* = 240212$, $d_1^+ = 9.733396$, $d_2^- = 0$, $d_3^+ = .8634557$, $d_1^- = d_2^+ = d_3^- = 0$. It should be noted here that the risk goal and capital adequacy goal are not fully achieved because $d_1^+ \neq 0$, $d_3^+ \neq 0$ [3]. Again if we choose $g_1 = 560$, $g_2 = 48700$, $g_3 = 80$ then we see that the risk goal and capital adequacy goal are not fully achieved. This means that the above Goal programming method of solution of the bank investment problem is not suitable as it does not permit the full achievement of all the considered goals, so it is necessary to allow some tolerances in the aspiration levels and this leads us to adopt fuzzy goal programming (F.G.P) technique for the above bank investment problem.

Fuzzy Goal Linear Programming Problem Formulation:

The fuzzy goal programming formula can be formulated as:

$$\text{Find}$$

$$X(x_1, x_2, \dots, x_n) \in R^n$$

$$\text{So as to satisfy}$$

$$Z_k(X) \begin{bmatrix} \gtrsim \\ \lesssim \end{bmatrix} g_k,\quad k = 1, 2, \dots, K$$

$$\text{Subject to}\quad AX \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} b_i,\quad X \geqq 0,\ b_i \in R^m$$

Where X is the vector of decision variables, g_k is the imprecise aspiration level of the k-th objective $Z_k(X)$ ($k = 1, 2, \dots, K$), A is the coefficient matrix, b is the vector of right-hand side values (resources).

$$A X \begin{bmatrix} \gtrsim \\ \cong \\ \lesssim \end{bmatrix} b_i$$ represents fuzzy constrained set

Where $\lesssim$ and $\gtrsim$ refer to the fuzzy version of $\leq$ and $\geq$.

Now, in a fuzzy decision-making situation, the fuzzy goals are to be characterized by their respective membership functions.. If p_k denote the tolerance ranges (subjectively chosen constants of admissible violations) for the aspiration level g_k, then the membership function, say $\mu(Z_k(X))$, for the corresponding fuzzy objective goals $Z_k(X)$ can be defined as follows:

For $\gtrsim$ type of restriction,

$\mu(Z_k(X))$ takes the form

$$\mu(Z_k(X)) = \begin{cases} 1 & \text{if } Z_k(X) \geq g_k \\ \dfrac{Z_k(X) - (g_k - p_k)}{p_k} & \text{if } (g_k - p_k) \leq Z_k(X) < g_k \\ 0 & \text{if } Z_k(X) < g_k - p_k, \end{cases} \quad k = 1, 2, \ldots, K$$

For $\lesssim$ type of restriction,

$\mu(Z_k(X))$ takes the form

$$\mu(Z_k(X)) = \begin{cases} 1 & \text{if } Z_k(X) \leq g_k \\ \dfrac{(g_k + p_k) - Z_k(X)}{p_k} & \text{if } g_k < Z_k(X) \leq (g_k + p_k) \\ 0 & \text{if } Z_k(X) > (g_k + p_k), \end{cases} \quad k = 1, 2, \ldots, K$$

Where $(g_k - p_k)$ and $(g_k + p_k)$ represent the lower tolerance limit and the upper tolerance limit for the achievement of the stated fuzzy objective goal. Let q_i (i = 1,2,…m) denotes the tolerance ranges (subjectively chosen constants of admissible violations) for the fuzzy constrained set. Then the membership function, say $\mu(a_i(X))$ for the corresponding fuzzy constraint $a_i(X)$ (a_i is the i th row of the matrix A)can be defined as follows:

For $\cong$ type of restrictions , $\mu(a_i(X))$ takes the form

$$\mu(a_i(X)) = \begin{cases} 1 & \text{if } a_i(X) = b_i \\ \dfrac{(b_i + q_i) - a_i(X)}{q_i} & \text{if } b_i < a_i(X) \leq (b_i + q_i) \\ \dfrac{a_i(X) - (b_i - q_i)}{q_i} & \text{if } (b_i - q_i) \leq a_i(X) < b_i \\ 0 & \text{if } a_i(X) < b_i - q_i \text{ or } a_i(X) > (b_i + q_i) \end{cases} \quad , i = 1,2,\ldots,m \qquad \ldots\ldots(1)$$

For $\lessapprox$ type of restriction, $\mu(a_i(X))$ takes the form

$$\mu(a_i(X)) = \begin{cases} 1 & \text{if } a_i(X) \leq b_i \\ \dfrac{(b_i + q_i) - a_i(X)}{q_i} & \text{if } b_i \angle a_i(X) \leq (b_i + q_i) \\ 0 & \text{if } a_i(X) \geq (b_i + q_i), \end{cases} \quad i = 1,2,\ldots, m \qquad \ldots(2)$$

For $\gtrapprox$ type of restriction, $\mu(a_i(X))$ takes the form

$$\mu(a_i(X)) = \begin{cases} 1 & \text{if } a_i(X) \geq b_i \\ \dfrac{a_i(X) - (b_i - q_i)}{q_i} & \text{if } (b_i - q_i) \leq a_i(X) \angle b_i \\ 0 & \text{if } a_i(X) \angle b_i - q_i, \end{cases} \quad i = 1,2,\ldots, m \qquad \ldots(3)$$

Where $(b_i + q_i)$ and $(b_i - q_i)$ represent the upper tolerance limit and lower tolerance limit for the achievement of the stated fuzzy constraint. It may be noted that $\mu(a_i(X))$ in (2) and (3) are actually the particular forms of $\mu(a_i(X))$ in (1) defined for both the right and left tolerance limits. The highest value of a membership function is unity. So for the membership function (of the $\geq$ type and $\leq$ type) defined as above, the membership goals having the aspired level unity can be represented as

$$\mu(Z_k(X)) + d_k^- - d_k^+ = 1 \text{ for the membership objective goals of the '}\geq\text{ type' and '}\leq\text{ type'}$$

$$\mu(a_i(X)) + d_k^- - d_k^+ = 1 \text{ for the membership constraint goals of the '}\geq\text{ type' and '}\leq\text{ type'}$$

$$d_k^-, d_k^+ \geq 0, d_k^- . d_k^+ = 0, k = 1,2,\ldots,K$$

Where d_k^-, d_k^+ are the under- and over-deviational variables respectively, of the k-th membership goals. Using the minsum method, the Fuzzy goal programming model is given by

$$\text{Find X so as to Minimize } Z = \Sigma d_k^-$$

$$\text{Subject to} \quad \frac{Z_k(X) - (g_k - p_k)}{p_k} + d_k^- - d_k^+ = 1 \quad \text{For the goal of the type} \geq$$

$$\frac{(g_k + p_k) - Z_k(X)}{p_k} + d_k^- - d_k^+ = 1 \quad \text{For the goal of the type} \leq$$

$$\frac{a_i(X) - (b_i - q_i)}{q_i} + d_k^- - d_k^+ = 1 \quad \text{For the goal of the type} \geq$$

$$\frac{(b_i + q_i) - a_i(X)}{q_i} + d_k^- - d_k^+ = 1 \quad \text{For the goal of the type} \leq$$

$$X, d_k^-, d_k^+ \geq 0, d_k^- . d_k^+ = 0, k = 1,2,\ldots,K$$

Now assigning aspiration level g_1 = 560, g_2 = 48700, g_3 = 90 to the risk, profit, capital adequacy objectives respectively, we formulate the F.G.P model of the bank present's allocation problem.

Fuzzy Goal Programming Model of the 'Bank AXN's Investment Problem:

$$\text{Find } X(x_1, x_2, \ldots, x_7)$$

$$\text{So as to satisfy } \frac{1}{530}(x_5 + x_6 + x_7) \lesssim 560$$

$$.040\,x_2 + .035\,x_3 + .070\,x_4 + .115\,x_5 + .120\,x_6 + .105\,x_7 \gtrsim 48700$$

$$\frac{1}{530}(.005\,x_2 + .050\,x_3 + .060\,x_4 + .160\,x_5 + .180\,x_6 + .100\,x_7) \lesssim 90$$

$$\text{Subject to } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \cong 600530$$

$$x_1 + .995\,x_2 + .96\,x_3 + .9\,x_4 \gtrsim 232500$$

$$\text{Where, } x_1 \geqq .14 \times 150{,}000 + .04 \times 450{,}000$$

$$x_j \geqq .05(530 + 150{,}000 + 450{,}000) \text{ for all } j = 2, \ldots, 6$$

$$x_7 \geqq .40(530 + 150{,}000 + 450{,}000)$$

Let the tolerances be p_1 = 16, p_2 = 850, p_3 = 10 for the first, second, third membership objective goals and d_1 = 3636, d_2 = 2000, d_3 = 2500 respectively. Under the framework of minsum G.P, the F.G.P problem can be formulated as:

$$\text{Minimize } d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6^-$$

$$\text{Subject to } \frac{(560+16) - \frac{1}{530}(x_5 + x_6 + x_7)}{16} + d_1^- - d_1^+ = 1$$

$$\frac{.040\,x_2 + .035\,x_3 + .070\,x_4 + .115\,x_5 + .120\,x_6 + .105\,x_7 - (48700 - 850)}{850} + d_2^- - d_2^+ = 1$$

$$\frac{(90+10) - \frac{1}{530}(.005\,x_2 + .050\,x_3 + .060\,x_4 + .160\,x_5 + .180\,x_6 + .100\,x_7)}{10} + d_3^- - d_3^+ = 1$$

$$\frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7 - (600530 - 3636)}{3636} + d_4^- - d_4^+ = 1$$

$$\frac{(600530 + 2000) - (x_1+x_2+x_3+x_4+x_5+x_6+x_7)}{2000} + d_5^- - d_5^+ = 1$$

$$\frac{x_1 + .995\,x_2 + .96\,x_3 + .9\,x_4 - (232500 - 2500)}{2500} + d_6^- - d_6^+ = 1$$

$$\text{Where, } x_1 \geqq .14 \times 150{,}000 + .04 \times 450{,}000$$

$$x_j \geqq .05(530 + 150{,}000 + 450{,}000) \text{ for all } j = 2, \ldots, 6$$

$$x_7 \geqq .40(530 + 150{,}000 + 450{,}000)$$

$$d_k^-,\ d_k^+ \geqq 0,\ d_k^- . d_k^+ = 0,\ k = 1,2,3,4,5,6$$

The optimal solutions are x_1^* =39000, $x_2^* = x_3^* = x_5^* = x_6^*$ = 30026.5, x_4^* = 201212,

x_7^* = 240212, d_1^- = .4086085, d_2^- = .09962941, d_3^- = .04799764, $d_4^- = d_5^- = d_6^-$ =0,

$d_1^+ = d_2^+ = d_3^+ = d_4^+ = d_5^+ = 0$, d_6^+ = 18.51704 with profit = 48615.32, risk = 566.54 and capital adequacy = 90.48. This solution is not acceptable because $d_6^+ > 1$.

If we choose $g_1 = 560$, $g_2 = 48700$, $g_3 = 90$ with the same tolerances (p_1, p_2, p_3) for the fuzzy objective goals and ($q_1 = 3636$, $q_2 = 2000$) for the fuzzy constraint goals whereas the liquidity constraint cannot be taken as fuzzy then the optimal solutions are x_1^* =39000, $x_2^* = x_3^*$ =

$x_5^* = x_6^*$ = 30026.5, x_4^* = 201212, x_7^* = 240212, d_1^- = .4086085, d_2^- = .09962941, d_3^- = .04799764, $d_4^- = d_5^- = 0$, $d_1^+ = d_2^+ = d_3^+ = d_4^+ = d_5^+ = 0$ with profit = 48615.32, risk = 566.54 and capital adequacy = 90.48. This solution is acceptable because all over – deviation variables cannot be greater than 1.

If we choose $g_1 = 560$, $g_2 = 48700$, $g_3 = 70$ with the same tolerances (p_1, p_2, p_3) for the fuzzy objective goals and ($q_1 = 3636$, $q_2 = 2000$) for the fuzzy constraint goals whereas the liquidity constraint cannot be taken as fuzzy we see that the solution is not acceptable because $d_3^- > 1$.

If we choose aspiration levels $g_1 = 560$, $g_2 = 48700$, $g_3 = 90$ with tolerances p_1 = 50,

$p_2 = 500$, $p_3 = 10$ for the fuzzy objective goals and ($q_1 = 3636$, $q_2 = 2000$) for the fuzzy constraint goals whereas the liquidity constraint cannot be taken as fuzzy then the optimal solutions are x_1^* =39000, $x_2^* = x_3^* = x_5^*$= 30026.5, x_4^* = 199518.3, x_6^* = 31720.2, x_7^* = 240212, d_1^- = .1946679, $d_2^- = 0$, d_3^- = .08634557, $d_4^- = d_5^-$ = 0, $d_1^+ = d_2^+ = d_3^+ = d_4^+ = d_5^+ = 0$ with profit = 48700, risk = 569 and capital adequacy = 90.86 which is best solution compared to the other solutions.

Conclusion

After solving the **'Bank AXN's** investment planning problem we conclude that two objectives of different types (maximum and minimum) i.e. risk and profit objectives can never be considered as constraints together with respect to the capital adequacy objective in M.O.L.P problem (by lexicographic method) but in fuzzy environment the two objectives (risk and profit) may be considered as constraint together. Similarly the goals are not fully satisfied in goal programming model. But in F.G.P model of the problem the fuzzy goals are fully satisfied with the same aspiration levels whereas the liquidity constraint can never be taken as fuzzy and the aspiration level for the third objective cannot be set smaller than 90 with different tolerance values. So we suggest that the **'Bank AXN'** may adopt the fuzzy goal programming technique for their allocation problem. In this paper we use the LINGO software (ver. 11.).

Acknowledgement

We are grateful to the Indian Statistical Institute, Kolkata for the library facilities.

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