

On Fuzzy Nearly Compact Regular Open Topology

¹A. Deb Ray and ²Pankaj Chettri

¹*Department of Mathematics, West Bengal State University,
Barasat, North 24 Paraganas, Kolkata - 700126, INDIA
E-mail: atasi@hotmail.com*

²*Department of Mathematics, Sikkim Manipal Institute of Technology
Majitar, Rangpoo, East Sikkim PIN - 737132, INDIA
E-mail: pankajct@gmail.com*

Abstract

In this paper, the concept of a new fuzzy topology called fuzzy nearly compact regular open topology on a collection of functions, \mathcal{F} , from a fuzzy topological space (*fts*, for short) X to another *fts* Y (denoted by F_{NR}) is introduced and studied. It is observed that the fuzzy $GS-T_2$ -ness (fuzzy almost regularity) of the range *fts* Y induces fuzzy $GS-T_2$ -ness (respectively, somewhat almost regularity) in \mathcal{F} . Moreover the fuzzy almost regularity of Y makes the evaluation map on (\mathcal{F}, F_{NR}) δ -continuous. The notion of jointly fuzzy δ -continuous on fuzzy near compacta is also introduced and studied in connection with the said F_{NR} topology.

AMS subject classification: 03E72, 54A40, 54C35.

Keywords: fuzzy nearly compact regular open topology, fuzzy δ -continuity on fuzzy nearly compact set, jointly fuzzy δ -continuous on fuzzy near compacta.

1. Introduction and Preliminaries

The concept of fuzzy sets and fuzzy logic was introduced by L.A Zadeh in 1965 [11]. C.L Chang [1] started developing the theory of fuzzy topological spaces in 1968. Following this line, several mathematicians could successfully generalize different concepts of general topology to their fuzzy settings. It is well known that function spaces play a very significant role in functional analysis, topology, differential equation, complex analysis, differential geometry and in almost all other branches of modern mathematics. So, an enormous study of functions between *fts*'s were also

made. However, the collection of functions between two *fts*, endowed with some fuzzy topology, has not been explored in great detail so far. Some of the fruitful attempts along this line that demand mention here, was due to Kohli and Prasanna [6], Gunther Jagar [5] and Peng [10].

In this paper we introduce a new fuzzy topology called F_{NR} topology on functions between two *fts*. The *NR* topology was introduced and studied in detail by Ganguly and Dutta [2] for functions between two topological spaces. Modifying the definition of fuzzy compact open topology, given by Gunther Jagar in [5], we introduce fuzzy nearly compact regular open (F_{NR}) topology and could characterize it imposing conditions on the respective range and domain *fts*'s.

In the last section of this article, we have introduced the notion of jointly fuzzy δ -continuous fuzzy topologies on functions between *fts*'s and find conditions for the F_{NR} topology to be jointly fuzzy δ -continuous.

To make this paper self sufficient, we state a few known definitions and results, that are required subsequently.

Let X be a non-empty set and I be the closed interval $[0,1]$. A fuzzy set μ on X is a function on X into I . Let I^X denote the collection of all fuzzy sets on X . The support of a fuzzy set μ , denoted by $supp(\mu)$, is the crisp set $\{x \in X : \mu(x) > 0\}$. A fuzzy set is called a fuzzy point, denoted by x_α , where $0 < \alpha \leq 1$, and defined as

$x_\alpha(z) = \begin{cases} \alpha, & \text{for } z = x \\ 0, & \text{otherwise} \end{cases}$. Let f be a function from a set X into a set Y . Let A, B be

fuzzy sets on X and Y respectively. Then $f(A)$ and $f^{-1}(B)$ are fuzzy sets on Y

and X respectively, defined by [11] $f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{when } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$

and $f^{-1}(B)(x) = B(f(x))$.

A collection $\tau \subseteq I^X$ is called a *fuzzy topology* [1] on X if (i) $0, 1 \in \tau$

(ii) $\forall \mu_1, \mu_2, \dots, \mu_n \in \tau \Rightarrow \bigwedge_{i=1}^n \mu_i \in \tau$ (iii) $\mu_\alpha \in \tau, \forall \alpha \in \Lambda$ (where Λ is an index set) $\Rightarrow \bigvee \mu_\alpha \in \tau$. Then (X, τ) is called a fuzzy topological space (*fts*, for short). In what follow, we always mean a *fts* (X, τ) as the underlying space, whenever we write X .

A fuzzy point x_α is said to q -coincident with a fuzzy set A , denoted by $x_\alpha qA$ if $\alpha + A(x) > 1$ and belong to a fuzzy set A (in notation, $x_\alpha \in A$) if $\alpha \leq A(x)$. A fuzzy set A is said to be a fuzzy neighbourhood (*nhd.*, for short) of a fuzzy point x_α if there is a fuzzy open set B such that $x_\alpha \in B \leq A$ and a fuzzy q -neighbourhood (q -*nhd.*, for short) of a fuzzy point x_α if there is a fuzzy open set B such that $x_\alpha qB \leq A$. If A is fuzzy open, the q -*nhd.* is called fuzzy open q -*nhd.* A fuzzy set

A is q -coincident with another fuzzy set B , if there exists $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not q -coincident, we write $A \bar{q} B$. [9]

Definition 1.1 A $fts (X, \tau)$ is called

- (1) fuzzy $GS - T_2$ [4] if for any two distinct fuzzy points x_α and y_β ; when $x \neq y$, x_α and y_β have fuzzy open nhd 's which are not q -coincident and when $x = y$ and $\alpha < \beta$ (say) x_α has a fuzzy open $nhd. U$ and y_β has a fuzzy open q - $nhd. V$ such that $U \bar{q} V$.
- (2) fuzzy regular [8], if for each fuzzy point x_α in X and each fuzzy open q - $nhd. U$ of x_α there exists a fuzzy open q - $nhd. V$ such that $\bar{V} \leq U$, where \bar{V} stands for the closure of V .
- (3) fuzzy almost regular [8], if for each fuzzy point x_α and any fuzzy regular open set A with $x_\alpha q A$, there exists a fuzzy regular open set B such that $x_\alpha q B$ and $\bar{B} \leq A$.

Definition 1.2 [7] A fuzzy set A is called fuzzy compact if for every family \mathcal{U} of fuzzy open sets on X such that $A \leq \sup\{U : U \in \mathcal{U}\}$ and for every $\varepsilon > 0$ there exist a finite subfamily \mathcal{U}_0 of \mathcal{U} such that $A \leq \sup\{U : U \in \mathcal{U}_0\} + \varepsilon$.

In a similar way fuzzy nearly compact set is defined as follows:

Definition 1.3 A fuzzy set A is called fuzzy nearly compact if for every family \mathcal{U} of fuzzy open sets on X such that $A \leq \sup\{U : U \in \mathcal{U}\}$ and for every $\varepsilon > 0$ there exist a finite subfamily \mathcal{U}_0 of \mathcal{U} such that $A \leq \sup\{\bar{U}^0 : U \in \mathcal{U}_0\} + \varepsilon$. where \bar{U}^0 denotes $int(clU)$.

Definition 1.4 [3] Let (X, τ) and (Y, σ) be two fuzzy topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be fuzzy δ -continuous if for each fuzzy point x_β on X and any fuzzy regular open q - $nhd. V$ of $(f(x))_\beta$ on Y , there exists fuzzy regular open q - $nhd. U$ of x_β on X such that $f(U) \leq V$.

The following are equivalent forms of fuzzy δ -continuous functions:

- (i) For each fuzzy regular closed set μ in Y , $f^{-1}(\mu)$ is fuzzy δ -closed in X .
- (ii) For each fuzzy point x_β on X and any fuzzy regular open $nhd. V$ of $(f(x))_\beta$ on Y , there exists fuzzy regular open $nhd. U$ of x_β on X such that $f(U) \leq V$.

2. The F_{NR} topology

Definition 2.1 Let (X, τ) and (Y, σ) be two fuzzy topological spaces and \mathcal{F} be a nonempty collection of functions from X to Y . For each fuzzy nearly compact set N on X and each fuzzy regular open set R on Y , a fuzzy set N_R on \mathcal{F} is given by

$$N_R(g) = \inf_{x \in \text{supp}(N)} R(g(x))$$

The collection of all such N_R forms a subbase for some fuzzy topology on \mathcal{F} , called fuzzy nearly compact regular open topology and it is denoted by F_{NR} .

Remark 2.1 If x_1 is a crisp fuzzy point and \mathcal{U} is any collection of fuzzy open sets on X with $x_1 \leq \text{sup}\{U : U \in \mathcal{U}\}$ then $1 \leq \text{sup}\{U : U \in \mathcal{U}\}(x)$ and for $y \neq x$, $0 \leq \text{sup}\{U : U \in \mathcal{U}\}(y)$. By definition of supremum, for any $\varepsilon > 0$, $\exists U_k \in \mathcal{U}$ such that $U_k(x) > 1 - \varepsilon$. Again for $y \neq x, U_k(y) \geq 0$, so that $(U_k + \varepsilon)(y) > 0$. Hence $\{U_k\}$ forms a finite subcollection of \mathcal{U} such that $x_1 < U_k + \varepsilon$. This proves that x_1 is fuzzy compact and hence fuzzy nearly compact too.

Remark 2.2 Here, in particular if we take $N = x_1$, we get $(x_1)_G(f) = Gf(x)$. Then the fuzzy topology generated by all $(x_1)_G$ is called fuzzy point fuzzy regular open (F_{PR}) topology.

Lemma 2.1 In a *fts* X ,

- (1) If x_α and y_β are any two fuzzy points with $x \neq y$ and A_1, B_1 are fuzzy open sets with $x_\alpha \in A_1$ and $y_\beta \in B_1$ and $A_1 q B_1$ then there exist fuzzy regular open sets A and B such that $x_\alpha \in A$, $y_\beta \in B$ and $A \bar{q} B$.
- (2) If x_α and x_β ($\alpha < \beta$) are any two fuzzy points and A_2, B_2 are fuzzy open sets with $x_\alpha \in A_2$, $x_\beta q B_2$ and $A_2 q B_2$ then there exist fuzzy regular open sets A and B such that $x_\alpha \in A$, $x_\beta q B$ and $A \bar{q} B$.

Proof. 1) Consider $A = \bar{A}_1^0$ and $B = \bar{B}_1^0$. $x_\alpha \in A_1 \Rightarrow \alpha \leq A_1(x)$. So, $\alpha \leq \bar{A}_1^0(x)$.

Hence $x_\alpha \in \bar{A}_1^0 = A$. Similarly, $y_\beta \in B_1 \Rightarrow y_\beta \in \bar{B}_1^0 = B$.

$$A_1 q B_1 \Rightarrow \forall x \in X, A_1(x) + B_1(x) \leq 1.$$

$$\Rightarrow \forall x \in X, A_1(x) \leq 1 - B_1(x).$$

$$\Rightarrow \forall x \in X, \bar{A}_1(x) \leq (1 - B_1)(x).$$

$$\Rightarrow \forall x \in X, \bar{A}_1^0(x) \leq \bar{A}_1(x) \leq (1 - B_1)(x).$$

$$\Rightarrow \forall x \in X, \bar{A}_1^0(x) + B_1(x) \leq 1.$$

$$\Rightarrow \forall x \in X, B_1(x) \leq 1 - \overline{A_1}^0(x).$$

As $\overline{A_1}^0$ is a fuzzy open set, by similar argument as above $\overline{B_1}^0(x) \leq \overline{B_1}(x) \leq 1 - \overline{A_1}^0(x), \forall x \in X$. Hence, $\overline{A_1}^0(x) + \overline{B_1}^0(x) \leq 1, \forall x \in X \Rightarrow \overline{A_1}^0 \overline{q} \overline{B_1}^0(x) \Rightarrow A \overline{q} B$. Hence, the result.

2) x_α and x_β ($\alpha < \beta$) are any two fuzzy points and A_2, B_2 are fuzzy open sets with $x_\alpha \in A_2, x_\beta q B_2$ and $A_2 q B_2, \alpha \leq A_2, \beta + B_2(x) > 1$ and $A_2(z) + B_2(z) \leq 1, \forall z \in X$. Choose $A = \overline{A_2}^0$ and $B = \overline{B_2}^0$. Then $\alpha \leq A_2(x) \leq \overline{A_2}^0(x) = A(x)$. So, $x_\alpha \in A$.

$$\beta + B(x) = \beta + \overline{B_2}^0(x) \geq \beta + B_2(x) > 1. \text{ So, } x_\beta q B \text{ and as in (1), } A \overline{q} B.$$

Theorem 2.1 Let (X, τ) and (Y, σ) be two *fts*, $\mathcal{F} \subseteq Y^X$, endowed with F_{NR} topology is fuzzy *GS*- T_2 when (Y, σ) is so.

Proof. Let f_λ and g_μ be two fuzzy points in \mathcal{F} .

Case (i): Suppose, $f \neq g$. Then $f(x) \neq g(x)$ for some $x \in X$. Now, $(f(x))_\lambda$ and $(g(x))_\mu$ be two fuzzy points in Y with $f(x) \neq g(x)$ and Y is fuzzy *GS*- T_2 , there exist fuzzy open sets U_1, V_1 with $(f(x))_\lambda \in U_1, (g(x))_\mu \in V_1$ and $U_1 q V_1$. By Lemma (2.1(1)), there exist fuzzy regular open sets U and V such that $(f(x))_\lambda \in U, (g(x))_\mu \in V$ and $U \overline{q} V$. So, $\lambda \leq U(f(x)), \mu \leq V(g(x))$ and $U \overline{q} V \Rightarrow \lambda \leq (x_1)_U(f), \mu \leq (x_1)_V(g)$ and $U \overline{q} V$. Hence, $f_\lambda \in (x_1)_U, g_\mu \in (x_1)_V$ and $U \overline{q} V$. To show $U \overline{q} V \Rightarrow (x_1)_U \overline{q} (x_1)_V$. $U \overline{q} V \Rightarrow \forall z \in X, U(z) + V(z) \leq 1$. Now, $\forall h \in \mathcal{F}$,

$$(x_1)_U(h) + (x_1)_V(h) = U(h(x)) + V(h(x)) \leq 1, \forall x \in X. \text{ Hence } (x_1)_U \overline{q} (x_1)_V.$$

Case (ii): Suppose $f = g$. with out loss of generality we assume $\lambda < \mu$. Then $f(x) = g(x), \forall x \in X$. $(f(x))_\lambda$ and $(g(x))_\mu$ with $\lambda < \mu$ are fuzzy points on Y with $f(x) = g(x)$. By fuzzy *GS*- T_2 ness of Y, \exists fuzzy open sets A_2, B_2 on Y such that $(f(x))_\lambda \in A_2, (g(x))_\mu q B_2$ and $A_2 q B_2$. By Lemma (2.1(2)), there exist fuzzy regular open sets A and B on Y such that $(f(x))_\lambda \in A, (g(x))_\mu q B$ and $A \overline{q} B \Rightarrow \lambda \leq A(f(x)), \mu + B(g(x)) > 1$ and $A(z) + B(z) \leq 1, \forall z \in X$.

$$\Rightarrow f_\lambda \in (x_1)_A, g_\mu(g) + (x_1)_B(g) > 1 \text{ and } A(z) + B(z) \leq 1, \forall z \in X.$$

$$\Rightarrow f_\lambda \in (x_1)_A, g_\mu q (x_1)_B \text{ and } A(z) + B(z) \leq 1, \forall z \in X.$$

Now, $\forall \psi \in \mathcal{F}, (x_1)_A(\psi) + (x_1)_B(\psi) = A(\psi(x)) + B(\psi(x)) \leq 1$. Hence $(x_1)_A \overline{q} (x_1)_B$. Hence, \mathcal{F} endowed with F_{NR} topology is fuzzy *GS*- T_2 .

As F_{PR} is a special case of F_{NR} topology, we have

Corollary 2.1 Let (X, τ) and (Y, σ) be two *fts*, $\mathcal{F} \subseteq Y^X$, endowed with F_{PR} topology is fuzzy $GS-T_2$ when (Y, σ) is fuzzy $GS-T_2$.

Lemma 2.2 Let (X, τ) and (Y, σ) be two *fts* and $\mathcal{F} \subset Y^X$ be endowed with fuzzy nearly compact regular open topology, F_{NR} , then $N_{\bar{F}} \geq \overline{N_F} \geq N_F$ where N is a crisp fuzzy point x_1 on X and F is fuzzy regular open on Y .

Proof. Suppose, $N = x_1$, for some $x \in X$. Then,

$$\begin{aligned} (1-(x_1)_F)(g) &= 1-(x_1)_F(g) \\ &= 1-F(g(x)) \\ &= \inf\{(1-F)(g(x)) : x \in \text{supp}(x_1)\} \\ &= N_{1-F}(g). \end{aligned}$$

Hence, $1-N_F = N_{1-F}$. Since $(x_1)_F$ is a member of F_{NR} , $1-(x_1)_F$ is fuzzy closed in \mathcal{F} . So, N_{1-F} is closed in \mathcal{F} . In particular if V is a fuzzy regular open set, \bar{V} is fuzzy regular closed in Y . Hence, $N_{\bar{V}} = N_{1-(1-\bar{V})} = 1-N_{1-\bar{V}}$. In other words, $N_{\bar{V}}$ is a fuzzy closed set in \mathcal{F} such that $N_V \leq N_{\bar{V}}$, as $V(f(x)) \leq \bar{V}(f(x))$. Consequently, $\overline{N_V} \leq N_{\bar{V}}$, as $\overline{N_V}$ is the smallest fuzzy closed set containing N_V .

Theorem 2.2 Let \mathcal{F} be endowed with F_{NR} topology, a fuction $e_x : \mathcal{F} \rightarrow Y$ defined by $e_x(f) = f(x)$ for all $f \in \mathcal{F}$, is fuzzy δ -continuous if Y is fuzzy almost regular.

Proof. Let f_λ be a fuzzy point on \mathcal{F} . As $e_x(f_\lambda)$ is a fuzzy set on Y , for any

$y \in Y$, $e_x(f_\lambda)(y) = \sup_{e_x(g)=y} [f_\lambda(g)]$

$$= \sup_{g(x)=y} [f_\lambda(g)] = \begin{cases} \lambda, & \text{for } f(x) = y \\ 0, & \text{otherwise} \end{cases}. \text{ Hence } e_x(f_\lambda) = (f(x))_\lambda. \text{ Let } (f(x))_\lambda \text{ be a}$$

fuzzy point on Y and U be a fuzzy regular open q -nhd of $(f(x))_\lambda$. As Y is fuzzy almost regular, there exist fuzzy regular open set V such that $(f(x))_\lambda q V$ and $\bar{V} \leq U$. Hence $\lambda + V(f(x)) > 1$ and $V \leq \bar{V} \leq U \Rightarrow 1 - \lambda < V(f(x)) \leq \bar{V}(f(x)) \leq U(f(x))$. $\Rightarrow (1 - f_\lambda)(f) < (x_1)_V(f) \leq (x_1)_{\bar{V}}(f) \leq (x_1)_U(f)$. Hence $\overline{(x_1)_V}^0 \leq \overline{(x_1)_V} \leq (x_1)_{\bar{V}} \leq (x_1)_U$ [using Lemma (2.2)]. Hence, $f_\lambda q \overline{(x_1)_V}^0$.

$$\text{Now, for each } y \in Y, e_x[\overline{(x_1)_V}^0](y) = \sup_{e_x(g)=y} [\overline{(x_1)_V}^0(g)]$$

$$\leq \sup_{e_x(g)=y} [(x_1)_{\bar{V}}(g)] = \sup_{e_x(g)=y} [\bar{V}(g)(x)] \leq U(y).$$

Hence $e_x[\overline{(x_1)_V}^0] \leq U$. So, $\overline{(x_1)_V}^0$ is fuzzy regular q -nhd of f_λ , as desired.

Theorem 2.3 If K_μ is a fuzzy open set in \mathcal{F} endowed with F_{NR} topology, then K_μ^- is a fuzzy δ -closed set in \mathcal{F} and $K_\mu \leq \overline{K_\mu} \leq \delta-cl(K_\mu) \leq K_\mu^-$.

Proof. Since μ is a fuzzy regular open set in Y , $\overline{\mu}$ is fuzzy regular closed and hence by fuzzy δ -continuity of $e_x: \mathcal{F} \rightarrow Y$ given by $e_x(f) = f(x), \forall x \in X$, $e_x^{-1}(\overline{\mu})$ is δ -closed in \mathcal{F} . Now, for all $f \in \mathcal{F}$, $K_\mu^-(f) = \inf\{\overline{\mu}(f(x)): x \in \text{supp}K\} = \inf\{e_x^{-1}(\overline{\mu})(f): x \in \text{supp}K\}$ i.e., $K_\mu^- = \inf\{e_x^{-1}(\overline{\mu}): x \in \text{supp}K\}$. Hence K_μ^- is fuzzy δ -closed set in \mathcal{F} . Again, $\forall x \in X$, $e_x^{-1}(\mu) \leq e_x^{-1}(\overline{\mu})$. Hence, $K_\mu \leq \overline{K_\mu} \leq \delta-cl(K_\mu) \leq K_\mu^-$.

Definition 2.2 A *fts* X is said to be fuzzy somewhat almost regular if for each fuzzy point x_α ($0 < \alpha < 1$) and any fuzzy regular open set A with $x_\alpha q A$, there exists a fuzzy regular open set B and γ with $0 < \alpha < \gamma < 1$ such that $x_\gamma q B$ and $\overline{B} \leq A$.

Theorem 2.4 Let (X, τ) and (Y, σ) be two *fts* and $\mathcal{F} \subset Y^X$ be endowed with F_{NR} topology. Then \mathcal{F} is fuzzy somewhat almost regular if Y is fuzzy almost regular.

Proof. Let (Y, σ) be fuzzy almost regular. Let g_λ be any fuzzy point in \mathcal{F} and N_R be a subbasic fuzzy open set in \mathcal{F} such that $g_\lambda q N_R$, where N is a fuzzy nearly compact on X and R is fuzzy regular open in Y . So,

$$\begin{aligned} g_\lambda q N_R &\Rightarrow g_\lambda(g) + N_R(g) > 1 \\ &\Rightarrow 1 - \lambda < \inf_{x \in \text{supp}(N)} R(g(x)) \quad (1) \end{aligned}$$

Then by (1), $(g(x))_\lambda q R \quad \forall x \in \text{supp}(N)$. By fuzzy almost regularity of Y there exist a fuzzy regular open set F in Y such that $(g(x))_\lambda q F$ and $\overline{F} \leq R$.

$$\begin{aligned} &\Rightarrow 1 - \lambda < F(g(x)) \leq \overline{F}(g(x)) \leq R(g(x)) \\ &\Rightarrow 1 - \lambda \leq N_F(g) \leq N_{\overline{F}}(g) \leq N_R(g) \end{aligned}$$

Choose any β such that $0 < \lambda < \beta < 1$.

Then $1 - \beta < 1 - \gamma$ and $1 - g_\beta(g) \leq N_F(g) \leq N_{\overline{F}}(g) \leq N_R(g)$. Hence, using Theorem(2.3) $1 - g_\beta(g) \leq N_F(g) \leq \overline{N_F}^0(g) \leq \overline{N_F}(g) \leq N_{\overline{F}}(g) \leq N_R(g)$. So, $g_\beta q \overline{N_F}^0$ and $\overline{N_F}^0 \leq N_R$.

Hence, (\mathcal{F}, F_{NR}) is fuzzy somewhat almost regular.

3. Jointly fuzzy δ -continuous fuzzy topologies on function spaces

Definition 3.1 Let (X, τ) and (Y, σ) be two fuzzy topological spaces and \mathcal{F} be a nonempty collection of functions from X to Y . A fuzzy topology on \mathcal{F} is said to be jointly fuzzy δ -continuous (jointly fuzzy δ -continuous on fuzzy near compacta) if a function $P: \mathcal{F} \times X \rightarrow Y$ given by $P(f, x) = f(x)$ is fuzzy δ -continuous (respectively, $P|_{\mathcal{F} \times \text{supp}(N)}$ is fuzzy δ -continuous for each fuzzy nearly compact set N on X).

Definition 3.2 Let (X, τ) and (Y, σ) be two fuzzy topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be fuzzy δ -continuous on a fuzzy nearly compact set N on X if for any fuzzy regular open set μ , $f|_{\text{supp}(N)}^{-1}(\mu)$ is fuzzy regular open on $\text{supp}(N)$.

Theorem 3.1 Each fuzzy topology on \mathcal{F} which is jointly fuzzy δ -continuous on fuzzy near compacta is larger than the fuzzy nearly compact regular open topology on \mathcal{F} .

Proof. Straight forward and hence omitted.

Theorem 3.2 If \mathcal{F} is endowed with the fuzzy topology which is jointly fuzzy δ -continuous then each $f \in \mathcal{F}$ is fuzzy δ -continuous.

Proof. Let Δ be a fuzzy topology on \mathcal{F} which is jointly fuzzy δ -continuous. Then P is fuzzy δ -continuous. Let x_β be any fuzzy point on X and V any fuzzy regular open *nhd* of $(f(x))_\beta$ on Y . Now,

$$(f_1 \times x_\beta)(h, t) = f_1(h) \wedge x_\beta(t) = \begin{cases} \beta & \text{if } h = f \text{ and } t = x \\ 0, & \text{otherwise} \end{cases}$$

Hence, $(f_1 \times x_\beta) = (f, x)_\beta$. So, $P(f_1 \times x_\beta) = P((f, x)_\beta) = (f(x))_\beta$. Using the fuzzy δ continuity of P , there exist fuzzy regular open *nhd*s. U_1 of f_1 on \mathcal{F} . and U_2 of x_β on X such that $P(U_1 \times U_2) \leq V$. Now, $P(U_1 \times U_2)(y) = \sup_{(h,t) \in P^{-1}(y)} [(U_1 \times U_2)(h, t)]$

$$\begin{aligned} \text{where } (h, t) \in \mathcal{F} \times X. \quad & P(U_1 \times U_2)(y) \geq \sup_{P(h,t)=y} [(f_1 \times U_2)(h, t)] \\ & = \sup_{h(t)=y} [(f_1 \times U_2)(h, t)] \\ & = \sup_{h(t)=y} [(f_1(h) \wedge U_2(t))] = \sup_{f(t)=y} [U_2(t)] = f(U_2)(y) \end{aligned}$$

Hence, $f(U_2) \leq P(U_1 \times U_2) \leq V$, as desired.

Definition 3.3 A family \mathcal{F} of functions from a *fts* X to a *fts* Y is said to be fuzzy δ -equicontinuous on fuzzy near compacta, if for each fuzzy nearly compact set N on X and any fuzzy regular open set V on Y with $(f(x))_\alpha q V$, for some $f \in \mathcal{F}$, $x \in \text{supp}(N)$, then there exist a fuzzy regular open set U on $\text{supp}(N)$ such that

$\forall h \in \mathcal{F}, h(U) < V$ and $t_\alpha q U, \forall t \in \text{supp}(N)$.

Theorem 3.3 Let (X, τ) and (Y, σ) be two *fts*. If \mathcal{F} is fuzzy δ -equicontinuous on fuzzy near compacta then \mathcal{F} endowed with F_{NR} topology is jointly fuzzy δ -continuous on fuzzy near compacta.

Proof. Let $(f, x)_\alpha$ be a fuzzy point on $\mathcal{F} \times \text{supp}(N)$ and V be any fuzzy regular open set on Y with $(P(f, x))_\alpha q V$ i.e $(f(x))_\alpha q V, x \in \text{supp}(N)$. Since \mathcal{F} is fuzzy δ -equicontinuous, there exist a fuzzy regular open set U on $\text{supp}(N)$ such that $\forall h \in \mathcal{F}, h(U) < V$ and $t_\alpha q U, \forall t \in \text{supp}(N)$. Now it is easy to see that $h(U) < V \Rightarrow U(z) < V(h(z)), \forall z \in X$. since $(N_{\bar{V}})^0$ and U are respectively fuzzy regular open in \mathcal{F} and $\text{supp}(N)$, we need to show that $(f, x)_\alpha q ((N_{\bar{V}})^0 \times U)$ and $P((N_{\bar{V}})^0 \times U) \leq V$. Now, $((N_{\bar{V}})^0 \times U)(f, x) + \alpha \geq (N_V(f) \wedge U(x)) + \alpha$

$$= [\inf_{t \in \text{supp}(N)} V(f(t)) \wedge U(x)] + \alpha$$

$$> [\inf_{t \in \text{supp}(N)} U(t) \wedge U(x)] + \alpha \geq (1 - \alpha) + \alpha$$

Hence, $(f, x)_\alpha q ((N_{\bar{V}})^0 \times U)$. Now, $P((N_{\bar{V}})^0 \times U)(y) = \sup_{P(h,t)=y} [((N_{\bar{V}})^0 \times U)(h,t)]$

$$\leq \sup_{P(h,t)=y} [(N_{\bar{V}} \times U)(h,t)]$$

$$= \sup_{h(t)=y} [\inf_{s \in \text{supp}N} \bar{V}(h(s)) \wedge U(t)]$$

$$< \sup_{h(t)=y} [\inf_{s \in \text{supp}N} \bar{V}(h(s)) \wedge V(h(t))]$$

$$\leq \sup_{h(t)=y} [V(h(t))] = V(y)$$

Hence, $P((N_{\bar{V}})^0 \times U) \leq V$.

References

- [1] **Chang, C. L.** : Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24(1968), 182-190.
- [2] **Ganguly, S. , Dutta, K.** : A new topology on function spaces, *J. Pure Math.*, 17(2000), 109-118
- [3] **Ganguly, S. , Saha, S.** : A note on δ continuity and δ connected sets in fuzzy set theory, *Simon stevin*, 62(1988), 127-141.
- [4] **Ganguly, S. , Saha, S.** : On separation axioms and $T, -$ fuzzy continuity, *Fuzzy sets and systems*, 16(1985), 265-275.
- [5] **Jagar, G.:** On fuzzy function spaces, *Internat J. Math. Math Sc.*, 22(4)(1999), 727-737
- [6] **Kohli, J. K. , Prasannan, A. R.** : Fuzzy topologies on function spaces, *Fuzzy*

- sets and systems*116(2000), 415-420.
- [7] **Lowen, R.** : Fuzzy Topological Spaces and Fuzzy Compactness, *Journal of Mathematical Analysis and Applications*,56,(1976), 621-633.
- [8] **Mukherjee, M.N. , Sinha, S.P.** : On some near-fuzzy continuous functions between fuzzy topological spaces, *Fuzzy sets and system*,34(1990),245-254.
- [9] **Pao-Ming, Pu and Ying-Ming, Liu** : Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence, *J. Math Anal. Appl.*,76(1980),571-599
- [10] **Peng, Y.** : Topological structure of a fuzzy function space-the point wise convergent topology and compact-open topology, *Kexue Tongbao(English Ed.)* , 29(3) (1984),289-292
- [11] **Zadeh, L.A.** : Fuzzy Sets, *Information and Control*,8(1965), 338-353.