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A New Structure and Construction of Q- Fuzzy Groups

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Abstract

In this paper, we fuzzify the new class of algebraic structures introduced by (5,9). In this fuzzification, we introduce the notion of Q- fuzzy groups (QFG) and investigate some of their related properties. The purpose of this study is to implement the fuzzy set theory and group theory in Q- fuzzy groups. This fuzzification leads to development of new notions over fuzzy groups. Characterisation of Q- fuzzy groups (QFCG) and normal Q- fuzzy groups (QFNG) are given.

Keywords: Q-fuzzyset,Q-fuzzygroupfamily, Level subset, Homomorphism, Q- fuzzy normal, Q- fuzzy characteristric.

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1. Introduction

The notion of fuzzy sets was first introduced by zadeh (10). Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics such as topological spaces, functional analysis, loop, group, ring, near- ring, vector spaces, automation. There have been wide ranging applications of the theory of fuzzy sets from the design of robots and computer simulation to Engineering and water resources planning. Rosenfeld (8) introduced the fuzzy sets in the realm of group theory.Recently, Davvaz at.al (2) considered intuitionistic fuzzification of the concept of the H_v- submodules in a H_v- module and Dudex at.al (1) consider the intuitionistic fuzzification of the concept of sub hyperquasigroups in a hyper quasigroup. The notion of an intuitionistic Q- fuzzy R- subgroups of a near rings is given by (6).The notion of intuitionistic Q- fuzzy semi primality in a semi group is given by kim(5). Also Rho.at.al (7) considered the intuitionistic Q- fuzzification of BCK / BCI algebra's. In this paper, we make some characterization of Q- fuzzy sets and then proved some results on Q- fuzzy groups.

2. Preliminaries

Definition2.1: A mapping μ: X→ [0,1], where X is an arbitrary non-empty set and is called a fuzzy set in X.

Definition2.2: Let G be any group. A mapping μ: G→ [0,1] is a fuzzy group if (FG1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ (FG2) $\mu(x^{-1}) = \mu(x)$ for all x,y ε G.

Definition 2.3: Let Q and G a set and a group respectively. A mapping μ: GxQ→ [0,1] is called Q – fuzzy set in G. For any Q- fuzzy set μ in G and t ε [0,1] we define the set $U(\mu; t) = \{x \varepsilon G / \mu(x,q) \geq t, q \varepsilon Q\}$ which is called an upper cut of 'μ' and can be used to the characterization of μ.

Definition 2.4: A Q- fuzzy set 'A' is called Q- fuzzy group of G if (QFG1) $A(xy,q) \geq \min\{A(x,q), A(y,q)\}$ (QFG2) $A(x^{-1},q) = A(x,q)$ for all x,y ε G and q ε Q.

3. Properties of Q- fuzzy groups

Proposition3.1: Let 'A' be a Q- fuzzy group of G then

(i) $A(x,q) \leq A(e,q)$ for all x εG and q ε Q.

(ii) The subset $G_A = \{x \varepsilon G / A(x,q) = A(e,q)\}$ is a Q- fuzzy group of G.

Proof: Let x be any element of G, then $A(x,q) = \min\{A(x,q), A(x,q)\} = \min\{A(x,q), A(x^{-1},q)\} \leq A(xx^{-1},q) = A(x,q)$ and (i) is proved. To prove (ii) we have e ε G_A, then $G_A \neq \Phi$. Now let x,y ε G_A and q ε Q.

$A(xy^{-1},q) \geq \min\{A(x,q), A(y^{-1},q)\} = \min\{A(x,q), A(e,q)\}=\min\{A(e,q). A(e,q)\} = A(e,q)$ but from (i) $A(xy^{-1},q) \leq A(e,q)$ for x,y ε G and q ε Q. Therefore $A(xy^{-1},q) = A(e,q)$ which means (xy^{-1},q) ε G_A and G_A is Q- fuzzy group of G.

Corollory 3.2: Let G be a finite group and A be a Q- fuzzy group of G. consider the subset H of G given by $H = \{x \varepsilon G / A(x,q) = A(e,q)\}$ then H is a crisp subgroup of G.

Proof: It is obivious.

Proposition 3.3: Let A and B be two Q- fuzzy groups of a group G then A∩B is Q- fuzzy group of G.

Proof: Let x,y ε G and q ε Q

(QFG1) $(A\cap B)(xy,q) = \min\{A(xy,q), B(xy,q)\}$
$\geq \min\{\min(A(x,q), A(y,q), \min\{B(x,q), B(y,q)\}$
$= \min\{\min\{A(x,q), B(x,q)\}, \min\{A(y,q), B(y,q)\}$
$= \min\{(A\cap B)(x,q), (A\cap B)(y,q)\}$.

(QFG2) $(A\cap B)(x^{-1},q) = \min\{A(x^{-1},q), B(x^{-1},q)\}$
$= \min\{A(x,q), B(x,q)\}$
$= (A\cap B)(x,q)$.

Proposition3.4: If 'A' is a Q- fuzzy group of G, then A^C is also Q- fuzzy group of G.

Proof: for any x,y ε G, q ε Q
(QFG1) $A^C(xy,q) = 1- A(xy,q)$
$\leq 1-\min\{A(x,q), A(y,q)\}$
$= \max\{1- A(x,q), 1- A(y,q)\}$
$= \max\{A^C(x,q), A^C(y,q)\}$

(QFG2) $A^C(x^{-1},q) = 1- A(x^{-1},q)$
$= 1- A(x,q) = A^C(x,q)$.

Proposition 3.5: If 'A' is Q- fuzzy group of G, then the set U(A; t) is also Q- fuzzy group for all q ε Q, t ε Im(A).

Proof: Let t ε Im(A) C [0,1] and let x,y ε U(A; t), q ε Q then $A(x,q) \geq t$, $A(y,q) \geq t$. since A is Q- fuzzy group of G we have
$A(xy,q) \geq \min\{A(x,q), A(y,q)\} \geq t$ hence xy ε U(A; t). Let x ε U(A; t) and q ε Q then $A(x^{-1},q) = A(x,q) \geq t$ which implies x^{-1} ε U(A ; t) therefore U (A; t) is Q – fuzzy group of G.

Definition 3.6: Let 'θ' be a mapping from X to Y. If A and B are Q- fuzzy sets in X and Y resply, then the inverse image of B under θ denoted by $\theta^{-1}(B)$ is Q- fuzzy set in X defined by $\theta^{-1}(B) = \mu_\theta{}^{-1}(B)$ where $\mu_\theta{}^{-1}{}_{(B)}(x,q) = \mu_B\theta(x,q)$ and $\mu_\theta{}^{-1}{}_{(B)}(x^{-1},q) = \mu_B(\theta(x, q)$ for all x ε Y, q ε Q and the image of A under θ denoted by θ(A), where
$\mu_{\theta(A)}(y,q) = \{V_{x\varepsilon\theta^{-1}(y)}\, \mu_A(x,q)$, if $\theta^{-1}(y) \neq 0$
0 otherwise for all y ε Y, q ε Q.

Proposition 3.7: Let G and G^1 be two groups and $\theta: G\to G^1$ a homomorphisms. If 'B' is Q – fuzzy group of G^1 then the pre image $\theta^{-1}(B)$ is Q- fuzzy group of G.

Proof: Assume that B is Q- fuzzy group of G^1 and let x,y ε G, q ε Q.
(QFG1) $\mu_\theta{}^{-1}{}_{(B)}(xy,q) = \mu_B(\theta(xy,q))$
$= \mu_B(\theta(x)\,\theta(y), q)$
$\geq \min\{\mu_B(\theta(x),q), \mu_B(\theta(y), q)\}$
$= \min\{\mu_\theta{}^{-1}{}_{(B)}(x,q), \mu_\theta{}^{-1}{}_{(B)}(y,q)\}$ and

(QFG2) $\mu_{\theta^{-1}(B)}(x^{-1},q) = \mu_B(\theta(x^{-1},q))$
$= \mu_B(\theta x^{-1},q)$
$= \mu_B(\theta x,q) = \mu_{\theta^{-1}(B)}(x,q)$ therefore $\theta^{-1}(B)$ is Q- fuzzy group of G.

Proposition 3.8: Let $\theta: G \to G^1$ be an epimorphisms and let 'B' is ssQ – fuzzy set in G^1. If $\theta^{-1}(B)$ is Q- fuzzy group of G then 'B' is Q- fuzzy group of G^1.

Proof: Let x, y, s ε G^1, q ε Q, then there exists a, b ε G such that θ(a) = x, θ(b) = y. It follows that

(QFG1) $\mu_B(xy,q) = \mu_B(\theta(a)\theta(b), q)$
$= \mu_B(\theta(ab), q)$
$= \mu_{\theta^{-1}(B)}(ab,q)$
$\geq \min\{\mu_{\theta^{-1}(B)}(a,q), \mu_{\theta^{-1}(B)}(b,q)\}$
$\geq \min\{\mu_B(\theta(a), q), \mu_B(\theta(b), q)\}$
$\geq \min\{\mu_B(x,q), \mu_B(y,q)\}$

(QFG2) $\mu_B(x^{-1},q) = \mu(\theta(a)^{-1}, q)$
$= \mu_B(\theta(a^{-1}), q)$
$= \mu_{\theta\text{-}1(B)}(a^{-1},q)$
$= \mu_{\theta\text{-}1(B)}(a, q)$
$= \mu_B(\theta(a),q) = \mu_B(x, q)$

Proposition 3.9: I f $\{A_i\}_{i\varepsilon A}$ is a family of Q- fuzzy groups of G, then $\bigcap_{i\varepsilon A} A_i$ is Q- fuzzy groups of G where $\bigcap_{i\varepsilon A} A_i = \{((x,q), \Lambda\, \mu_{Ai}(x,q)) / x \,\varepsilon\, G, q \,\varepsilon\, Q\}$

Proof: Let x,y ε G, q ε Q, then

(QFG1) $(\bigcap_{i\varepsilon A}\mu_{Ai})(xy,q) = \Lambda_{i\varepsilon A}\, \mu_{Ai}(xy,q) \geq \Lambda_{i\varepsilon A} \min\{\mu_{Ai}(x,q), \mu_{Ai}(y,q)\}$
$= \min\{(\Lambda_{i\varepsilon A}\, \mu_{Ai}(x,q)), (\Lambda_{i\varepsilon A}\mu_{Ai}(y,q))\}$
$= \min\{(\bigcap_{i\varepsilon A}\mu_{Ai})(x,q), (\bigcap_{i\varepsilon A}\mu_{Ai})(y,q)$

(QFG2) Let x ε G and q ε Q, then

$(\bigcap_{i\varepsilon A}\mu_{Ai})(x^{-1},q) = \Lambda_{i\varepsilon A}\mu_{Ai}(x^{-1},q) \geq \Lambda_{i\varepsilon A}\mu_{Ai}(x,q) = (\bigcap_{i\varepsilon A}\mu_{Ai})(x,q)$

hence $\bigcap_{i\varepsilon A} A_i$ is Q- fuzzy group of G.

Proposition 3.10: If 'A' is Q- fuzzy set in G such that all non- empty level subset U(A; t) is Q- fuzzy group of G then A is Q- fuzzy group of G.

Proof: Assume that the non-empty level set U(A; t) is Q- fuzzy group of G. If min {A(x,q), A(y,q)} and for x,y ε G, q ε Q, then x,y ε U(A; t_0) hence A(xy,q) $\geq t_0$ = min {A(x,q), A(y,q)} which implies that the condition (QFG1) is valid. For x^{-1} ε G and q ε Q, then x^{-1} ε U(A; t_0) hence A(x^{-1},q) = t_0 = A(x,q) which implies that the condition (QFG2) is valid and therefore A is Q- fuzzy group of G.

Proposition 3.11: A set of necessary and sufficient conditions for a Q- fuzzy set of a group G to be a Q- fuzzy group of G is that

A(xy^{-1}, q) ≥ min (A(x,q), A(y,q)} for all x,y in G and q in Q.

Proof: Let A be a Q- fuzzy group of G. Then

A(xy^{-1},q) ≥ min{A(x,q), A(y^{-1},q)}= min {A(x,q), A(y,q)} for x,y ε G and q ε Q.

For the converse part suppose that A be a Q- fuzzy set of the group G of which e is the identity element.

Now A(yy^{-1},q) ≥ min {A(y,q), A(y,q)}

Or A(e,q) ≥ A(y,q)

Now A(ey^{-1},q) ≥ min {A(e,q), A(y,q)}

Or A(y^{-1},q) ≥ A(y,q) also A(xy,q) ≥ min {A(x,q), A(y^{-1},q)} ≥ min{A(x,q), A(y,q)}.

4. Characteristric of Q- fuzzy normal group

Our work in this section is to define Q- fuzzy characteristric group(QFCG) and to study their properties. For this first of all we define the notions of μ_A^θ which will be useful in our discussion.

Definition 4.1: Let 'μ_A' be a Q- fuzzy set of G. Let θ: GxQ → G be a map and define the map μ_A^θ: GxQ → [0,1] by $\mu_A^\theta(x,q) = \mu_A(\theta(x,q))$.

Definition 4.2: A Q- fuzzy group 'A' of G is called Q- fuzzy characteristric of G if $\mu_A^\theta = \mu_A$.

Definition 4.3: A Q- fuzzy group 'A' of G is said to be normal if there exists x ε G and q ε Q such that A(x,q) = 1. Note that if μ is normal Q- fuzzy group of G, then A(e,q) = 1 and hence A is normal if and only if A(e,q) = 1.

Definition 4.4: Let 'A' be a Q- fuzzy group of G. Then A is called Q- fuzzy normal group (QFNG) if for all x, y ε G, A(xy,q) = A(yx,q), qεQ.Alternatively, we can say that a Q- fuzzy group A is said to be Q- fuzzy normal if A(x,q) = A(yxy^{-1},q) for x,y ε G and q ε Q. we shall use the notation [x,y] to stand for the expression $x^{-1}y^{-1}xy$. We now prove the following propositions.

Proposition 4.5:If 'A' is Q- fuzzy group of G and θ is a homomorphism of G then the Q- fuzzy set A^θ of G given by A^θ = {< (x,q), μ_A^θ (x,q) > ; xε G, q ε Q} is Q- fuzzy group of G.

Proof: Let x,y ε G, then

(QFG1) $\mu_A^\theta(xy,q)$ $= \mu_A(\theta(xy,q))$
$= \mu_A(\theta(x)\theta(y), q)$
$\geq \min\{\mu_A(\theta(x),q), \mu_A(\theta(y),q)\}$
$= \min\{\mu_A^\theta(x,q), \mu_A^\theta(y,q)\}$

(QFG2) $\mu_A^\theta(x^{-1},q)$ $= \mu_A(\theta(x^{-1},q))$
$= \mu_A(\theta(x^{-1}), q)$
$= \mu_A^\theta(x,q)$ Thus A^θ is Q- fuzzy group of G.

Proposition 4.6: Let 'A' be Q – fuzzy group of G. Let A^+ be a Q- fuzzy set in G defined by $A^+(x,q) = A(x,q) + 1 - A(e,q)$ for all x ε G, then A^+ is normal Q- fuzzy group of G which contains A.

Proof: for any x, y ε G, we have $A^+(x,q) = A(x,q) + 1 - A(e,q) = 1$

(QFG1) $A^+(xy,q)$ $= A(xy,q) + 1 - A(e,q)$
$\geq \min\{A(x,q), A(y,q)\} + 1 - A(e,q)$
$= \min\{A(x,q) + 1 - A(e,q), A(y,q) + 1 - A(x,q)\}$
$= \min\{A^+(x,q), A^+(y,q)\}$

(QFG2) $A^+(x^{-1},q)$ $= A(x^{-1},q) + 1 - A(x,q)$
$= A(x,q) + 1 - A(e,q)$
$= A^+(x,q)$ hence A^+ is normal Q- fuzzy group of G. clearly A CA^+.

Proposition 4.7: Let 'A' be a QFNG of a group G. Then for all x,y ε G, A([x,y], q) = A(e,q).

Proof: since A is QFNG of G, we have $A(x,q) = A(yxy^{-1},q)$, for all x,y ε G, q ε Q.
Replacing x by x^{-1} and y by y^{-1}, we get $A(y^{-1},q) = A(x^{-1}y^{-1}xy, q)$
Or $A(x^{-1}y^{-1}xyy^{-1}, q) = A(y^{-1},q)$
Or $A([x,y]y^{-1},q) = A(y^{-1},q)$
Or A([x,y],q) = A(e,q).

Proposition 4.8: If 'A' is QFCG of a group G, then A is QFNG of G.

Proof: Let x,y ε G. consider the map θ: G → G given by $\theta(x,q) = a^{-1}xa$, for all a ε G, q ε Q. clearly θ is an automorphism of G.

Now $\mu_A(xy,q)$ $= \mu_A^\theta(xy,q)$
$= \mu_A(\theta(xy, q)$
$=\mu(a^{-1}xya) = \mu_A(a^{-1}yxa) = \mu_A(yx,q)$. Therefore A is QFNG of G.

Conclusion

Group theory has vast and potential applications in many core areas like physics, chemistry,communications, coding theory, computer science etc. In this paper, we have studied the concept of Q- fuzzy groups and their properties. We have also proved a result on classical groups with the help of Q- fuzzy group theory.

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