Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 3, Number 1 (2008), pp. 27–42
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http://www.ripublication.com/afm.htm

Intutionistic Fuzzy δ-Continuity and $(\lambda_1,\lambda_2)-$near Compactness in Intutionistic Fuzzy Topological Spaces

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Abstract

The concept of a fuzzy δ-continuity in fuzzy topological spaces was introduced by Soha [20] and (Allam and Zahran [1]). In [14, 16, 18] the notion of nearly compactness has been introduced. In this paper, we introduce the notion of fuzzy δ-continuity in intutionistic fuzzy topological spaces (IFTS$^{'}$ s, for short) and give a characterization of such notion. Also, we introduce the notion of (λ_1,λ_2)-near compact which depends on the notion of (λ_1,λ_2)-shading . Finally, we investigate the image of (λ_1,λ_2)-nearly compact spaces under some types of continuous functions.

Keywords: Fuzzy topological spaces, intutionistic fuzzy sets, intutionistic fuzzy topological spaces, $\delta-$ continuity, near compactness.

1. Introduction

In [2,3,5], Atanassov introduced the fundamental concepts of intutionistic fuzzy sets. Latter this concepts was generalized to intutionistic L-fuzzy sets by [4]. Coker [9], introduced the notion IFTS$^{'}$s.

In this paper, we introduce the notion of δ-continuity between IFTS$^{'}$s. We define the new notion of (λ_1,λ_2)-shading of X and introduce the notion of (λ_1,λ_2)-near compact and study some of its properties. We investigate the image of (λ_1,λ_2)-nearly compact under some types of intutionistic fuzzy continuous functions.

2. Preliminaries

In this section we introduce some concepts which will be needed in the sequel. For more information see [8, 9,10,11,12,15].

Definition 2.1. Let X be a non-empty fixed set. An intutionistic fuzzy set (IFS, for short) is an object of the form

$$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$$

where $\mu_A : X\to I=[0,1]$ and $\gamma_A : X\to I=[0,1]$ denote the membership (non-membership) functions of A and $\mu_A(x)(\gamma_A(x))$ denote the degree (non-degree) of membership of each element $x\in X$ to the set A such that $0\le\mu_A(x)+\gamma_A(x)\le 1$.

Remark 2.2 [8]. (a) An intuitionistic fuzzy set $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ in X can be identified to an ordered pair $\langle\mu_A,\gamma_A\rangle$ in $I^X\times I^X$ or to an element in $(I\times I)^X$.

(b) For the sake of simplicity, we use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$ for the intuitionistic fuzzy set $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$.

(c) Every fuzzy set A on a nonempty set X is obviously an IFS having the form $A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$.

Definition 2.3. Let $X\neq\phi$, A and B be IFS$^{'}$s in the form

$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$

Then:

(a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$ $\forall x\in X$.

(b) $A^c=\{\langle x,\gamma_A(x),\mu_A(x) : x\in X\rangle\}$ is the complement of A.

(c) $A=B$ *iff* $A\subseteq B$ and $B\subseteq A$.

(d) $A\bigcap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\ \gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$.

(e) $A\bigcup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\ \gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$.

Definition 2.4 . Let $\{A_i : i\in J\}$ be an arbitrary family of IFS$^{'}$s in X. Then:

(a) $\bigcap A_i=\{\langle x,\wedge\mu_{A_i}(x),\vee\gamma_{A_i}(x)\rangle : x\in X\}$.

(b) $\bigcup A_i=\{\langle x,\vee\mu_{A_i}(x),\wedge\gamma_{A_i}(x)\rangle : x\in X\}$.

Definition 2.5. $\underset{\sim}{0}=\{\langle x,0,1\rangle : x\in X\}$ and $\underset{\sim}{1}=\{\langle x,1,0\rangle : x\in X\}$.

Definition 2,6. Let A,B,C,D and $\{A_i : i\in J\}$ be IFS$^{'}$s in X. Then:

(a) $(\bigcup A_i)^c = \bigcap A_i^c$ and $(\bigcap A_i)^c = \bigcup A_i^c$.

(b) $A \subseteq B$ implies $B^c \subseteq A^c$.

(c) $\left(A^c\right)^c = A$.

(d) $(\underset{\sim}{\varphi})^c = \underset{\sim}{1}$ and $(\underset{\sim}{1})^c = \underset{\sim}{\varphi}$.

Definition 2.7. Let X and Y be two non-empty sets and $f : X \to Y$ be a function. If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by:

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\lambda_B)(x)\rangle : x \in X\}.$$

Also, if $A = \{x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ is a IFS in X, then the image $f(A)$ is the IFS in Y defined by:

$$f(A) = \{y, f(\mu_A)(y), \underset{\sim}{f}(\gamma_A)(y)\rangle : y \in Y\}$$

where, $\underset{\sim}{f}(\gamma_A)(y) = 1 - f(1-\gamma_A)$.

Theorem 2.8. Let A, $A_i\,(i \in J)$ be IFS's in X, B, $B_i\,(i \in K)$ IFS's in Y and $f : X \to Y$. Then:

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$.

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(c) $A \subseteq f^{-1}f(A)$ (if f is 1-1, then $A = f^{-1}f(A)$).

(d) $ff^{-1}(B) \subseteq B$ (if f is onto, then $ff^{-1}(B) = B$).

(e) $f^{-1}(\bigcup B_i) = \bigcup f^{-1}(B_i)$.

(f) $f\ (\bigcup A_i) = \bigcup f(A_i)$.

(g) $f^{-1}(\bigcap B_i) = \bigcap f^{-1}(B_i)$.

(h) $f\ (\bigcap A_i) \subseteq \bigcap f(A_i)$ (if f is 1-1, then $f\ (\bigcap A_i) = \bigcap f(A_i)$).

(i) $f^{-1}(\underset{\sim}{1}) = \underset{\sim}{1}$ and $f^{-1}(\underset{\sim}{\varphi}) = \underset{\sim}{\varphi}$. If f is onto, then

$f(\underset{\sim}{1}) = \underset{\sim}{1}$ *and* $f(\underset{\sim}{\varphi}) = \underset{\sim}{\varphi}$.

(j) If f is onto, then $f(A^c) \supseteq (f(A))^c$.

(k) $f^{-1}(B^c) = (f^{-1}(B))^c$.

Definition 2.9. An IFTS on a non-empty set X is a family τ of IFS's in X satisfying the following axioms:

(a) $\underset{\sim}{0}, \underset{\sim}{1} \in \tau$.

(b) $G_1 \bigcap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(c) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case (X, τ) is called IFTS and each element of τ is called intutionistic fuzzy open set (IFO, for short) and its complement is said to be intutionistic fuzzy closed set (IFC, for short).

Definition 2.10. Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \gamma_A \rangle$ be IFS in X. Then, the fuzzy closure and fuzzy interior of A defined by:

$Cl(A) = \bigcap\{K : K \ is\ IFCS,\ K \supseteq A\}.$

$Int(A) = \bigcup\{G : G \ is\ IFOS,\ G \subseteq A\}.$

It can be also shown that $Cl(A)$ is IFC set and $Int(A)$ is IFO set in X . Also,

(a) A is IFCS if and only if $A = Cl(A)$.

(b) A is IFOS if and only if $A = Int(A)$.

Definition 2.11. Let $f : (X, \tau) \to (Y, \varphi)$ be a function. Then

(a) f is said be fuzzy continuous if, the preimage of each IFS in φ is an IFS in τ.

(b) f is said to be fuzzy open if and only if the image of each IFS in τ is an IFS in φ.

Definition 2.12. Let (X, τ) be an IFTS. Then

(a) If a family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle : i \in J\}$ of IFO sets in X satisfies the condition $\bigcup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle : i \in J\} = \underset{\sim}{1}$, then it is called an open cover of $\underset{\sim}{X}$.

(b) An IFTS (X, τ) is said to be compact if every open cover of $\underset{\sim}{X}$ has a finite subcover.

Definition 2.13. Let A be IFS in X. Then

(a) A is said to be regular open iff $Int(Cl(A)) = A$

(b) A is said to be regular closed iff $Cl(Int(A)) = A$.

The following definition is a natural generalization of fuzzy points given by Pu-Liu [19].

Definition 2.14 [11, 12]. Let c be a fixed point in X, $\alpha, \beta \in I$ and

($\alpha \in (0,1]$, $\beta \in [0,1)$) such that $\alpha+\beta \leq 1$. The intutionistic fuzzy point (IFP, for short) is of the form $c(\alpha,\beta)=\langle x, c_\alpha, 1-c_{1-\beta}\rangle$ where α denotes the degree of membership of $c(\alpha,\beta)$, β the degree of non-membership of $c(\alpha,\beta)$ and $c \in X$ is the support of $c(\alpha,\beta)$.

Remark 2.15 [12]. (a) IFP's in X sometimes be inconvenient when we express an IFS in X in terms of IFP's . This situation will occur if an IFS A contains some points $x \in X$ such that $\mu_A(x)=0$ and $\gamma_A(x)\in[0,1)$. Therefore, we define vanishing IFP as follows:

(b) Let $X \neq \phi, c \in X$ be fixed element in X . If $\beta \in [0,1)$, then the IFS $c(\beta)=\langle x, 0, 1-c_{1-\beta}\rangle$ is called vanishing IFP (VIVP, for short) in X , where β denotes the degree of nonmembership of $c(\beta)$.

The point $c(\alpha,\beta)$ is said to be quasi-coincident with A, in symbol $c(\alpha,\beta)\,q\,A$ if $\alpha > \gamma_A(c)$ or $\beta < \mu_A(c)$.

Also, two IFS's A and B are said to be quasi-coincident and denoted by AqB if and only if there is $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

3. Neighborhood of IFP's

In this section we introduce the notion of Q-neighbourhood (Q-nbd, for short) in IFTS's by using the relation of quasi-coincident [19] and we study some of its properties.

Definition 3.1. Let (X,τ) be an IFTS and A be IFS. Then A is said to be Q-neighbourhood (Q-nbd, for short) of $c(\alpha,\beta)$ if there is an IFO set G such that $c(\alpha,\beta)\,q\,G \subseteq A$.

It is easy to prove the following lemma.

Lemma 3.2. Let A and B be an IFS's. Then

(a) $A\overline{q}B \text{ iff } A \subseteq B^c$.

(b) $A \subseteq B \text{ iff } c(\alpha,\beta)\,q\,B$ for each $c(\alpha,\beta)$, $c(\alpha,\beta)\,q\,A$.

(c) $c(\alpha,\beta)\ q(\bigcup A_i)$ iff there is $i_0 \in J$ such that $c(\alpha,\beta)\ q\,A_{i_0}$.

Proposition 3.3. Let (X,τ) be an IFTS and $V \in \tau$. Then, $V\,q\,A \Leftrightarrow V\,q\,Cl(A).$ for each IFS A.

Proof: First, we prove that $V\,\overline{q}\,A \Rightarrow V\,\overline{q}\,Cl(A).$ Let $V\,\overline{q}\,A$. Then $A \subseteq V^c$ and this implies $Cl(A) \subseteq V^c$, that is $V\,\overline{q}\ Cl(A).$ On the other hand if $V\,\overline{q}\ Cl(A).$, we have $V\,\overline{q}\ A.$

Lemma 3.4. Let f be a function from X into Y and $c(\alpha,\beta)$ be IFP in X. Then:

(a) If B is IFS in Y and $f(c(\alpha,\beta)) = f(c)(\alpha,\beta)\, q\, B,$ then $c(\alpha,\beta)\, q\, f^{-1}(B).$

(b) If A is IFS in X and $c(\alpha,,\beta)\, q\, A$, then $f(c(\alpha,\beta))\, q\, f(A)$.

Proof: (a) Let $f(c(\alpha,\beta)) = f(c)(\alpha,\beta)\, q\, B.$ Then $\alpha > \gamma_B(f(c))$ or $\beta < \mu_B(f(c))$. Next, $\alpha > f^{-1}(\gamma_B)(c)$ or $\beta < f^{-1}(\mu_B)(c)$. This implies $c(\alpha,\beta)\, q\, f^{-1}(B) = \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle$.

(b) Let $f(c(\alpha,\beta))\, \overline{q}\, f(A)$. Then $f(c)(\alpha,\beta)\, \overline{q}\, \langle y, f(\mu_A), 1 - f(1-\gamma_A)\rangle.$ So,

$\alpha \le [1 - f(1-\gamma_A)](f(c))$ and $\beta \ge f(\mu_A)(f(c))$. Then, $\beta \ge f^{-1}[f(\mu_A)](c) \ge \mu_A(c)$, that is $\beta \ge \mu_A(c)$. Also,

$$\begin{aligned}\alpha \le [1 - f(1-\gamma_A)](f(c)) &\le f^{-1}[1 - f(1-\gamma_A)](c)\\ &= [f^{-1}(f(1-\gamma_A)]^c(c)\\ &\le (1-\gamma_A)^c(c)\\ &= \gamma_A(c)\end{aligned}$$

Finally we have, $\alpha \le \gamma_A(c)$ and $\beta \ge \mu_A(c)$, that is $c(\alpha,\beta)\, \overline{q}\, A$ which contradicts the assumption.

Definition 3.5. Let $f : X \to Y$ be a function from IFTS X into IFTS Y. Then:

(a) f is said to be fuzzy almost continuous if $f^{-1}(V)$ is IFO set in X for each intutionistic fuzzy regular open (IFR, for short) open set V in Y [8].

(b) f is said to be fuzzy almost open if $f^{-1}(Cl(U)) \le Cl(f^{-1}(U))$ for each IFO set U in Y.

(c) f is said to be fuzzy weakly continuous if $f^{-1}(V) \le Int(f^{-1}Cl(V))$ for each IFO set V in Y [8].

(d) f is said to be fuzzy almost open S if $f(V)$ is IFO set in Y for each IFO set V in X.

Lemma 3.6. If $f : X \to Y$ is fuzzy weakly continuous, then

$f^{-1}(Int(A)) \le Int(f^{-1}(A))$

for each IFC set A in Y.

Proof: Let A be IFC set in Y. Since $Int(A)$ is an IFO set in Y, then by definition 3.5 (c), $f^{-1}(Int(A)) \le Int(f^{-1}(Cl(Int(A)))$. Hence $f^{-1}(Int(A)) \le Int(f^{-1}(Cl(A)) = Int(f^{-1}(A)$ (Since A in IFC set).

4. Fuzzy $\delta-$closed sets and fuzzy $\delta-$open sets.

In this section we introduce the notion of IF-$\delta-$open sets and IF-$\delta-$closed sets by using the quasi-coincident relation [19] and pseudo-quasi coincident relation [12].

Note that the point $c(\alpha,\beta)$ in X is said to be pseudo-coincident with the IFS $A=\langle x,\mu_A,\gamma_A\rangle$, denoted by $c(\alpha,\beta)\,\mathrm{p}\,A$, iff $\alpha\geq\gamma_A(c)$ or $\beta\leq\mu_A(c)$. Also, two IFS's A and B in X are said to be pseudo-coincident, denoted by $A\,p\,B$ iff there exists $x\in X$ such that $\mu_A(x)\geq\gamma_B(x)$ or $\gamma_A(x)\leq\mu_B(x)$ [12].

Definition 4.1. (a) An IFS A of an IFTS X is said to be RQ-nbd of an IFP $c(\alpha,\beta)$ if and only if there is a IFR open set B such that $c(\alpha,\beta)\ \overline{q}\,B\subseteq A$.

(b) A is said to be RP-nbd of a vanishing point $c(\beta)$ if and only if there is a IFR open set B such that $c(\beta)\,p\,B\subset A$.

Definition 4.2. (a) An IFP $c(\alpha,\beta)$ is said to be fuzzy Q$\delta-$cluster point of a IFS A in an IFTS X if and only if each IFR open Q-nbd of $c(\alpha,\beta)$ is quasi-coincident with A.

(b) $c(\beta)$ is said to be $P\delta-$cluster point of a IFS A in an IFTS X if and only if each IFR open P-nbd of $c(\beta)$ is pseudo-coincident with A.

(c) The union of all Q$\delta-$cluster point and P$\delta-$cluster point of A is denoted by $\delta-Cl(A)$. If $A=\delta-Cl(A)$, then A is called a IF-$\delta-$closed. The complement of a IF-$\delta-$closed is called IF-$\delta-$open.

It is easy to see that $\delta..Cl(A)\geq Cl(A)$ for each IFS A in IFTS X. For, if $c(\alpha,\beta)\leq Cl(A)$ and V be IFR open Q-nbd of $c(\alpha,\beta)$, then it is IFO Q-nbd of $c(\alpha,\beta)$ such that $V\,q\,A$. Hence $c(\alpha,\beta)\leq\delta.Cl(A)$ and there fore $\delta..Cl(A)\geq Cl(A)$.

Theorem 4.3. An IFS U is $\delta-$open if and only if:

(a) for each IFP $c(\alpha,\beta)$ with $c(\alpha,\beta)\,q\,U$, there is IFR open set V in X such that $c(\alpha,\beta)\ q\ V\subseteq U$.

(b) for each IVP $c(\beta)$ with $c(\beta)\,p\,U$ there is IFR open set V in X such that $c(\beta)\,pV\subset U$.

Proof: $(\Rightarrow)$: Let U be IF-$\delta-$open set such that $c(\alpha,\beta)\,q\,U$ (resp. $c(\beta)\,p\,U$). Then $c(\alpha,\beta)\not\leq U^c$ $(c(\beta)\notin U^c)$ (see [12]). Since U^c is $\delta-$closed, there is IFR open Q-nbd V of $c(\alpha,\beta)$ (resp. P-nbd of $c(\beta)$) such that $c(\alpha,\beta)\,q\,V\,(c(\beta)\,p\,V)$ and $V\,\overline{q}\,U^c$ $(V\,\overline{p}U^c)$. Hence is $V\subseteq U$ (resp. $V\subset U$).

Conversely, let U be any IFS such that $c(\alpha,\beta)\,q\,U$ (resp. $c(\beta)\,p\,U$). Then there is IFR open Q-nbd (resp. p.nbd) V of $c(\alpha,\beta)(resp.\,c(\beta))$ such that $V\subseteq U$ (resp. $V\subset U$) Hence $c(\alpha,\beta)\not\leq U^c(resp.\,c(\alpha,\beta)\notin U^c)$. Therefore $V\,\overline{q}U^c$ (resp. $V\,\overline{p}\,U^c$.

So, $c(\alpha,\beta)\not\leq \delta.Cl(U^c)(resp.\ c(\beta)\notin \delta.Cl(U^c)$. Hence $\delta.Cl(U^c)\subseteq U^c\,(resp. \delta.Cl(U^c)\subset U^c)$. This implies that U^c is $\delta-$closed and thereby U is $\delta-$open.

Theorem 4.4. (a) Each IFR closed (IFR open) set is IF-$\delta-$closed (IF-$\delta-$open) set.

(b) Each IF-$\delta-$closed (IF-$\delta-$open) set is IF-closed (IF-open) set.

Proof: (a) Let A be IFR closed set and $c(\alpha,\mathrm{B})\not\leq A$ (resp. $c(\beta)\notin A)$. Since A is IFR closed , $Cl(Int(A))=A$. Then, there exits $Q-nbd$ (resp. p.nbd) V of $c(\alpha,\beta)$ (resp. $c(\beta)$) such that $Int(A)\ \overline{q}\ V$ ($Int(A)\ \overline{p}\ V)$. By proposition 3.3 $Int(A)\,\overline{q}\,Cl(V)\,(resp.\ Int(A)\,\overline{p}\,Cl(V)),\ Cl(V)\supseteq Cl(In(V))$. Then $Int(A)\ \overline{q}\,Int(Cl\,(V)(resp.\ Int(A)\,\overline{p}\ Int(Cl(V))$ and $A\ \overline{q}\ Int(Cl(V))=U\,(resp.\ A\ \overline{p}\ \ Int(Cl(V)))=U)$. Since U is IR-open , $c(\alpha,\mathrm{B})\not\leq\delta.Cl(A)\,(c(\beta)\notin \delta.Cl(A))$. Then $\delta.Cl(A)\subseteq A(resp. \delta.Cl(A)\subset A)$. Hence, $A=\delta.Cl(A)$ and A is $\delta-$closed.

The converse of the above theorem is not true in general (see [1]).

Theorem 4.5. Let A and B be IFTS$^{'}$s in an IFTS X . The following statements are true:

(a) $\delta.Cl\left(\underset{\sim}{\phi}\right)=\underset{\sim}{\phi}$.

(b) If $A\subseteq B,$ then $\delta.Cl(A)\subseteq\delta.Cl(B)$.

(c) If V is IFR open in X , then $V\,q\,A \Leftrightarrow V\,q\,\delta.C(A).$

(d) $\delta.Cl(\delta.Cl(A))=\delta.Cl(A)$.

(e) $\delta.Cl(A\bigcup B)=\delta.Cl(A)\bigcup\delta.Cl(B).$

(f) $\delta.Cl(A\bigcap B)\subseteq\delta.Cl(A)\bigcap\delta.Cl(B).$

Proof: Straightforward.

5. On IF-$\delta-$Continuous Functions.

Definition 5.1. A function f from an IFTS (X,τ) into IFTS (Y,θ) is fuzzy $\delta-$continuous if and only if $f^{-1}(V)$ is $\delta-$open in τ for all $\delta-$open set V in θ.

Now we give a characterization of the fuzzy $\delta-$continuous functions.

Theorem 5.2. Let f be a function from an IFTS X into an IFTS Y. The following are equivalent:

(a) f is fuzzy $\delta-$continuous.

(b) For each IFR open set V in Y, $f^{-1}(V)$ is IF-$\delta-$open in X.

(c) For each IFR closed set V in Y, $f^{-1}(V)$ is IF-$\delta-$closed in X.

(d) For each IFP $x(\alpha,\beta)$ in X and each IFR open set V in Y with $f(x(\alpha,\beta))qV$, there is IFR open set U in X with $x(\alpha,\beta)qU$ such that $f(U)\subseteq V$.

Proof: $(a)\Rightarrow(b)$ Let (a) holds and let V be IFR open in Y. Then V is $\delta-$open (by theorem 4.4). By part (a) $f^{-1}(V)$ is IF-$\delta-$open in X.

$(b)\Leftrightarrow(c)$: Obvious.

$(c)\Rightarrow(d)$: Let © holds and let $x(\alpha,\beta)$ be IFP in X and V be IFR open in Y with $f(x(\alpha,\beta))qV$. Since V is IFR open in Y, $f^{-1}(V)$ is IF-$\delta-$ open. By lemma 3.4, $x(\alpha,\beta)q\,f^{-1}(V)$. Also, by theorem 4.3, there is IFR open set U in X such that $x(\alpha,\beta)q\,U\subseteq f^{-1}(V)$. Hence $U\subseteq f^{-1}(V)$. Therefore, $f(U)\subseteq V$. The proof in the case of vanishing points is similar.

$(d)\Rightarrow(a)$: Let (d) holds and let V be IF-$\delta-$open in Y , we show that $f^{-1}(V)$ is IF-$\delta-$open in X. So, let $x(\alpha,\beta)\ q\ f^{-1}(V)$. Then, $f(x(\alpha,\beta))q\,ff^{-1}V$. Therefore, $f(x(\alpha,\beta))qV$. Using part (d), there is IFR open set U in X such that $x(\alpha,\beta)qU$ and $f(U)\subseteq V$. Then, $f^{-1}f(U)\subseteq f^{-1}(V)$ or $U\subseteq f^{-1}(V)$. By Theorem 4.3, $f^{-1}(V)$ is IF- $\delta-$open in X. Hence f is $\delta-$ continuous.

Definition 5.3. (a) A net $S=\{S(n)=A_n : n\in D\}$ in an IFTS (X,τ) is a function $S:D\rightarrow\Im(X)$, where D is a directed set and $\Im(X)$ is the set of IFS on X.

(b) A net S is said to be R-convergent to IFP $x(\alpha,\beta)$ (resp. $x(\beta)$) in X relative to τ if and only if S is eventually quasi-coincident with each IFR open Q-nbd (P-nbd.) of $x(\alpha,\beta)$ (resp. $x(\beta)$). We denote this convergence by $S_n \xrightarrow{R} x(\alpha,\beta)$.

Lemma 5.4. Let U and V be IFR Q-nbd (resp. P-nbd) of $x(\alpha,\beta)$ (resp. $x(\beta)$) Then $U\bigcap V$ is a IFR Q-nbd (P-nbd.)of $x(\alpha,\beta)$ (resp. $x(\beta)$).

Proof. See [1].

Definition 5.5. A function $f:X\rightarrow Y$ is said to be fuzzy weakly almost open (FW-almost open, for short) if $f^{-1}(Cl(U))\subseteq Cl(f^{-1}U)$ for all IFR open set U in Y.

Remark 5.6: It is clear that each F-almost open function is FW-almost open, but the converse is not true as shown in the following example.

Example 5.7. Let $X=\{a,b\}$. Also, let A,B,C and D be IFS's on X such that:

$$A=\left\langle x,\begin{pmatrix} a & b \\ 0 & 0.2\end{pmatrix},\begin{pmatrix} a & b \\ 0.3 & 0.1\end{pmatrix}\right\rangle,\quad B=A^c=\left\langle x,\begin{pmatrix} a & b \\ 0.3 & 0.1\end{pmatrix},\begin{pmatrix} a & b \\ 0 & 0.2\end{pmatrix}\right\rangle,$$

$$C=\left\langle x,\begin{pmatrix} a & b \\ 0 & 0.1\end{pmatrix},\begin{pmatrix} a & b \\ 0.4 & 0.2\end{pmatrix}\right\rangle \text{ and } D=C^c=\left\langle x,\begin{pmatrix} a & b \\ 0.4 & 0.2\end{pmatrix},\begin{pmatrix} a & b \\ 0 & 0.1\end{pmatrix}\right\rangle.$$

It is easy to show that:

$$A\wedge B=\left\langle x,\begin{pmatrix} a & b \\ 0 & 0.1\end{pmatrix},\begin{pmatrix} a & b \\ 0.3 & 0.2\end{pmatrix}\right\rangle \text{ and } A\vee B=\left\langle x,\begin{pmatrix} a & b \\ 0..3 & 0.2\end{pmatrix},\begin{pmatrix} a & b \\ 0 & 0.1\end{pmatrix}\right\rangle$$

Next, let $\tau=\{\underset{\sim}{\phi},\underset{\sim}{1},B,D\}$ and $\theta=\{\underset{\sim}{\phi},\underset{\sim}{1},\ A,B,C,A\bigcap B,A\bigcup B\}$.

Let $f:X\to X$ be a mapping defined by: $f(a)=a$ and $f(b)=b$. It is obvious that $A,B,A\wedge B$ and $A\bigcup B$ are fuzzy IRO sets of (X,θ) while C is not. Now, it is clear that f is an FW-almost open function, but not F-almost open, since $C\in\theta,\ \theta.Cl(C)=A\bigcap B,\ \tau.Cl(C)=D$ and $f^{-1}(\theta.Cl(C))=A\bigcap B\not\subseteq D=\tau.Cl(C)=\tau.Cl(f^{-1}(C))$.

Theorem 5.8. A function $f:X\to Y$ is $\delta-$continuous if and only if for each IFP $x(\alpha,\beta)$ in X and every IF-net $\{x(\alpha_i,\beta_i):i\in D\}$ which R-converges to $x(\alpha,\beta)$, if M is a RQ-nbd of $f(x(\alpha,\beta))$, then there is a $m\in D$ such that $f(x(\alpha_i,\beta_i))\,q\,M$ for each $i\geq m$.

Proof: Let $x(\alpha,\beta)$ be a IFP in X, $x(\alpha_i,\beta_i)\xrightarrow{R}x(\alpha,\beta)$ and M be an RQ-nbd. Of $f(x(\alpha,\beta))$. There is IFR open set U in X with $x(\alpha,\beta)\,\mathrm{q}\,U$ and $f(U)\leq M$ (by theorem 5.2). Since $x(\alpha_i,\beta_i)\xrightarrow{R}x(\alpha,\beta)$, there is $m\in D$ such that $x(\alpha_i,\beta_i)\,qU$ (for each IFR Q-nbd U of $x(\alpha,\beta)$) and for all $i\geq m$. Hence $f(x(\alpha_i,\beta_i))\,q\ f(U)\subseteq M$, $i\geq m$ and therefore $f(x(\alpha_i,\beta_i))\,qM$.

Conversely, let suppose that f is not IF- $\delta-$continuous. Then there is at least one IFP $x(\alpha,\beta)$ in X and IFR open set M in Y with $f(x(\alpha,\beta))\ qM$ such that for each IFR open set U in X with $x(\alpha,\beta)qU$ we have $f(U)\not\subseteq M$, that is $U\,q\ f^{-1}(M^c)=\langle x,f^{-1}(\gamma_M)(x),f^{-1}(\mu_M)(x)\rangle,\ x\in X$. Then there is $x\in X$ such that $\mu_U(x)>f^{-1}(\mu_M)(x)$ or $\gamma_U(x)<f^{-1}(\gamma_M)(x)$. For all IFR open set U in X, $x(\alpha,\beta)qU$, choose $\sigma_U^1,\sigma_U^2>0$ such that $\alpha_U=f^{-1}(\gamma_M)(x)-\sigma_U^1$ and $\beta_U=f^{-1}(\mu_M)(x)+\sigma_U^2$. Let ζ be the family of all RQ-nbd of $x(\alpha,\beta)$. Clearly $(\zeta,\geq)$ is a directed set , where $(U\geq V\Leftrightarrow U\subseteq V$. Now, the set $\{x(\alpha_U,\beta_U):U\in\zeta\}$ is a net in X which R-converges to $x(\alpha,\beta)$, on the other hand $f(x(\alpha_U,\beta_U))\,\overline{q}\,M$ which a contradiction.

Theorem 5.9. If $f : X \to Y$ is an FW-almost open and FW-continuous, then $f^{-1}(V)$ is IFR open (closed) set in X for each IFR open (closed) set V in Y.

Proof: (See [1], Theorem 3.10).

Theorem 5.10. If $f : X \to Y$ is an FW-almost open and fuzzy weakly continuous, then f is a almost continuous.

Proof. Straightforward.

Corollary 5.11. If $f : X \to Y$ is an IFW-almost open and fuzzy weakly continuous, then f is $\delta-$continuous.

Proof: It follows from theorem 4.4, theorem 5.2 and theorem 5.9.

Corollary 5.12. If $f : X \to Y$ is an F-almost open and fuzzy weakly continuous, then f is $\delta-$continuous.

Proof: Ii follows from corollary 5.11 and the fact that each fuzzy almost open is FW-almost open [1].

Definition 5.13. A function $f : X \to Y$ is said to be IFW-open if $f(U) \subseteq Int(f(Cl(U))$ for each IF-open set U of X.

Note that each F-almost open S function is fuzzy weakly open but the converse is not true (See [1], Example 3.16 provided that each fuzzy set if IFS).

Theorem 5.14. If a function $f : X \to Y$ is fuzzy weakly open and fuzzy almost continuous then, $f^{-1}(V)$ is IFR open (closed) set in X for each IFR open (closed) set V in Y.

Proof: Let $V \in RO(Y)$. Since f is almost continuous, $f^{-1}(V)$ is IFO in X. Also, $f^{-1}(V) \leq Int(Cl\ f^{-1}V)$. Next, $Int(Cl\ f^{-1}V) \leq Cl\,(f^{-1}V) \leq f^{-1}Cl(V)$. Moreover, f is fuzzy weakly open, we obtain

$$\begin{aligned} f(Int(Cl\ f^{-1}V) &\leq Int(f(Cl(Int(Cl\ f^{-1}V))) \leq Int\ f(Cl(Int\ f^{-1}Cl(V))) \\ &\leq Int\ f(Cl(f^{-1}Cl(V)) \\ &\leq Int(f\ f^{-1}Cl(V)) \\ &= Int(Cl(V)) \\ &= V \end{aligned}$$

since V is IFR open in Y. Then, $Int(Cl\ f^{-1}V) \leq f^{-1}(V)$. Since $f^{-1}(V) \leq Int(Cl\ f^{-1}V)$, we have $f^{-1}(V) = Int(Cl\ f^{-1}V)$ and therefore $f^{-1}(V)$ is IFR open.

Corollary 5.15. If $f : X \to Y$ is a fuzzy weakly open and fuzzy almost continuous, then it is fuzzy $\delta-$continuous.

Proof: It follows from theorem 4.4 and theorem 5.2.

Corollary 5.16. If $f : X \to Y$ is a fuzzy almost open S and fuzzy almost continuous, then f is IF-$\delta-$continuous.

Proof: It follows from corollary 5.15 and the fact that each fuzzy almost open S mapping is FW-open [1].

Definition 5.17. A function $f : X \to Y$ is said to be IWF-$\theta-$continuous if for all IFP $x(\alpha,\beta)$ and for each IFO set V with $f(x(\alpha,\beta))\ qV$, there is a IFO set U of $x(\alpha,\beta)$ with $x(\alpha,\beta)q\ U$ and $f(Int(Cl(U))) \subseteq V$.

Note that , fuzzy almost continuity implies fuzzy weakly $\theta-$ continuity and this implies fuzzy weakly continuity. But the converse need not be true in general (see Example 3.22 in [1])

Theorem 5.18. If a function $f : X \to Y$ is fuzzy weakly $\theta-$continuous and fuzzy almost open S, then f is IF-$\delta-$continuous.

Proof: Let $x(\alpha,\beta)$ be IFP in X and let V be IFR open in Y such that $f(x(\alpha,\beta))\ qV$. Then, there is IF-open set U such that $x(\alpha,\beta)qU$ and $f(Int(Cl(U))) \le Cl(V)$. Since f is a fuzzy almost open S and $Int(Cl(U))$ is IFR open in X, $f(Int(Cl(U))) \le Int(Cl(V)) = V$. Hence the result.

6. Functions and $(\lambda_1,\lambda_2)-$nearly compact Intuitionistic Fuzzy Topological Spaces

In this section we introduce the notion of $(\lambda_1,\lambda_2)-$shading and we use this definition to define the concept of $(\lambda_1,\lambda_2)-$nearly compact in intuitionistic fuzzy topological spaces. Also, we study some of its properties.

Definition 6.1. Let $\eta \subseteq \Im^X$, be a family of IFS's on X. Then :

(a) If A is IFS , then the support of A denoted by $S(A)$ and

defined as follows:

$S(A) = \{x \in X : \mu_A(x) > 0,\ \gamma_A(x) < 1\}$

(b) $\eta \subseteq \Im^X$, is said to be $(\lambda_1,\lambda_2)-$shading of $S(A)$ if and only if for all $x \in S(A)$ there is $U \in \eta$ such that $\mu_U(x) > \lambda_1\ or\ \gamma_U(x) < \lambda_2,\ 0 \le \lambda_1 + \lambda_2 \le 1$.

Definition 6.2 [17]. Let A be an IFS and $\langle\lambda_1,\lambda_2\rangle \in \langle I\rangle$, $\langle\lambda_1,\lambda_2\rangle \ne \langle 0,1\rangle$. Then:

$A_{\langle\lambda_1,\lambda_2\rangle} = \{x \in X : \mu_A(x) > \lambda_1, \gamma_A(x) < \lambda_2\}$.

is the strong $\langle\lambda_1,\lambda_2\rangle-$cut of A.

Definition 6.3. An IFS A of an IFTS (X,τ) is said to be fuzzy $\langle\lambda_1,\lambda_2\rangle-$ nearly compact iff every IF-open $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ has a finite subfamily such that the fuzzy interior of the fuzzy closure of its member are $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$.

Theorem 6.4. A IFS A is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to an IFTS (X,τ) if and only if each IFR open $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ has a finite $\langle\lambda_1,\lambda_2\rangle-$sub-shading.

Proof: It follows from the above definition and the definition of IFR open $\langle\lambda_1,\lambda_2\rangle-$shading.

Theorem 6.5. An IFS A is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to an IFTS X if and only if each $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ by IF-$\delta-$ open sets has a finite $\langle\lambda_1,\lambda_2\rangle-$sub-shading.

Proof: Let $\{V_i : i\in J\}$ be an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ by IF-$\delta-$open sets of X. Then for all $x\in S(A)$, there is m such that $V_m\in\{V_i : i\in J\}$ and

$\mu_{V_m}(x)>\lambda_1$ or $\gamma_{V_m}(x)<\lambda_2$.

Now, the IFP $x\langle\lambda_1,\lambda_2\rangle=\langle y,x_{\lambda_1},1-x_{1-\lambda_2}\rangle$ such that $x\langle\lambda_2,\lambda_1\rangle\ qV_m=\langle y,\mu_{V_m},\gamma_{V_m}\rangle$ Since V_m is $\delta-$open , there is IR-open set H_x such that

$x(\lambda_2,\lambda_1)qH_x\le V_m$. Hence $\lambda_1<\mu_{H_x}(x)$ or $\lambda_2>\gamma_{H_x}(x)$. Hence, the family $\{H_x : x\in S(A)\}$ is an IR-open $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$. By the above theorem there is a finite subfamily $\{H_{x_i} : i=1,2,...n\}$ which is $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$). Hence $\{V_i : i=1,2,...,n\}$ is an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$.

Conversely, since each IFR open set is IF-$\delta-$open , the converse follows immediately.

Corollary 6.6. An IFTS X is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact if and only if each IF-$\delta-$open shading of X has a finite $\langle\lambda_1,\lambda_2\rangle-$shading.

Theorem 6.7. If $f : X\to Y$ is IF-$\delta-$continuous and A is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to X, then $f(A)$ is a fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to Y.

Proof: Let $\{V_i : i\in J\}$ be IR-open $\langle\lambda_1,\lambda_2\rangle-$shading of the support of $f(A)$ ($S(f(A))$, for short). Then $\{f^{-1}(V_i) : i\in J\}$ is an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ by IF-$\delta-$open sets of X. For, if $x\in S(A)$, $f(x)\in f(S(A))=S(f(A))$ and hence there is m such that $\mu_{V_m}(f(x))>\lambda_1\ or\ \gamma_{V_m}(f(x))<\lambda_2$. Hence $f^{-1}(\mu_{V_m})(x)>\lambda_1\ or\ f^{-1}(\gamma_{V_m})(x)<\lambda_2$. Since A is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to X, then by theorem 4.3 there is a finite subfamily $\{f^{-1}(V_i) : i=1,2,...,n\}$

which forms an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$. For $y\in S(f(A))$, $f(x)=y$, for some $x\in S(A)$. Hence there is m such that $\mu_{f^{-1}(V_m)}(x)>\lambda_1\ or\ \gamma_{f^{-1}(V_m)}<\lambda_2$ which implies $\mu_{V_m}(f(x))=\mu_{V_m}(y)>\lambda_1\ or\ \gamma_{V_m}(f(x))=\gamma_{V_m}(y)<\lambda_2$. Hence $\{V_i : i=1,2,...,n\}$ is an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(f(A))$. Therefore $f(A)$ is $\langle\lambda_1,\lambda_2\rangle-$nearly compact.

Corollary 6.8. If $f:X\to Y$ is a fuzzy $\delta-$continuous surjective function and X is $\langle\lambda_1,\lambda_2\rangle-$nearly compact , then so is Y.

Corollary 6.9. If $f:X\to Y$ is a fuzzy weakly open, fuzzy almost continuous surjective and X is $\langle\lambda_1,\lambda_2\rangle-$nearly compact, then so is Y.

Proof: It follows from corollary 5.15 and corollary 6.8.

Corollary 6.10. If $f:X\to Y$ is an FW-almost open, fuzzy weakly continuous surjective function and X is $\langle\lambda_1,\lambda_2\rangle-$nearly compact, then so is Y.

Proof: It follows from corollary 5.11 and corollary 6.8.

Corollary 6.11. If $f:X\to Y$ is an F-almost open, fuzzy weakly continuous surjective and X is $\langle\lambda_1,\lambda_2\rangle-$nearly compact, then so is Y.

Proof: It follows from corollary 6.10 and the fact that each fuzzy almost open function is FW-almost open [1].

Corollary 6.12. If $f:X\to Y$ is a fuzzy almost open S, fuzzy weakly $\theta-$continuous surjective function and X is $\langle\lambda_1,\lambda_2\rangle-$nearly compact, then so is Y.

Proof: It follows from theorem 5.19 and theorem 6.8.

Lemma 6.13. If A is IF- $\delta-$closed of an IFTS X and $x\notin S(A)$, then there is IFR open V in X such that $\mu_V(x)>\lambda_1\ or\ \gamma_V(x)<\lambda_2$ for all $\langle\lambda_1,\lambda_2\rangle\in\langle I\rangle$.

Proof: Let $x\notin S(A)$. Then $\mu_A(x)=0\ or\ \gamma_A(x)=1$. Hence for all $\langle\lambda_1,\lambda_2\rangle\in\langle I\rangle$ we have $x(\lambda_2,\lambda_1)\,q\,\langle\gamma_A,\mu_A\rangle=A^c$. Since A^c is $\delta-$open, so there is IFR open set V in X such that $x(\lambda_2,\lambda_1)\,q\,V\subseteq A^c$. Hence $\lambda_2>\gamma_V(x)\ or\ \lambda_1<\mu_V(x)$.

Theorem 6.14. If A is a IF- $\delta-$closed set of an $\langle\lambda_1,\lambda_2\rangle-$nearly compact IFTS X. Then A is fuzzy $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to X.

Proof: Let $\eta=\{U_i : i\in J\}$ be an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$ by IFR open sets of X. For $x\notin S(A)(x\in S(A)')$ there is IFR open set V_x in X such that $\mu_{V_x}(x)>\lambda_1\ or\ \lambda_2>\gamma_{V_x}(x)$ (by the above lemma). Now, the family

$\eta\bigcup\{V_x : x\in(S(A))'\}$ is an $\langle\lambda_1,\lambda_2\rangle-$shading of X by IFR open sets of X. Since X is $\langle\lambda_1,\lambda_2\rangle-$ nearly compact, there is a finite subfamily $\{U_1,U_2,...,U_n\}\bigcup\{V_{x_i} : i=1,2,...,n\}$ which is an $\langle\lambda_1,\lambda_2\rangle-$shading of X.

Consequently, $\{U_1,U_2,...,U_n\}$ is an $\langle\lambda_1,\lambda_2\rangle-$shading of $S(A)$. Hence A is $\langle\lambda_1,\lambda_2\rangle-$nearly compact relative to X .

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