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On Strongly Fuzzy α-Preirresolute Functions

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Abstract

The purpose of the present paper is to introduce new class of function called strongly fuzzy α-preirresolute (in brief, stFα-preirresolute) functions in fuzzy topology. Some properties and several characterizations are also obtained. We investigate the relationship between this function and other classes of non-continuous functions in fuzzy topology.

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1. Introduction

Yalvac [13] introduced the notion of fuzzy irresolute functions . Recently, the present authors [10] introduced the notion of fuzzy α-preirresolute functions. The purpose of this not is to introduce and investigate the notion of strongly fuzzy α-preirresolute functions and give several characterizations and their properties . Relation between this function and other classes of functions are obtained. The newly defined function is stronger than fuzzy α-preirresolutefunctions and is a generalization of strongly M fuzzy β-continuous functions.

2. Preliminaries

Throughout this paper, by $(X, \Im Á)$ (or simply X) we mean a fuzzy topological space in Chang's sense [5] and $f : X \to Y$ denotes a function from a space (X, τ) into a space (Y, σ). The closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.1. A fuzzy subset A of a space X is said to be:

(i) fuzzy preopen [4] (in brief, FP-open) if $A \le Int(Cl(A))$,

(ii) fuzzy α-open [4] (in brief, $F\alpha$-open) if $A \le Int(Cl(Int(A)))$,

(iii) fuzzy β-open [2](in brief, $F\beta$-open) if $A \le Cl(Int(Cl(A)))$.

The family of all fuzzy α-open (resp. fuzzy preopen, fuzzy β-open) set of a space (X, τ) is denoted by $FpO(X)$ (resp. $F\alpha O(X)$, $F\beta O(X)$). The complement of Fp-open (resp. $F\alpha$-open , $F\beta$-open)set is called Fp-closed (resp. $F\alpha$-closed, $F\beta$-closed).

Definition 2.2. [2] The intersection of all $F\beta$-closed sets containing A is called the β-closure of A and is denoted by $\beta Cl(A)$. The union of all $F\beta$-open sets contained in A is called the β-interior of A and is denoted by $\beta Int(A)$.

Definition 2.3. A function $f : X \to Y$ is called:

(i) strongly M fuzzy -precontinuous if $f^{-1}(A)$ is fuzzy open in X for every Fp-open set A of Y,

(ii) fuzzy α-irresolute [10] if $f^{-1}(A)$ is $F\alpha$-open in X for every Fp-open set A of Y

(iii) M fuzzy precontinuous [3] if $f^{-1}(A)$ is Fp-open in X for every Fp-open set A of Y

(iv) fuzzy β-preirresolute[11] if $f^{-1}(A)$ is $F\beta$-open in X for every Fp-open set A of Y

Definition 2.4. A function $f : X \to Y$ is said to be strongly M fuzzy β-continuous (resp. M fuzzy β-continuous [9]) if $f^{-1}(A)$ is fuzzy open (resp. $F\beta$-open) in X for every $F\beta$-open set A of Y.

Definition 2.5. A function $f : X \to Y$ is said to be fuzzy strongly continuous [6] if $f^{-1}(A)$ is fuzzy clopen in X for every fuzzy subset A of Y.

Definition 2.6. [8] A fuzzy point x_t is said to be quasi-coincident with a fuzzy set A in X if $t + A(x) > 1$. A fuzzy set A in X is said to be quasi-coincident with a fuzzy set B in X, denoted by AqB, if there exists a point x in X, such that $A(x) + B(x) > 1$.

Lemma 2.7. [13] Let $f : X \to Y$ be a function and x_t be a fuzzy point of X. Then,

(a) $f(x_t)qB \Rightarrow x_t q f^{-1}(B)$, for every fuzzy set B of Y,

(b) $x_t q A \Rightarrow f(x_t) q f(A)$, for every fuzzy set A of X.

3. St. Fαp-Irresolute Functions

Definition 3.1. A function $f : X \to Y$ is said to be strongly fuzzy α-preirresolute (shortly St-$F\alpha p$-irresolute) if $f^{-1}(A)$ is $F\alpha$-open in X, for every $F\beta$-open set A of Y.

Equivalently, we may say that f is St-$F\alpha$p-irresolute if $f^{-1}(A)$ is $F\alpha$-closed in X, for every $F\beta$-closed set A of Y. From the definition we obtain the following diagram:

St.F$\alpha-$preirresolute

$\nearrow \quad \searrow$

St.MF$\beta-$continuity $\quad F\alpha-$preirresolute

$\searrow \quad \nearrow$

St.MF $-$ precontinuity

Converse of these implications are not true in general. For,

Example 3.2. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Where,

$$A(a) = 0,\ A(b) = 0.8;$$
$$B(x) = 0.6,\ B(y) = 0.2.$$

Clearly, τ and σ are fuzzy topology on X and Y, respectively. Define $f : (X, \tau) \to (Y, \sigma)$ as $f(a) = x$ and $f(b) = y$. Then f is St.$F\alpha$ p-irresolute but neither St.$MF\beta$-continuous nor St.MFp-continuous

Example 3.3. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Let $\tau = \{0, A, B, 1\}$ and $\sigma = \{0, H, 1\}$ where,

$$A(a) = 0,\ A(b) = 0.1;$$
$$B(a) = B(b) = 0.1;$$
$$H(x) = 0,\ H(y) = 0.1.$$

Clearly, τ and σ are fuzzy topology on X and Y, respectively. Define $f : (X, \tau) \to (Y, \sigma)$ as $f(a) = x$ and $f(b) = y$. Then f is St.MFp-continuous but neither St.$MF\beta$-continuous nor St.$MF\alpha$p-irresolute.

Theorem 3.4. For a function $f : X \to Y$, the following are equivalent:

(a) f is St.$F\alpha$-preirresolute;

(b) For every fuzzy point x_t in X and every $F\beta$-open set V of Y containing $f(x_t)$, there exists an $F\alpha$-open set U of X containing x_t such that $F(U) \leq V$;

(c) For every fuzzy point x_t of X and every $F\beta$-open set V of Y containing $f(x_t)$, there exists an $F\alpha$-open set U of X such that $x_t \in U \leq f^{-1}(V)$;

(d) For every fuzzy point x_t in X, the inverse image of each β-neighbourhood of $f(x_t)$ is α-neighbourhood of x_t;

(e) For every fuzzy point x_t in X and each β-neighbourhood B of $f(x_t)$, there exists an α-neighbourhood A of x_t such that $f(A) \leq B$;

(f) $f^{-1}(V) \leq Int(Cl(Int(f^{-1}(V))))$, for every $F\beta$-open set V of Y;

(g) $f^{-1}(H)$ is $F\alpha$-closed in X, for every $F\beta$-closed set H of Y;

(h) $Cl(Int(Cl(f^{-1}(B)))) \leq f^{-1}(\beta Cl(B))$, for every fuzzy subset B of Y;

(i) $f(Cl(Int(Cl(A)))) \leq \beta Cl(f(A))$, for every fuzzy subset A of X.

Proof. $(a) \Leftrightarrow (b) \Leftrightarrow (c)$; $(d) \Leftrightarrow (e)$: Obvious.

$(b) \Rightarrow (f)$: Let $V \in F\beta O(Y)$ and $x_t \in f^{-1}(V)$. By (b), there exists $U \in F\alpha O(X)$ containing x_t such that $f(U) \leq V$ thus we have $x_t \in U \leq Int(Cl(Int(U))) \leq Int(Cl(Int(f^{-1}(V))))$ and hence $f^{-1}(V) \leq Int(Cl(Int(f^{-1}(V))))$.

$(f) \Rightarrow (g)$: Let H be any $F\beta$-closed set of Y. Set $V = Y \setminus H$ then $V \in F\beta O(Y)$. By (f), we obtain $f^{-1}(V) \leq Int(Cl(Int(f^{-1}(V))))$ and hence, $f^{-1}(H) = X - f{-}1(Y \setminus H) = X \setminus f^{-1}(V)$ is $F\alpha$-closed in X.

$(g) \Rightarrow (h)$: Let B be any fuzzy set of Y.Since $\beta Cl(B)$ is fuzzy β-closed subset of Y, $f^{-1}(\beta Cl(B))$ is $F\alpha$-closed in X and hence $Cl(Int(Cl(f^{-1}(\beta Cl(B))))) \leq f^{-1}(\beta Cl(B))$.Therefore, we obtained $Cl(Int(Cl(f^{-1}(B)))) \leq f^{-1}(\beta Cl(B))$.

$(h) \Rightarrow (i)$: Let A be any fuzzy subset of X. By (h) we have $Cl(Int(Cl(A))) \leq Cl(Int(Cl(f^{-1}(f(A))))) \leq f^{-1}(\beta Cl(f(A)))$ and hence $f(Cl(Int(Cl(A))) \leq \beta Cl(f(A))$.

$(i) \Rightarrow (a)$: Let $V \in F\beta O(Y)$. Since $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is fuzzy subset of X and by (i), we obtain $f(Cl(Int(Cl(f^{-1}(Y \setminus V))))) \leq \beta Cl(f(f^{-1}(Y \setminus V))) \leq \beta Cl(Y \setminus V) = Y \setminus \beta Int(V) = Y \setminus V$ and hence $X \setminus Int(Cl(Int(f^{-1}(V)))) = Cl(Int(Cl(X \setminus f^{-1}(V)))) = Cl(Int(Cl(f^{-1}(Y \setminus V)))) \leq f^{-1}(f(Cl(Int(Cl(f^{-1}(Y \setminus V)))))) \leq f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$.Therefore, we have $f^{-1}(V) \leq Int(Cl(Int(f^{-1}(V))))$ and hence $f^{-1}(V) \in F\alpha O(X)$.

$(a) \Rightarrow (d)$: Letx_t be a fuzzy point in X and V be β-neighbourhood of $f(x_t)$, then there exists $G \in F\beta O(Y)$ such that $f(x_t) \in G \leq V$. Now, $f^{-1}(G) \in F\alpha O(X)$ and $x_t \in f^{-1}(G) \leq f^{-1}(V)$. Thus, $f^{-1}(V)$ is α-neighbourhood of x_t in X.

$(e) \Rightarrow (b)$: Let x_t be a fuzzy point in X and $V \in F\beta O(Y)$ such that $f(x_t) \in V$.Then V is β-neighbourhood of $f(x_t)$, so there is a α-neighbourhood A of x_t such that $x_t \in A$

and $f(A) \leq V$. Hence there exists a $U \in F\alpha L(X)$ such that $x_t \in U \leq A$ and so $f(U) \leq f(A) \leq V$. ■

Theorem 3.5. For a function $f : X \to Y$, the following are equivalent:

(a) f is $St.F\alpha$-preirresolute.

(b) For each fuzzy point x_t of X and every $B \in F\beta O(Y)$, such that $f(x_t)qB$, there exist a $A \in F\alpha O(X)$ such that $x_t qA$ and $f(A) \leq B$.

(c) For every fuzzy point x_t of X and every $B \in F\beta O(Y)$, such that $f(x_t)qB$, there exists $A \in F\alpha O(X)$ such that $x_t qA$ and $A \leq f^{-1}(B)$

Proof. $(a) \Rightarrow (b)$: Let x_t be a fuzzy point of X and $B \in F\beta O(Y)$ such that $f(x_t)qB$. Then $f^{-1}(B) \in F\alpha O(X)$ and $x_t q f^{-1}(B)$ by Lemma 2.6. If we take $A = f^{-1}(B)$, then $x_t qA$ and $f(A) = f(f^{-1}(B)) \leq B$

$(b) \Rightarrow (c)$: Let x_t be a fuzzy point of X and $B \in F\beta O(Y)$ such that $f(x_t qB$. Then by (b), there exists $A \in F\alpha O(X)$ such that $x_t qA$ and $f(A) \leq B$. Hence $x_t qA$ and $A \leq f^{-1}(f(A)) \leq f^{-1}(B)$.

$(c) \Rightarrow (a)$: Let $B \in F\beta O(Y)$ and x_t be a fuzzy point of X such that $x_t \in f^{-1}(B)$. Then $f(x_t) \in B$. Choose the fuzzy point $x_t^c = 1 - x_t$. Then $f(x_t^c)qB$. And so by (c), there exists $A \in F\alpha O(X)$ such that $x_t^c qA$ and $f(A) \leq B$. Now, $x_t^c qA \to x_t^c(x) + A(x) = 1 - x_t(x) + A(x) > 1$. It follows that $x_t \in A$. Thus $x_t \in A \leq f^{-1}(B)$. Hence $f^{-1}(B) \in F\alpha O(X)$. ■

Theorem 3.6. A function $f : X \to Y$ is $St.F\alpha$-preirresolute if the graph function $g : X \to X \times Y$, defined $g(x) = (x, f(x))$ for each $x \in X$, is $St.F\alpha$-preirresolute.

Proof. Let $x_t \in X$ and $V \in F\beta O(Y)$ containing $f(x_t)$. Then $X \times V \in F\beta O(X \times Y)$ containing $g(x_t)$. Since g is $St.F\alpha$-preirresolute there exists$U \in F\alpha O(X)$ containing x_t such that $g(U) \leq X \times V$ and hence $f(U) \leq V$. ■

Theorem 3.7. If $f : X \to Y$ is $St.F\alpha$-preirresolute and A is Fp-open in X,then the restriction $f|A : A \to Y$ is $St.F\alpha$-preirresolute.

Proof. Let $V \in F\beta O(Y)$. Since f is $St.F\alpha$-preirresolute, then $f^{-1}(V) \in F\alpha O(X)$. Since A is Fp-open in X, $(F|A)^{-1}(V) = A \cap f^{-1}(V)$ is $F\alpha$-open in A. Hence $f|A$ is $St.F\alpha$-preirresolute. ■

Theorem 3.8. Let $f : X \to Y$ be a function and $\{A_i : i \in A\}$ be a cover of X by $F\alpha$-open sets of X. Then f is $St.F\alpha$-preirresolute if $f|A_i : A_i \to Y$ is $St.F\alpha$-preirresolute for each $i \in A$.

Proof. Easy. ■

4. Preservation Theorems

In this section we firstly give some compositions and then preservations of some fuzzy topological structures under St.Fαp-irresolute functions:

Theorem 4.1. $f : X \to Y$ be a function is St.Fαp-irresolute and $g : Y \to Z$ is M-fuzzy β-continuous, then $gof : X \to Z$ is St.Fαp-irresolute.

Proof. Straightforward. ∎

Corollary 4.2. The composition of two St.Fαp-irresolute functions are St.Fαp-irresolute.

Corollary 4.3. If $f : X \to Y$ is fuzzy strongly continuous and $g : Y \to Z$ is St.Fαp-irresolute, then $gof : X \to Z$ is St.Fαp-irresolute.

Theorem 4.4. If $f : X \to Y$ is $F\alpha$-irresolute [7] and $g : Y \to Z$ is St.Fαp-irresolute, then $gof : X \to Z$ is St.Fαp-irresolute.

Proof. Obvious. ∎

Let us recall the following.

Definition 4.5. [1] A fuzzy topological space X is fuzzy β-compact iff for every $F\beta$-open cover of X has a finite subcover.

Definition 4.6. A fuzzy topological space X is fuzzy α-compact iff for every $F\alpha$-open cover of X has a finite subcover.

Theorem 4.7. Every surjective St.Fαp-irresolute image of fuzzy α-compact space is fuzzy β-compact.

Proof. Let $f : X \to Y$ be $St.F\alpha P$-irresolute function of a fuzzy α-compact space X onto a fuzzy topological space Y. Let $\{G_a : a \in \Lambda\}$ be any $F\beta$-open cover of Y. Then, $\{f^{-1}(G_a) : a \in \Lambda\}$ is a $F\alpha$-open cover of X. Since X is $F\alpha$-compact, there exists a finite subfamily $\{f^{-1}(G_{a_i} : i = 1, 2, 3, \ldots\}$ of $\{f^{-1}(G_a) : a \in \Lambda\}$ which cover X. It follows that $\{G_{a_i} : i = 1, 2, 3, \ldots\}$ is a finite subfamily of Let $\{G_a : a \in \Lambda\}$ which covers Y. Hence Y is $F\beta$-compact. ∎

We recall the following.

Definition 4.8. [1] Two non-empty fuzzy subsets G and F are fuzzy β-separated iff there exist two fuzzy β-open subsets U and V such that $G \leq U$, $H \leq V$, $G\bar{q}V$ and $H\bar{q}U$.

Definition 4.9. [1] A fuzzy subset which cannot be expressed as the union of two fuzzy β-separated subsets is said to be fuzzy β(in short $F\beta$)-connected.

Theorem 4.10. Let $f : X \to Y$ be $St.F\alpha P$-irresolute function from a space X onto a space Y. If A is $F\beta$-connected subset in X, then $f(A)$ is also $F\beta$-connected in Y.

Proof. Suppose $f(A)$ is not $F\beta$-connected in Y. Then there exist $F\beta$-separated subset G and H in Y such that $f(A) = G \cup H$. Since G and H are $F\beta$-separated, there exist two $F\beta$-open sets U and V such that $G \leq U$, $H \leq V$, $G\bar{q}V$ and $H\bar{q}U$. Now, f being $St.F\alpha p$-irresolute surjection, so $f^{-1}(G)$ and $f^{-1}(H)$ are $F\alpha$-open in X, and hence $F\beta$-open in A and $A = f^{-1}(f(A)) = f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$. It is easy to see that $f^{-1}(G)$ and $f^{-1}(H)$ are $F\beta$-separated in X. Hence A is not $F\beta$-connected in X. Hence the theorem.

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