

## Somewhat Fuzzy pre $\alpha$ -irresolute Functions

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### Abstract

In this paper the concepts of somewhat fuzzy pre  $\alpha$ -irresolute functions and strongly somewhat fuzzy pre-open functions are introduced and studied. Besides giving characterizations of these functions, some interesting properties of these functions are also given.

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### 1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L. A. Zadeh [13]. Fuzzy sets have applications in many fields such as information [6] and control [8]. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of somewhat continuous functions was introduced by Karl R.Gentry and Hughes B.Hoyle III in [4] and this concept was studied in connection with the idea of feeble continuous function and feebly open function introduced and studied by Zdenek Frolik in [14]. The concepts of somewhat fuzzy continuous functions and somewhat fuzzy pre-continuous functions was introduced and studied by G.Thangaraj and G.Balasubramanian in [9] and [10] respectively. In this paper we introduce the concepts of somewhat fuzzy pre  $\alpha$ -irresolute functions and strongly somewhat fuzzy pre-open functions and study their properties.

## 2. Preliminaries

By a fuzzy topological space we shall mean a non-empty set  $X$  together with fuzzy topology  $T$  [3] and denote it by  $(X, T)$ . A fuzzy point in  $X$  with support  $x \in X$  and value  $p$  ( $0 < p \leq 1$ ) is denoted by  $x_p$ . If  $\lambda$  is a fuzzy set in  $X$  and  $\mu$  is a fuzzy set in  $Y$ , Then  $\lambda \times \mu$  is a fuzzy set in  $X \times Y$ , defined by  $(\lambda \times \mu)(x, y) = \min(\lambda(x), \mu(y))$ , for every  $(x, y)$  in  $X \times Y$  [1]. A fuzzy topological space  $X$  is product related to a fuzzy topological space  $Y$  [1] if for fuzzy sets  $\gamma$  in  $X$  and  $\xi$  in  $Y$  whenever  $\lambda' (= 1 - \lambda) \not\leq \gamma$  and  $\mu' (= 1 - \mu) \not\leq \xi$  (in which case  $(\lambda' \times 1) \vee (1 \times \mu') \geq (\gamma \times \xi)$ ) where  $\lambda$  is fuzzy open set in  $X$  and  $\mu$  is a fuzzy open set in  $Y$ , there exists a fuzzy open set  $\lambda_1$  in  $X$  and a fuzzy open set  $\mu_1$  in  $Y$  such that  $\lambda'_1 \geq \gamma$  or  $\mu'_1 \geq \xi$  and  $(\lambda'_1 \times 1) \vee (1 \times \mu'_1) = (\lambda' \times 1) \vee (1 \times \mu')$ . Let  $f : X \rightarrow Y$  be a mapping from  $X$  to  $Y$ . If  $\lambda$  is a fuzzy set in  $X$ ,  $f(\lambda)$  is defined as follows:

$$f(\lambda)(y) = \begin{cases} \text{Sup } \lambda(x), & \text{if } f^{-1}(y) \neq \phi \\ \quad \quad \quad x \in f^{-1}(y) \\ 0, & \text{otherwise,} \end{cases}$$

for each  $y \in Y$ ; and if  $\mu$  is a fuzzy set in  $Y$ ,  $f^{-1}(\mu)$  is defined by  $f^{-1}(\mu)(x) = \mu f(x)$  for each  $x \in X$  [3]. Let  $f$  be a mapping from  $X$  to  $Y$ . Then the graph  $g$  of  $f$  is mapping from  $X$  to  $X \times Y$  takes  $x$  in  $X$  to  $(x, f(x))$ . For two mappings  $f_1 : X_1 \rightarrow Y_1$  and  $f_2 : X_2 \rightarrow Y_2$ , we define the product  $f_1 \times f_2$  of  $f_1$  and  $f_2$  to be a mapping from  $X_1 \times X_2$  to  $Y_1 \times Y_2$  takes  $(x_1, x_2)$  in  $X_1 \times X_2$  to  $(f_1(x_1), f_2(x_2))$ . For any fuzzy set  $\lambda$  in a fuzzy topological space, it is shown in [1] that (i)  $1 - \text{cl } \lambda = \text{int}(1 - \lambda)$ , (ii)  $\text{cl}(1 - \lambda) = 1 - \text{int } \lambda$ . A fuzzy set  $\lambda$  in fuzzy topological space  $(X, T)$  is called fuzzy  $\alpha$ -dense if there exists no fuzzy  $\alpha$ -closed set  $\mu$  such that  $\lambda < \mu < 1$ .

**Definition 2.1.** [5] A function  $f : (X, T) \rightarrow (Y, S)$  is said to be **fuzzy  $\alpha$ -irresolute** if  $f^{-1}(\lambda)$  is fuzzy  $\alpha$ -open set in  $(X, T)$  for every fuzzy  $\alpha$ -open set  $\lambda$  of  $(Y, S)$ .

**Definition 2.2.** [7] A function  $f$  from a fuzzy topological space  $(X, T)$  to a fuzzy topological space  $(Y, S)$  is said to be **fuzzy pre  $\alpha$ -irresolute** if  $f^{-1}(\lambda)$  is fuzzy pre-open set in  $(X, T)$  for each fuzzy  $\alpha$ -open set  $\lambda$  in  $(Y, S)$ .

**Definition 2.3.** [10] Let  $(X, T)$  and  $(Y, S)$  be fuzzy topological spaces. A function  $f : (X, T) \rightarrow (Y, S)$  is called **somewhat fuzzy pre-open function** if and only if for all  $\lambda \in T$ ,  $\lambda \neq 0$  there exists a fuzzy semi-open set  $\mu$  of  $Y$  such that  $\mu \neq 0$  and  $\mu \leq f(\lambda)$ .

**Definition 2.4.** Let  $(X, T)$  be a fuzzy topological space and  $\lambda$  be any fuzzy set in  $X$ .

1.  $\lambda$  is called fuzzy  $\alpha$ -open set [2] if  $\lambda \leq \text{int } \text{cl } \text{int } \lambda$
2.  $\lambda$  is called fuzzy pre-open set [2] if  $\lambda \leq \text{int } \text{cl } \lambda$

The complement of fuzzy  $\alpha$ -open (fuzzy pre-open) set is called fuzzy  $\alpha$ -closed (fuzzy pre-closed) set.

### 3. Somewhat Fuzzy pre $\alpha$ -irresolute Functions

The concept of fuzzy pre  $\alpha$ -irresolute functions was introduced and studied in [7]. In this section we shall introduce the concept of somewhat fuzzy pre  $\alpha$ -irresolute functions and study their properties.

**Definition 3.1.** Let  $(X, T)$  and  $(Y, S)$  be any two fuzzy topological spaces. A function  $f : (X, T) \rightarrow (Y, S)$  is said to be **somewhat fuzzy pre  $\alpha$ -irresolute** if for any non zero fuzzy  $\alpha$ -open set  $\lambda$  in  $Y$  and  $f^{-1}(\lambda) \neq 0$  then there exists a fuzzy pre-open set  $\mu \neq 0$  in  $X$  such that  $\mu \leq f^{-1}(\lambda)$ .

**Definition 3.2.** [3] The complement  $\mu'$  of a fuzzy set  $\mu$  is  $1 - \mu$ , defined by  $\mu'(x) = (1_X - \mu)(x) = 1 - \mu(x)$  for all  $x \in X$ .

Clearly fuzzy pre  $\alpha$ -irresolute function is somewhat fuzzy pre  $\alpha$ -irresolute, but the converse is not true as the following example shows:

**Example 3.3.** Let  $\mu_1, \mu_2$  and  $\mu_3$  be fuzzy sets on  $I = [0, 1]$ .

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x - 1), & \frac{1}{4} \leq x \leq 1. \end{cases}$$

Clearly  $T_3 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$  is a fuzzy topology on  $I$ . Let  $f : (I, T_3) \rightarrow (I, T_3)$  be defined  $f(x) = \frac{x}{2}$  for each  $x \in I$ . It can be easily seen that  $\text{int } \mu_2' = \mu_1$ ;  $\text{int } \mu_3' = \mu_2$ ;  $\text{int } (\mu_1 \vee \mu_2)' = 0$ ;  $\text{cl } \mu_1 = \mu_2'$ ;  $\text{int } \mu_1' = \mu_2$ ;  $\text{cl } \mu_2 = \mu_1'$ ;  $\text{cl } (\mu_1 \vee \mu_2)' = (\mu_1 \vee \mu_2)'$ ;  $\text{cl } \mu_1' = \mu_1'$  and  $\text{int } \mu_3 = \mu_1$  in  $(I, T_3)$ . Let  $\lambda, \rho$  and  $\delta$  be fuzzy sets such that  $0 < \lambda < \mu_1, \mu_1 < \rho < \mu_2$  and  $\mu_2 < \delta < (\mu_1 \vee \mu_2)$ . Then  $\lambda, \rho$  and  $\delta$  are not fuzzy  $\alpha$ -open sets in  $(I, T_3)$ . Therefore the only non-zero fuzzy  $\alpha$ -open sets in  $(I, T_3)$  are  $1, \mu_1, \mu_2, \mu_1 \vee \mu_2$  and fuzzy sets  $\mu$  such that  $(\mu_1 \vee \mu_2) < \mu < 1$ . Now  $f^{-1}(1) = 1$ ;  $f^{-1}(\mu_1) = 0$ ;  $f^{-1}(\mu_2) = \mu_1' = f^{-1}(\mu_1 \vee \mu_2)$  and  $f^{-1}(\mu) = 1$ . The fuzzy pre-open set  $\mu_2$  in  $(I, T_3)$  is contained in  $f^{-1}(1), f^{-1}(\mu_2), f^{-1}(\mu_1 \vee \mu_2)$  and  $f^{-1}(\mu)$ . This

proves  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute function.  $\mu_2$  is a fuzzy  $\alpha$ -open set in  $(I, T_3)$ . Since  $\mu'_1 \not\leq \text{int cl } \mu'_1$ ,  $\mu'_1$  is not a fuzzy pre-open set in  $(I, T_3)$ . Then  $f^{-1}(\mu_2) = \mu'_1$ , which is not a fuzzy pre-open set in  $(I, T_3)$ . Hence  $f$  is not fuzzy pre  $\alpha$ -irresolute.

**Theorem 3.4.** Let  $f : (X, T) \rightarrow (Y, S)$  and  $g : (Y, S) \rightarrow (Z, Q)$  be any two functions. If  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute and  $g$  is fuzzy  $\alpha$ -irresolute, then  $g \circ f$  is somewhat fuzzy pre  $\alpha$ -irresolute.

*Proof.* Let  $\lambda$  be non-zero fuzzy  $\alpha$ -open set in  $(Z, Q)$ . Since  $g$  is fuzzy  $\alpha$ -irresolute then  $g^{-1}(\lambda) \neq 0$  is fuzzy  $\alpha$ -open in  $(Y, S)$ . Now  $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda)) \neq 0$ . Since  $g^{-1}(\lambda)$  is fuzzy  $\alpha$ -open in  $(Y, S)$  and  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute, we conclude that there exists a fuzzy pre-open set  $\mu \neq 0$  of  $(X, T)$  such that  $\mu \leq f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ . Hence  $g \circ f$  is somewhat fuzzy pre  $\alpha$ -irresolute. ■

**Remark 3.5.** In above Theorem 3.4 if  $f$  is either fuzzy  $\alpha$ -irresolute or fuzzy pre  $\alpha$ -irresolute and  $g$  is somewhat fuzzy pre  $\alpha$ -irresolute, then it is not necessarily true that  $g \circ f$  is somewhat fuzzy pre  $\alpha$ -irresolute as the following example shows:

**Example 3.6.** Let  $\mu_1, \mu_2$  and  $\mu_3$  be fuzzy sets in  $I$  described in Example 3.3. Let  $T_1 = \{0, \mu_1, 1\}$ ;  $T_2 = \{0, \mu_3, 1\}$  and  $T_3 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ . Then  $T_1, T_2$  and  $T_3$  are fuzzy topologies on  $I$ . Let  $f : (I, T_1) \rightarrow (I, T_2)$  be defined by  $f(x) = x$ , for each  $x \in I$ . It can be easily seen that  $\text{int } \mu_3 = \mu_1$ ;  $\text{int } \mu'_2 = \mu_1$   $\text{cl } \mu_3 = 1$ ;  $\text{cl } \mu'_2 = 1$  and  $\text{cl } \mu_1 = 1$  in  $(I, T_1)$ . Let  $\lambda$  be fuzzy set such that  $0 < \lambda < \mu_3$ . Then  $\lambda$  is not fuzzy  $\alpha$ -open set in  $(I, T_2)$ . Therefore the only fuzzy  $\alpha$ -open set in  $(I, T_2)$  are  $0, 1, \mu_3$  and fuzzy sets  $\mu$  such that  $\mu_3 < \mu < 1$ . Now  $f^{-1}(0) = 0$ ;  $f^{-1}(1) = 1$ ;  $f^{-1}(\mu_3) = \mu_3$  and  $f^{-1}(\mu) = \mu'_2$  are fuzzy  $\alpha$ -open set and also fuzzy pre-open set in  $(I, T_1)$ . Then  $f$  is fuzzy  $\alpha$ -irresolute and also  $f$  is fuzzy pre  $\alpha$ -irresolute from  $(I, T_1)$  to  $(I, T_2)$ . Let  $g : (I, T_2) \rightarrow (I, T_3)$  be defined by  $g(x) = \frac{x}{2}$  for each  $x \in I$ . Let  $\lambda, \rho$  and  $\delta$  be fuzzy sets such that  $0 < \lambda < \mu_1, \mu_1 < \rho < \mu_2$  and  $\mu_2 < \delta < (\mu_1 \vee \mu_2)$ . Then  $\lambda, \rho$  and  $\delta$  are not fuzzy  $\alpha$ -open sets in  $(I, T_3)$ . Therefore the only non-zero fuzzy  $\alpha$ -open sets in  $(I, T_3)$  are  $1, \mu_1, \mu_2, \mu_1 \vee \mu_2$  and fuzzy sets  $\mu$  such that  $(\mu_1 \vee \mu_2) < \mu < 1$ . Now  $g^{-1}(1) = 1$ ;  $g^{-1}(\mu_1) = 0$ ;  $g^{-1}(\mu_2) = \mu'_1 = g^{-1}(\mu_1 \vee \mu_2)$  and  $g^{-1}(\mu) = 1$ . It can be easily seen that  $\text{cl } \mu'_1 = 1$  and  $\text{int } \mu'_1 = 0$  in  $(I, T_2)$ . Since  $\mu'_1 \leq \text{cl int cl } \mu'_1$ ,  $\mu'_1$  is fuzzy pre-open set in  $(I, T_2)$ . Now  $\mu'_1$  is a fuzzy pre-open set in  $(I, T_2)$  such that  $\mu'_1 \leq g^{-1}(\mu_2), \mu'_1 \leq g^{-1}(1), \mu'_1 \leq g^{-1}(\mu)$  and  $\mu'_1 \leq g^{-1}(\mu_1 \vee \mu_2)$ . Hence  $g$  is somewhat fuzzy pre  $\alpha$ -irresolute function.

Now consider the functions  $(g \circ f) : (I, T_1) \rightarrow (I, T_3)$ . Then  $(g \circ f)^{-1}(0) = 0$ ;  $(g \circ f)^{-1}(1) = 1$ ;  $(g \circ f)^{-1}(\mu_1) = f^{-1}(g^{-1}(\mu_1)) = 0$ ;  $(g \circ f)^{-1}(\mu_2) = f^{-1}(g^{-1}(\mu_2)) = f^{-1}(\mu'_1) = \mu'_1$  and  $(g \circ f)^{-1}(\mu_1 \vee \mu_2) = f^{-1}(g^{-1}(\mu_1 \vee \mu_2)) = f^{-1}(\mu'_1) = \mu'_1$ . But  $(g \circ f)^{-1}(\mu_2) = \mu'_1$  and if  $\lambda$  is a fuzzy set such that  $0 < \lambda \leq \mu'_1$ , then  $\lambda$  is not a fuzzy pre-open set in  $(I, T_1)$ . Hence there is no non-zero fuzzy pre-open set in  $(I, T_1)$  such that it is contained in  $(g \circ f)^{-1}(\mu_2) = \mu'_1$ . This shows that  $(g \circ f)$  is not somewhat fuzzy pre  $\alpha$ -irresolute function.

**Definition 3.7.** [10] A fuzzy set  $\lambda$  in fuzzy topological space  $(X, T)$  is called fuzzy pre-dense if there exists no fuzzy pre-closed set  $\mu$  such that  $\lambda < \mu < 1$ .

**Theorem 3.8.** Let  $(X, T)$  and  $(Y, S)$  be any two fuzzy topological spaces and  $f : (X, T) \rightarrow (Y, S)$  be a function. Then the following assertions are equivalent.

- (1)  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute.
- (2) If  $\lambda$  is a fuzzy  $\alpha$  closed set of  $Y$  such that  $f^{-1}(\lambda) \neq 1$ , then there exists a fuzzy pre-closed set  $\mu \neq 1$  of  $X$  such that  $\mu \geq f^{-1}(\lambda)$ .
- (3) If  $\lambda$  is a fuzzy pre-dense set in  $X$ , then  $f(\lambda)$  is fuzzy  $\alpha$ -dense set in  $Y$ .

*Proof.* (1)  $\Rightarrow$  (2): Suppose  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute and  $\lambda$  is a fuzzy  $\alpha$ -closed in  $Y$  such that  $f^{-1}(\lambda) \neq 1$ . Therefore clearly  $1 - \lambda$  is fuzzy  $\alpha$ -open set in  $(Y, S)$  and  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda) \neq 0$  (since  $f^{-1}(\lambda) \neq 1$ ). By (1), there exists a fuzzy pre-open set  $\eta$  in  $X$  such that  $\eta \leq f^{-1}(1 - \lambda)$ . That is,  $\eta \leq 1 - f^{-1}(\lambda)$  which implies that  $f^{-1}(\lambda) \leq 1 - \eta$ . Clearly  $1 - \eta$  is fuzzy pre-closed set and taking  $\mu = 1 - \eta$ , we have therefore  $f^{-1}(\lambda) \leq \mu$ . Thus we find that (1)  $\Rightarrow$  (2) is proved. (2)  $\Rightarrow$  (3) Let  $\lambda$  be a fuzzy pre-dense in  $X$  and suppose  $f(\lambda)$  is not fuzzy  $\alpha$ -dense in  $Y$ . Then there exists a fuzzy  $\alpha$ -closed set  $\eta$  (say) in  $Y$  such that

$$f(\lambda) < \eta < 1. \quad (\text{A})$$

Since  $\eta < 1$ ,  $f^{-1}(\eta) \neq 1$  and so by (2) there exists a fuzzy pre-closed set  $\delta$  ( $\delta \neq 1$ ) such that  $\delta \geq f^{-1}(\eta) > f^{-1}(f(\lambda)) \geq \lambda$  (from (A)). That is, there exists a fuzzy pre-closed set  $\delta$  such that  $\delta > \lambda$  which is contradiction to the assumption on  $\lambda$ . Therefore (2)  $\Rightarrow$  (3) is proved. (3)  $\Rightarrow$  (1) Suppose  $\lambda$  is fuzzy  $\alpha$ -open set in  $Y$  and  $f^{-1}(\lambda) \neq 0$  and therefore  $\lambda \neq 0$ . Suppose there exists no fuzzy pre-open set  $\mu$  in  $X$  such that  $\mu \leq f^{-1}(\lambda)$ . Then  $1 - f^{-1}(\lambda)$  is fuzzy set in  $X$  such that there is no fuzzy pre-closed set  $\delta$  in  $X$  with  $1 - f^{-1}(\lambda) < \delta < 1$  (otherwise  $1 - f^{-1}(\lambda) < \delta \Rightarrow 1 - \delta \leq f^{-1}(\lambda)$  and  $1 - \delta$  is fuzzy pre-open, which is a contradiction). This means  $1 - f^{-1}(\lambda)$  is fuzzy pre-dense in  $X$ . Then by (3),  $f(1 - f^{-1}(\lambda))$  is fuzzy  $\alpha$ -dense in  $Y$ . But  $f(1 - f^{-1}(\lambda)) = f(f^{-1}(1 - \lambda)) < 1 - \lambda < 1$  (since  $\lambda \neq 0$ ). This is a contradiction to the fact that  $f(1 - f^{-1}(\lambda))$  is fuzzy  $\alpha$ -dense. Therefore, there exists a fuzzy pre-open set  $\mu$  in  $X$  such that  $\mu \leq f^{-1}(\lambda)$ . Therefore  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute function.  $\blacksquare$

**Theorem 3.9.** Let  $(X_1, T_1)$ ,  $(X_2, T_2)$ ,  $(Y_1, S_1)$  and  $(Y_2, S_2)$  be fuzzy topological spaces such that  $X_1$  is product related to  $X_2$  and  $Y_1$  is product related to  $Y_2$ . Let  $f_1 : X_1 \rightarrow Y_1$  and  $f_2 : X_2 \rightarrow Y_2$  be somewhat fuzzy pre  $\alpha$ -irresolute functions. Then  $f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is somewhat fuzzy pre  $\alpha$ -irresolute function.

*Proof.* Let  $\lambda = \bigvee_{i,j} (\lambda_i \times \mu_j)$  be fuzzy  $\alpha$ -open set in  $Y_1 \times Y_2$  (where  $\lambda_i$  and  $\mu_j$  are fuzzy  $\alpha$ -open sets in  $Y_1$  and  $Y_2$  respectively). We can assume that  $\lambda_i$ 's and  $\mu_j$ 's are

not all zeros. If any one is zero, that factor can be omitted. Now  $(f_1 \times f_2)^{-1}(\lambda) =$

$$(f_1 \times f_2)^{-1} \left( \bigvee_{i,j} (\lambda_i \times \mu_j) \right) = \bigvee_{i,j} (f_1 \times f_2)^{-1}(\lambda_i \times \mu_j) = \bigvee_{i,j} (f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j)).$$

Since  $f_1 : X_1 \rightarrow Y_1$  is somewhat fuzzy pre  $\alpha$ -irresolute and  $\lambda_i$  is fuzzy  $\alpha$ -open set in  $Y_1$  and  $f_1^{-1}(\lambda_i) \neq 0$ , there exists a fuzzy pre-open set  $\delta_i$  in  $X_1$  such that  $\delta_i \leq f_1^{-1}(\lambda_i)$ . Also since  $f_2 : X_2 \rightarrow Y_2$  is somewhat fuzzy pre  $\alpha$ -irresolute and  $\mu_j$  is fuzzy  $\alpha$ -open set in  $Y_2$  and  $f_2^{-1}(\mu_j) \neq 0$ , there exists a fuzzy pre-open set  $\eta_j$  in  $X_2$  such that  $\eta_j \leq f_2^{-1}(\mu_j)$ . Therefore  $\delta_i \times \eta_j \leq f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j) = (f_1 \times f_2)^{-1}(\lambda_i \times \mu_j)$ . Then by Theorem 1.4 and Theorem 1.5 in [2]  $\bigvee_{i,j} (\delta_i \times \eta_j)$  is a fuzzy pre-open set and  $\bigvee_{i,j} (\delta_i \times \eta_j) \leq \bigvee_{i,j} (f_1 \times$

$$f_2)^{-1}(\lambda_i \times \mu_j) = (f_1 \times f_2)^{-1} \left( \bigvee_{i,j} (\lambda_i \times \mu_j) \right) = (f_1 \times f_2)^{-1}(\lambda). \text{ This proves } f_1 \times f_2$$

is somewhat fuzzy pre  $\alpha$ -irresolute.

The following Lemma which is established in [1] is required to prove the Theorem 3.11. ■

**Lemma 3.10.** [1] Let  $g : X \rightarrow X \times Y$  be the graph of a function  $f : X \rightarrow Y$ . If  $\lambda$  is a fuzzy set in  $X$  and  $\mu$  is a fuzzy set in  $Y$ , then  $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$ .

**Theorem 3.11.** Let  $f : (X, T) \rightarrow (Y, S)$  be a function from fuzzy topological space  $(X, T)$  to another fuzzy topological space  $(Y, S)$ . If the graph  $g : X \rightarrow X \times Y$  of  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute, then  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute.

*Proof.* Let  $\lambda$  be a non zero fuzzy  $\alpha$ -open in  $Y$ . Then, by Lemma 3.10, we have  $f^{-1}(\lambda) = 1 \wedge f^{-1}(\lambda) = g^{-1}(1 \times \lambda)$ . Since  $g$  is somewhat fuzzy pre  $\alpha$ -irresolute and  $1 \times \lambda \neq 0$  is a fuzzy  $\alpha$ -open set in  $X \times Y$ , there exists a fuzzy pre-open  $\mu (\neq 0)$  (say) of  $X$  such that  $\mu \leq g^{-1}(1 \times \lambda) = f^{-1}(\lambda)$ . This proves that  $f$  is somewhat fuzzy pre  $\alpha$ -irresolute function. ■

## 4. Strongly Somewhat Fuzzy pre-open Functions

The concept of somewhat fuzzy pre-open function was introduced and studied in [10]. In this section we shall introduce an strongly notion as follows:

**Definition 4.1.** Let  $(X, T)$  and  $(Y, S)$  be fuzzy topological spaces. A function  $f : (X, T) \rightarrow (Y, S)$  is called **strongly somewhat fuzzy pre-open** if and only if for each non-zero fuzzy  $\alpha$ -open set  $\lambda$  of  $(X, T)$ , there exists a fuzzy pre-open set  $\mu$  in  $(Y, S)$  such that  $\mu \neq 0$  and  $\mu < f(\lambda)$ .

Clearly every strongly somewhat fuzzy pre-open function is somewhat fuzzy pre-open function. However the converse is not true as the following example shows:

**Example 4.2.** Let  $\mu_1, \mu_2$  and  $\mu_3$  be fuzzy sets in  $I$  described in Example 3.3. Clearly  $T_1 = \{0, \mu_1, 1\}$  and  $T_2 = \{0, \mu_2, 1\}$  are fuzzy topologies on  $I$ . Let  $f : (I, T_1) \rightarrow (I, T_2)$  be defined by  $f(x) = \min\{2x, 1\}$  for each  $x \in I$ . It can be easily seen that  $\text{int } \mu_3 = \mu_1$ ;  $\text{cl } \mu_1 = 1$  in  $(I, T_1)$ . Simple computations gives  $f(0) = 0$ ;  $f(1) = 1$ ;  $f(\mu_1) = 0$ . Thus  $f$  is somewhat fuzzy pre-open function. Since  $\mu_3 \leq \text{int cl int } \mu_3$  in  $(I, T_1)$ ,  $\mu_3$  is fuzzy  $\alpha$ -open set in  $(I, T_1)$ . But

$$f(\mu_3)(y) = \begin{cases} 0, & 0 \leq y \leq \frac{1}{2}, \\ \frac{1}{3}, & \frac{1}{2} \leq y \leq 1. \end{cases}$$

Let  $\lambda$  be any non-zero fuzzy set such that  $\lambda \leq f(\mu_3)$  in  $(I, T_2)$ . Then  $\text{int cl } \lambda = 0$ , show that  $\lambda$  is not a fuzzy pre-open set. Thus there is no non-zero fuzzy pre-open set such that it is contained in  $f(\mu_3)$ . Hence  $f$  is not strongly somewhat fuzzy pre-open function.

**Theorem 4.3.** Suppose  $(X, T)$  and  $(Y, S)$  be fuzzy topological spaces. Let  $f : (X, T) \rightarrow (Y, S)$  be an onto function. If  $f$  is strongly somewhat fuzzy pre-open and  $\lambda$  is a fuzzy pre-dense set in  $Y$ , then  $f^{-1}(\lambda)$  is fuzzy  $\alpha$ -dense in  $X$ .

*Proof.* Suppose  $\lambda$  is a fuzzy pre-dense set in  $Y$ . We want to show that  $f^{-1}(\lambda)$  is a fuzzy  $\alpha$ -dense set in  $X$ . Suppose not. Then there exists a fuzzy  $\alpha$ -closed set  $\mu$  in  $X$  such that  $f^{-1}(\lambda) < \mu < 1$ . Then  $1 - f^{-1}(\lambda) > 1 - \mu > 0$ .  $f(1 - f^{-1}(\lambda)) > f(1 - \mu)$  which implies that  $f(f^{-1}(1 - \lambda)) > f(1 - \mu)$ . That is,  $f(1 - \mu) < f f^{-1}(1 - \lambda) = 1 - \lambda$ . Now  $\mu$  is fuzzy  $\alpha$ -closed  $\Rightarrow 1 - \mu$  is fuzzy  $\alpha$ -open in  $X$ . Since  $f$  is strongly somewhat fuzzy pre open,  $1 - \mu$  is fuzzy  $\alpha$ -open in  $X \Rightarrow$  there exists a fuzzy pre-open set  $\delta \neq 0$  in  $(Y, S)$  such that  $\delta < f(1 - \mu)$ . Therefore  $\delta < f(1 - \mu) < 1 - \lambda \Rightarrow \delta < 1 - \lambda \Rightarrow \lambda < 1 - \delta$ . Now  $1 - \delta$  is fuzzy pre-closed set and  $\lambda < 1 - \delta \Rightarrow \lambda$  is not a fuzzy pre-dense set in  $Y$ , which is a contradiction to our hypothesis. Therefore  $f^{-1}(\lambda)$  must be a fuzzy  $\alpha$ -dense in  $(X, T)$ . ■

**Theorem 4.4.** Suppose  $(X, T)$  and  $(Y, S)$  be fuzzy topological spaces. Let  $f : (X, T) \rightarrow (Y, S)$  be a 1 - 1 and onto function. Then the following conditions are equivalent.

- (1)  $f$  is strongly somewhat fuzzy pre-open.
- (2) If  $\lambda$  is a fuzzy  $\alpha$ -closed set in  $X$  such that  $f(\lambda) \neq 1$ , then there exists a fuzzy pre-closed set  $\mu$  in  $Y$  such that  $\mu \neq 1$  and  $f(\lambda) < \mu$ .

*Proof.* (1)  $\Rightarrow$  (2). Let  $\lambda$  be a fuzzy  $\alpha$ -closed set in  $X$  such that  $f(\lambda) \neq 1$ . Then  $1 - \lambda$  is fuzzy  $\alpha$ -open and since  $f$  is 1 - 1 and onto  $f(1 - \lambda) = 1 - f(\lambda) \neq 0$  [3]. As  $f$  is strongly somewhat fuzzy pre-open, there exists a fuzzy pre-open set  $\eta$  in  $Y$  such that  $\eta \neq 0$  and  $\eta < f(1 - \lambda) = 1 - f(\lambda)$ . That is  $f(\lambda) < 1 - \eta = \mu$  (say) and  $\mu$  is fuzzy pre-closed set. This proves (1)  $\Rightarrow$  (2).

(2)  $\Rightarrow$  (1). Let  $\lambda$  be a fuzzy  $\alpha$ -open set in  $X$  such that  $\lambda \neq 0$ . Then  $1 - \lambda$  is fuzzy  $\alpha$ -closed and  $1 - \lambda \neq 1$ . Now  $f(1 - \lambda) = 1 - f(\lambda) \neq 1$  (for, if  $1 - f(\lambda) = 1$ , then

$f(\lambda) = 0 \Rightarrow \lambda = 0$ ). Hence by (2) there exists a fuzzy pre-closed set  $\mu$  in  $Y$  such that  $\mu > f(1 - \lambda)$ . Then  $\mu > 1 - f(\lambda)$ . That is,  $f(\lambda) > 1 - \mu = \delta$  (say). Clearly  $\delta$  is fuzzy pre-open set in  $Y$  such that  $\delta < f(\lambda)$  and  $\delta \neq 0$  (since  $\mu \neq 1$ ). This completes the proof of (2)  $\Rightarrow$  (1). ■

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