Advances in Fuzzy Mathematics.
ISSN 0973-533X Vol. 1 No. 1 (2006), pp. 35–44

http://www.ripublication.com/afm.htm

On Fuzzy Contra-Continuities

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Abstract

This paper is devoted to introduce and investigate all types of fuzzy contra-continuities. Properties and relationships of fuzzy contra-β-continuous functions are investigated.

AMS Subject Classification: 54A40, 03E72.
Keywords: Fuzzy contra-continuous, fuzzy S-closed space, fuzzy graph.

1. Introduction

In 1976, Thompson [12] introduced the notion of S-closed spaces via Levine's semi-open sets [6]. A topological space X is called S-closed if every semi-open cover has a finite subfamily the closures of whose members cover X or equivalently if every regular closed cover has a finite subcover. In 1989, Ganster and Reilly [4] introduced the notion of LC-continuous functions via the concept of locally closed sets. In 1996, Dontchev [2] studied a stronger form of LC-continuity called contra-continuity. In that paper, it was proved that contra-continuous images of strongly S-closed spaces are compact as well as that contra-continuous, β-continuous images of S-closed spaces are also compact.

In this paper, types of fuzzy contra-continuities are introduced. The notion of fuzzy contra-β-continuous functions is studied. Moreover, properties and relationships of fuzzy contra-β-continuous functions are investigated.

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2. Preliminaries

The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc.

A family τ of fuzzy sets in X is called a fuzzy topology for X if and only if (1) $\emptyset$, $X \in \tau$, (2) $\mu \wedge \rho \in \tau$ whenever $\mu, \rho \in \tau$, (3) If $\mu_i \in \tau$ for each $i \in I$, then $\vee \mu_i \in \tau$. Moreover, the pair (X, τ) is called a fuzzy topological space (fts). Every member of τ is called a fuzzy open set [8].

In this paper, (X, τ_1) and (Y, τ_2) are fuzzy topological spaces.

Let μ be a fuzzy set in a fts (X, τ). We denote the interior and the closure of a fuzzy set μ by $int(\mu)$ and $cl(\mu)$, respectively.

A fuzzy set in a fts (X, τ) is called a fuzzy singleton if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is ε $(0 < \varepsilon \leq 1)$ we denote this fuzzy singleton by x_ε, where the point x is called its support [8]. For any fuzzy singleton x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ if and only if $\varepsilon \leq \mu(x)$.

A fuzzy singleton x_ε is called quasi-coincident with a fuzzy set ρ, denoted by $x_\varepsilon q \rho$, iff $\varepsilon + \rho(x) > 1$. A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu q \rho$, if and only if there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$.

Let $f : X \to Y$ a function from a fuzzy topological space (X, τ_1) to a fuzzy topological space (Y, τ_2). Then the function $g : X \to X \times Y$ defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ is called the fuzzy graph function of f [1].

A fuzzy set μ in a fts (X, τ) is called fuzzy preopen [10] (resp. fuzzy semi-open [1]) if $\mu \subseteq int(cl(\mu))$ (resp. $\mu \subseteq cl(int(\mu))$). The complement of a fuzzy preopen (resp. fuzzy semi-open) set is said to be fuzzy preclosed (resp. fuzzy semi-closed).

A fuzzy set μ in a fts (X, τ) is called fuzzy β-open [7] or fuzzy semipreopen [11] if $\mu \subseteq cl(int(cl(\mu)))$. The complement of a fuzzy β-open set is said to be fuzzy β-closed.

Definition 2.1. Let μ be a fuzzy set in a fts (X, τ). The fuzzy β-closure and β-interior of μ are defined as follows:

$$\begin{aligned} &\wedge\{\rho \;:\; \mu \subseteq \rho, \rho \text{ is } \beta\text{-closed}\}, \\ &\vee\{\rho \;:\; \mu \supseteq \rho, \rho \text{ is } \beta\text{-open}\}, \end{aligned}$$

and denoted by $\beta\text{-}cl(\mu)$ and $\beta\text{-}int(\mu)$, respectively.

3. Fuzzy contra-β-continuous Functions

In this section, several types of fuzzy contra-continuous functions and in particular characterizations of fuzzy contra-β-continuous functions are introduced.

Definition 3.1. Let X and Y be fuzzy topological spaces. A function $f : X \to Y$ is said to be fuzzy contra-β-continuous if for each fuzzy singleton $x_\varepsilon \in X$ and each fuzzy closed set ρ in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \subseteq \rho$.

Theorem 3.2. For a function $f : X \to Y$, the following statements are equivalent:

(1) f is fuzzy contra-β-continuous,

(2) for every fuzzy closed set μ in Y, $f^{-1}(\mu)$ is fuzzy β-open,

(3) for every fuzzy open set ρ in Y, $f^{-1}(\rho)$ is fuzzy β-closed,

(4) for any fuzzy closed set μ in Y and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$,

(5) for any fuzzy closed set μ in Y and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then there exists a fuzzy β-open set η such that $x_\varepsilon q\eta$ and $f(\eta) \subseteq \mu$.

Proof. (1) $\Leftrightarrow$ (2) : Let ρ be a fuzzy closed set in Y and let $x_\varepsilon \in f^{-1}(\rho)$. Since $f(x_\varepsilon) \in \rho$, by (1), there exists a fuzzy β-open set μ_{x_ε} in X containing x_ε such that $\mu_{x_\varepsilon} \subseteq f^{-1}(\rho)$. Thus, $f^{-1}(\rho)$ is fuzzy β-open.

Conversely, let $x_\varepsilon \in X$ and ρ be a fuzzy closed set containing $f(x_\varepsilon)$. By (2), $f^{-1}(\rho)$ is fuzzy β-open. If we take $\mu = f^{-1}(\rho)$, it follows that $x_\varepsilon \in \mu$ and $f(\mu) \subseteq \rho$.

(2) $\Leftrightarrow$ (3) : Let ρ be a fuzzy open set in Y. Then, $Y \backslash \rho$ is fuzzy closed. By (2), $f^{-1}(Y \backslash \rho) = X \backslash f^{-1}(\rho)$ is fuzzy β-open. Thus, $f^{-1}(\rho)$ is fuzzy β-closed.

Converse is similar.

(2) $\Leftrightarrow$ (4) : Let μ be a fuzzy closed set in Y and $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q f^{-1}(\mu)$ and from (2), $f^{-1}(\mu) \subseteq \beta\text{-}int(f^{-1}(\mu))$. Hence, $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$. Thus, (4) holds.

The reverse is obvious.

(4) $\Rightarrow$ (5) : Let μ be any fuzzy closed set in Y and let $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$. Take $\eta = \beta\text{-}int(f^{-1}(\mu))$, then $f(\eta) = f(\beta\text{-}int(f^{-1}(\mu))) \subseteq f(f^{-1}(\mu)) \subseteq \mu$.

(5) $\Rightarrow$ (4) : Let μ be any fuzzy closed set in Y and let $f(x_\varepsilon)q\mu$. From (5), there exists fuzzy β-open set η such that $x_\varepsilon q\eta$ and $f(\eta) \subseteq \mu$. Hence, $\eta \subseteq f^{-1}(\mu)$ and then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$. ■

Definition 3.3. A function $f : X \to Y$ is called fuzzy β-irresolute if the inverse image of each fuzzy β-open set is fuzzy β-open.

Theorem 3.4. Let X, Y, Z be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be functions.

(1) If f is fuzzy β-irresolute and g is fuzzy contra-β-continuous, then $g \circ f$ is a fuzzy contra-β-continuous function,

(2) If f is fuzzy contra-β-continuous and g is fuzzy continuous, then $g \circ f$ is a fuzzy contra-β-continuous function.

Proof.

(1) Let μ be any fuzzy closed set in Z and let $(g \circ f)(x_\varepsilon) \in \mu$. Then $g(f(x_\varepsilon)) \in \mu$. Since g is a fuzzy contra-β-continuous function, it follows that there exists a fuzzy β-open set ρ containing $f(x_\varepsilon)$ such that $g(\rho) \subseteq \mu$. Since f is fuzzy β-irresolute, it follows that there exists a fuzzy β-open set η containing x_ε such that $f(\eta) \subseteq \rho$. Hence, we obtain that $(g \circ f)(\eta) = g(f(\eta)) \subseteq g(\rho) \subseteq \mu$, so that $g \circ f$ is fuzzy contra-β-continuous.

(2) This proof is similar to the above. ■

Definition 3.5. A function $f : X \to Y$ is called fuzzy β-open [3] if the direct image of each fuzzy β-open set is fuzzy β-open.

Theorem 3.6. If $f : X \to Y$ is a surjective fuzzy β-open function and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is fuzzy contra-β-continuous, then g is fuzzy contra-β-continuous.

Proof. Suppose that x_ε is a fuzzy singleteon in X. Let η be a closed set in Z containing $(g \circ f)(x_\varepsilon)$. Then there exists a fuzzy β-open set μ in X containing x_ε such that $g(f(\mu)) \subseteq \eta$. Since f is fuzzy β-open, $f(\mu)$ is a fuzzy β-open set in Y containing $f(x_\varepsilon)$ such that $g(f(\mu)) \subseteq \eta$. This implies that g is fuzzy contra-β-continuous. ■

Corollary 3.7. Let $f : X \to Y$ be a surjective fuzzy β-irresolute and fuzzy β-open function and let $g : Y \to Z$ be a function. Then, $g \circ f : X \to Z$ is fuzzy contra-β-continuous if and only if g is fuzzy contra-β-continuous.

Proof. It can be obtained from Theorem 3.4 and Theorem 3.6. ■

Definition 3.8. A function $f : X \to Y$ is called fuzzy weakly contra-β-continuous if for each $x \in X$ and each fuzzy closed set η of Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $int(f(\mu)) \subseteq \eta$.

Definition 3.9. A function $f : X \to Y$ is called fuzzy (β, s)-open if the direct image of each fuzzy β-open set is fuzzy semi-open.

Theorem 3.10. If a function $f : X \to Y$ is fuzzy weakly contra-β-continuous and fuzzy (β, s)-open, then f is fuzzy contra-β-continuous.

Proof. Let $x_\varepsilon \in X$ and η be a fuzzy closed set containing $f(x_\varepsilon)$. Since f is fuzzy weakly contra-β-continuous, there exists a fuzzy β-open set μ in X containing x_ε such that $int(f(\mu)) \subseteq \eta$. Since f is fuzzy (β, s)-open, $f(\mu)$ is a semi-open set in Y and $f(\mu) \subseteq cl(int(f(\mu))) \subseteq \eta$. This shows that f is fuzzy contra-β-continuous. ■

Definition 3.11. Let X and Y be fuzzy topological spaces. A function $f : X \to Y$ is said to be fuzzy contra-continuous (resp. fuzzy contra-precontinuous, fuzzy contra-semicontinuous) if for each fuzzy singleton $x_\varepsilon \in X$ and each fuzzy closed set η in Y

containing $f(x_\varepsilon)$, there exists a fuzzy open (resp. fuzzy preopen, fuzzy semi-open) set μ in X containing x_ε such that $f(\mu) \subseteq \eta$.

Remark 3.12. The following diagram holds for a function $f : X \to Y$:

$$\begin{tikzcd}
f \text{ is fuzzy contra-continuous} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & f \text{ is fuzzy contra-semicontinuous} \arrow[d, Rightarrow] \\
f \text{ is fuzzy contra-precontinuous} \arrow[r, Rightarrow] & f \text{ is fuzzy contra-}\beta\text{-continuous}
\end{tikzcd}$$

None of the above implications is reversible as shown in the following examples.

Example 3.13. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\begin{array}{ll} \lambda(a) = 0,3 & \lambda(b) = 0,4 \\ \mu(x) = 0,7 & \mu(y) = 0,5 \end{array}$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy contra-β-continuous but not fuzzy contra-precontinuous.

Example 3.14. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\begin{array}{ll} \lambda(a) = 0,3 & \lambda(b) = 0,6 \\ \mu(x) = 0,3 & \mu(y) = 0,5 \end{array}$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy contra-β-continuous but not fuzzy contra-semicontinuous.

Example 3.15. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\begin{array}{ll} \lambda(a) = 0,3 & \lambda(b) = 0,6 \\ \mu(x) = 0,7 & \mu(y) = 0,5 \end{array}$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy contra-precontinuous but not fuzzy contra-continuous.

Example 3.16. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\begin{array}{ll} \lambda(a) = 0,5 & \lambda(b) = 0,3 \\ \mu(x) = 0,5 & \mu(y) = 0,3 \end{array}$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy contra-semicontinuous but not fuzzy contra-continuous.

Theorem 3.17. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ for every $x_\varepsilon \in X$. If g is fuzzy contra-β-continuous, then f is fuzzy contra-β-continuous.*

Proof. Let η be a fuzzy closed set in Y, then $X \times \eta$ is a fuzzy closed set in $X \times Y$. Since g is fuzzy contra-β-continuous, then $f^{-1}(\eta) = g^{-1}(X \times \eta)$ is fuzzy β-open in X. Thus, f is fuzzy contra-β-continuous. ■

Definition 3.18. A fuzzy filter base Λ is said to be fuzzy β-convergent to a fuzzy singleton x_ε in X [3] if for any fuzzy β-open set η in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \subseteq \eta$.

Definition 3.19. A fuzzy filter base Λ is said to be fuzzy c-convergent to a fuzzy singleton x_ε in X if for any fuzzy closed set η in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \subseteq \eta$.

Theorem 3.20. If a function $f : X \to Y$ is fuzzy contra-β-continuous, then for each fuzzy singleton $x_\varepsilon \in X$ and each fuzzy filter base Λ in X β-converging to x_ε, the fuzzy filter base $f(\Lambda)$ is fuzzy c-convergent to $f(x_\varepsilon)$.

Proof. Let $x_\varepsilon \in X$ and Λ be any fuzzy filter base in X β-converging to x_ε. Since f is fuzzy contra-β-continuous, then for any fuzzy closed set λ in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \subseteq \lambda$. Since Λ is fuzzy β-converging to x_ε, there exists a $\xi \in \Lambda$ such that $\xi \subseteq \mu$. This means that $f(\xi) \subseteq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy c-convergent to $f(x_\varepsilon)$. ■

4. Properties of Fuzzy Contra-β-Continuous Functions

In this section, properties and preservation theorems of fuzzy contra-β-continuous functions are investigated.

Definition 4.1. A fts (X, τ) is said to be

(1) fuzzy β-compact [5] (fuzzy strongly S-closed) if every fuzzy β-open (respectively fuzzy closed) cover of X has a finite subcover,

(2) fuzzy countably β-compact (fuzzy strong countably S-closed) if every countable cover of X by fuzzy β-open (respectively fuzzy closed) sets has a finite subcover,

(3) fuzzy β-Lindelöf (fuzzy strongly S-Lindelöf) if every fuzzy β-open (respectively fuzzy closed) cover of X has a countable subcover.

Theorem 4.2. The fuzzy contra-β-continuous image under a surjection of a fuzzy β-compact (fuzzy β-Lindelöf, fuzzy countably β-compact) space is strongly S-closed (respectively fuzzy strongly S-Lindelöf, fuzzy strong countably S-closed).

Proof. Suppose that $f : X \to Y$ is a fuzzy contra-β-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy closed cover of Y. Since f is fuzzy contra-β-continuous, then $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy β-open cover of X and hence there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Therefore, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy strongly S-closed.

The other proofs can be obtained similarly. ■

Definition 4.3. A fts (X, τ) is called fuzzy β-connected if X is not the union of two disjoint nonempty fuzzy β-open sets.

Definition 4.4. A fts (X, τ) is called fuzzy connected [9] if X is not the union of two disjoint nonempty fuzzy open sets.

Theorem 4.5. If $f : X \to Y$ is a fuzzy contra-β-continuous surjection and X is fuzzy β-connected, then Y is fuzzy connected.

Proof. Suppose that Y is not a fuzzy connected space. There exist nonempty disjoint fuzzy open sets η_1 and η_2 such that $Y = \eta_1 \vee \eta_2$. Therefore, η_1 and η_2 are fuzzy clopen in Y. Since f is fuzzy contra-β-continuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy β-open in X. Moreover, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are nonempty disjoint and $X = f^{-1}(\eta_1) \vee f^{-1}(\eta_2)$. This shows that X is not fuzzy β-connected and hence a contradiction. ■

Definition 4.6. A fuzzy topological space is called

(1) fuzzy β-ultra-connected if every two non-void fuzzy β-closed subsets of X intersect,

(2) fuzzy hyperconnected if every fuzzy open set is dense.

Theorem 4.7. If (X, τ) is fuzzy β-ultra-connected and $f : X \to Y$ is fuzzy contra-β-continuous and surjective, then Y is fuzzy hyperconnected.
Proof. Assume that Y is not fuzzy hyperconnected. Then there exists a fuzzy open set η such that η is not dense in Y. Then there exist disjoint non-empty fuzzy open subsets ξ_1 and ξ_2 in Y, namely $int(cl(\eta))$ and $Y \backslash cl(\eta)$. Since f is fuzzy contra-β-continuous and onto, by Theorem 3.2, $\rho_1 = f^{-1}(\xi_1)$ and $\rho_2 = f^{-1}(\xi_2)$ are disjoint non-empty fuzzy β-closed sets in X. By assumption, the fuzzy β-ultra-connectedness of X implies that ρ_1 and ρ_2 must intersect. By contradiction, Y is fuzzy hyperconnected. ■

Definition 4.8. A fts X is said to be

(1) fuzzy weakly T_2 if each element of X is an intersection of fuzzy regular closed sets,

(2) fuzzy β-T_2 [3] if for each pair of distinct fuzzy singletons x_ε and y_ν in X, there exist fuzzy β-open set μ containing x_ε and fuzzy β-open set η containing y_ν such that $\mu \wedge \eta = \emptyset$,

(3) fuzzy β-T_1 [3] if for each pair of distinct fuzzy singletons x_ε and y_ν in X, there exist fuzzy β-open sets μ and η containing x_ε and y_ν, respectively, such that $y_\nu \notin \mu$ and $x_\varepsilon \notin \eta$.

Recall that for a function $f : X \to Y$, the subset $\{(x_\varepsilon, f(x_\varepsilon)) : x_\varepsilon \in X\} \subseteq X \times Y$ is called the fuzzy graph of f and is denoted by $G(f)$.

Definition 4.9. The fuzzy graph $G(f)$ of a function $f : X \to Y$ is said to be fuzzy contra-β-closed if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy closed set η in Y containing y_ν such that $(\mu \times \eta) \wedge G(f) = \emptyset$.

Lemma 4.10. The following properties are equivalent for the fuzzy graph $G(f)$ of a function f:

(1) $G(f)$ is fuzzy contra-β-closed;

(2) for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy closed set η containing y_ν such that $f(\mu) \wedge \eta = \emptyset$.

Proof. Obvious. ■

Theorem 4.11. If $f : X \to Y$ is fuzzy contra-β-continuous and Y is fuzzy Urysohn, $G(f)$ is fuzzy contra-β-closed in $X \times Y$.

Proof. Suppose that Y is fuzzy Urysohn. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$. It follows that $f(x_\varepsilon) \neq y_\nu$. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $y_\nu \in \rho$ and $cl(\eta) \wedge cl(\rho) = \emptyset$. Since f is fuzzy contra-β-continuous, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \subseteq cl(\eta)$. Therefore, $f(\mu) \wedge cl(\rho) = \emptyset$ and $G(f)$ is fuzzy contra-β-closed in $X \times Y$. ■

Theorem 4.12. Let $f : X \to Y$ have a fuzzy contra-β-closed graph. If f is injective, then X is fuzzy β-T_1.

Proof. Let x_ε and y_ν be any two distinct fuzzy singletons in X. Then, we have $(x_\varepsilon, f(y_\nu)) \in (X \times Y)\backslash G(f)$. By Lemma 4.10, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy closed set ρ in Y containing $f(y_\nu)$ such that $f(\mu) \wedge \rho = \emptyset$; hence $\mu \wedge f^{-1}(\rho) = \emptyset$. Therefore, we have $y_\nu \notin \mu$. This implies that X is fuzzy β-T_1. ■

Theorem 4.13. If $f : X \to Y$ is a fuzzy contra-β-continuous injection and Y is fuzzy Urysohn, then X is fuzzy β-T_2.

Proof. Suppose that Y is fuzzy Urysohn. By the injectivity of f, it follows that $f(x_\varepsilon) \neq f(y_\nu)$ for any distinct fuzzy singletons x_ε and y_ν in X. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $f(y_\nu) \in \rho$ and $cl(\eta) \wedge cl(\rho) = \emptyset$. Since f is fuzzy contra-β-continuous, there exist fuzzy β-open sets μ and ξ in X containing x_ε and y_ν, respectively, such that $f(\mu) \subseteq cl(\eta)$ and $f(\xi) \subseteq cl(\rho)$. Hence $\mu \wedge \xi = \emptyset$. This shows that X is fuzzy β-T_2. ■

Theorem 4.14. If $f : X \to Y$ is a fuzzy contra-β-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy β-T_1.

Proof. Suppose that Y is fuzzy weakly T_2. For any distinct fuzzy singletons x_ε and y_ν in X, there exist fuzzy regular closed sets η, ρ in Y such that $f(x_\varepsilon) \in \eta$, $f(y_\nu) \notin \eta$, $f(x_\varepsilon) \notin \rho$ and $f(y_\nu) \in \rho$. Since f is fuzzy contra-β-continuous, by Theorem 3.2,

$f^{-1}(\eta)$ and $f^{-1}(\rho)$ are fuzzy β-open subsets of X such that $x_\varepsilon \in f^{-1}(\eta)$, $y_\nu \notin f^{-1}(\eta)$, $x_\varepsilon \notin f^{-1}(\rho)$ and $y_\nu \in f^{-1}(\rho)$. This shows that X is fuzzy β-T_1. ■

Theorem 4.15. Let (X_i, τ_i) be fuzzy topological space for all $i \in I$ and I be finite. Suppose that $(\prod_{i \in I} X_i, \sigma)$ is a product space and $f : (X, \tau) \to (\prod_{i \in I} X_i, \sigma)$ is any function. If f fuzzy contra-β-continuous, then $pr_i \circ f$ is fuzzy contra-β-continuous where pr_i is projection function for each $i \in I$.

Proof. Let $x_\varepsilon \in X$ and $(pr_i \circ f)(x_\varepsilon) \in \rho_i$ and ρ_i be a fuzzy closed set in (X_i, τ_i). Then $f(x_\varepsilon) \in pr_i^{-1}(\rho_i) = \rho_i \times \prod_{j \neq i} X_j$ a fuzzy closed set in $(\prod_{i \in I} X_i, \sigma)$. Since f is fuzzy contra-β-continuous, there exists a fuzzy β-open set μ containing x_ε such that $f(\mu) \subseteq \rho_i \times \prod_{j \neq i} X_j = pr_i^{-1}(\rho_i)$ and hence $\mu \subseteq (pr_i \circ f)^{-1}(\rho_i)$ and we obtain that $pr_i \circ f$ is fuzzy contra-β-continuous for each $i \in I$. ■

5. Conclusion

In this paper we introduced several forms of contra-continuities and we investigated their properties. Most of these properties have been proved for fuzzy contra β-continuous functions, but they can easily been proved for other types of contra-continuity such as fuzzy contra-continuity, fuzzy contra-precontinuity and fuzzy contra-semicontinuity.

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