

Model Reduction Using improved Bilinear Routh Approximation Technique

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Abstract

In recent years, model reduction has become an omnipresent tool in a variety of application regions and, consequently, a research emphasis for many mathematicians and engineers. Model reduction and real-time control find applications in different areas. In present technological world, physical activities are depicted mainly applying mathematical models that are accustomed simulating the behavior of the fundamental processes. Frequently, they are also accustomed modify or control system conduct. In this framework, there is an ever enhancing necessity for bettered accuracy, which results to models of higher complexity. The fundamental motivation for system approximation is the demand, in many cases, towards an oversimplified model of a dynamical system, which catches the primary features of the original complex model. This demand originates from limited accuracy, computational or storage potentialities. The simplified model may then be utilized in place of the novel complex model, either for control, or simulation.

In this research paper a method is applied, which overcomes drawbacks and limitations of some of the methods available in paper, for high order system simplification. The method using the Improved bilinear Routh approximations has been suggested in the paper for high order system reduction. This method yields the reduced models which conserves the stability and first few time-moments of the novel ones. Recently, improvements have been attained so that the impulse response energies of original models are also preserved in the reduced models.

Keywords: Model reduction; real-time control; Improved bilinear Routh approximations.

1. Introduction

In practice the order of the system is extremely higher. Therefore there are several disadvantages associated with high order systems in the prospects like, designing, memory requirement application of available methods for analyzing the stability. Therefore for realization, computation, control, and other purposes, it is often in demand adequately to comprise a high-order system by a low-order model without giving the significant features of the novel system.

In the former decades model reduction has turned a tool in analysis and simulation of high-order dynamic systems, circuit simulation, control design and numerous other fields dealing with complex physical systems.

Order reduction is frequently significant to accelerate the simulation of such large-scale systems. In fact, the system may be so comprehensive that simulations or even storage of data depicting such systems are preventative without first substituting the original large scale systems by appropriately accurate reduced models of smaller dimension.

2. Model Reduction Techniques for Continuous-Time Systems

Although there are so many methods available in the literature for the reduction of high-order continuous time systems, here we introduce some of the well-known methods viz.

- Pade Approximation method
- Moment Matching method
- Continuous fraction expansion method
- Error Minimization technique
- Routh Approximation method

3. Higher Order Discrete Time System Reduction Methods

There are numerous methods obtainable for the simplification of high order linear continuous time systems, where very fewer are obtainable for the reduction of linear discrete time system some of the well-known methods obtainable in the literature and considered here are

- Continued Fraction method
- Stability equation method
- Error minimization technique
- Bilinear Routh Approximation
- Improved Bilinear Routh Approximation method

- Sub-optimal Bilinear Routh Approximation method

4. Particle Swarm Optimization

This theme reviews a recent method of optimization technique called particle swarm optimization (PSO) developed by James Kennedy, Russel Eberhart in 1995.

4.1 Particle Swarm Optimization

Particle swarm optimization is a population based stochastic optimization technique. Like numerous of the other biologically urged on algorithms (such as neural networks, hereditary algorithm, and simulated annealing), Particle swarm optimization also have a natural motivation. Two specific inspirations are that of fish schooling and bird flocking.

In PSO, an aggregation of particles (or agents) swarm through an N-dimensional space. The principles for how the particles pass through the space are based on simple flocking rules that cause the particles to orbit around the amplest found solution in the promise of finding improved one. This algorithm seems to be simple but it is efficient and can be used in many types of optimization troubles.

5. Proposed Ambraj Method

5.1 The flow chart for the proposed AMBRAJ method

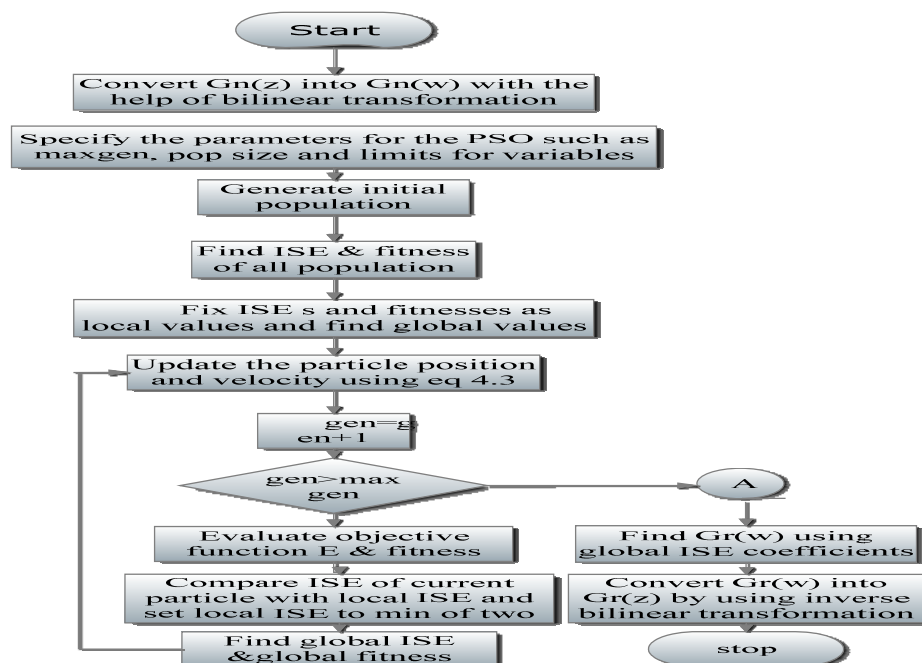


Figure 1: The AMBRAJ Method's Flowchart.

EXAMPLE 5.1: Consider the discrete time system given by its transfer function

$G(z) = \frac{2z^4 + 1.8z^3 + .8z^2 + 0.1z - 0.1}{z^4 - 1.2z^3 + 0.3z^2 + 0.1z + 0.02}$ By using Bilinear transformation $z = \frac{1+w}{1-w}$, the transfer function $G(z)$ can be written as

$$G(w) = \frac{0.8w^4 + 5w^3 + 9.8w^2 + 11.8w + 4.6}{2.42w^4 + 6.52w^3 + 5.52w^2 + 1.32w + 0.22}$$

5.2 Application of Proposed AMBRAJ Method

It is proposed to employ the method suggested here utilizing particle swarm optimization to receive second order model. Parameters believed for the proposed PSO method of model reduction are Maxgen = 500; Swarm Size or Population size = 20; Limits for variable = [0.001, 100]. By simulating the digital computer algorithm formulated for the proposed reduction technique based on PSO, the second order reduced model is found as

$$R_2(w) = \frac{1.3091w + 4.6}{w^2 + 0.2437w + .0054} \text{ By applying inverse bilinear transformation}$$

$w = \frac{z-1}{z+1}$, the transfer

function $R_2(w)$ can be written as

$$R_2(z) = \frac{2.439z^2 + 2.26z - 0.179}{1.2977z^2 - 1.892z + 0.8103}$$

Integral square error (ISE) between the original system and its reduced order model $R_2(z)$ is ISE = 0.123622.

5.3 Comparison with other Method

(i) The second order reduced model is obtained using “Suboptimal Bilinear Routh Approximation method” as

$$R_2'(z) = \frac{2.54733z^2 + 2.36893z - 1.6465}{1.27367z^2 - 1.92181z + 0.80453} \text{ (SUBOPTIMAL BRAM)}$$

Integral square error (ISE) between the original system and its reduced order model $R_2'(z)$ is ISE = 2.52363

(ii) The second order reduced model is derived using “Bilinear Routh Approximation method” as

$$R_2''(z) = \frac{2.69474z^2 + 4.08421z - 2.62857}{1.34737z^2 - 1.90075z + 0.75190} \text{ (BRAM)}$$

Integral square error (ISE) between the original system and its reduced order model $R_2''(z)$ is ISE = 5.687473

(iii) The second order reduced model is acquired using “Improved Bilinear Routh Approximation method” as

$$R_2'''(z) = \frac{2.77569z^2 + 5.02618z - 3.16789}{1.38784z^2 - 1.88919z + 0.72297} \text{ (IBRAM)}$$

Integral square error (ISE) between the original system and its reduced order model $R_2'''(z)$ is ISE=9.171604.

Table 5.1

S. no	REDUCTION METHOD	ISE
1	Proposed AMBRAJ	0.123622
2	Suboptimal BRAM	2.52363
3	BRAM	5.687473
4	IBRAM	9.171604

It can be determined that the proposed AMBRAJ method gives better approximation of the original system when equated to BRAM, IBRAM and Suboptimal BRAM methods.

5.4 Advantages

- This method always establishes stable reduced order models if the novel system is stable.
- This method gives better approximation of the original system.
- This method yields zero steady state error.
- This method is simple and efficient.
- This method retains both time moments and markov parameters.

6. Conclusions

A novel method AMBRAJ is proposed for the reduction of high-order discrete- time systems to conquer few of critical limitations and drawbacks of some of the present methods of discrete- time systems reduction. This order reduction method is based on Particle Swarm Optimization technique for getting the coefficients of both numerator and denominator of the reduced order models. AMBRAJ method is used for the stability analysis and design of dead beat compensators for high-order discrete time systems. This method holds the stability of the original high-order system in the reduced order models. And it is computationally simple compared to many of the available methods of order reduction of high-order discrete-time systems.

The flexibility and effectiveness of the proposed AMBRAJ method is shown through typical numerical cases deliberated from the literature by simulating on digital computer applying digital computer algorithm formulated for this method in C-language. The outcomes are equated with the results derived by some of other known

methods and successfully asserted to establish the superiority of this method over other methods.

References

- [1] C .F. Chen and L .S. Shieh,' A Novel approach to linear model simplification', International Journal of Control, 1968, p 561-570.
- [2] C. Hwang and C. Hsieh,'Order reduction of discrete time system via Bilinear Routh approximation', ASME, J.Dyn.Syst, Meas, Control, 1990, Vol 112,p 292-297.
- [3] G.V.K.R Sastry, G. Lakshmi Narayana et. al "Particle Swarm Optimization technique for Model reduction of High-Order Discrete Time Systems" , PCID-2008, Bannari Amman Institute of Technology, p 64-66.
- [4] M. F. Hutton and B. Friedland, 'Routh approximants for reducing order of linear time-invariant systems, IEEE Trans, vol AC-20, June 1975, p 329-337.
- [5] Mohd.Jamshidi, "Large scale Systems Modelling and Control", North-Holland Publs., 1983.
- [6] Parmer et.al "Relative Mapping Errors of Linear time invariant system caused by particle swarm optimized reduced order model", International Journal of Control, 2007
- [7] Proceedings of the National Conference PCID-2008, held at B.A. Institute of Technology, Tamil Nadu, February 22-23, 2008
- [8] R. Prasad," Order reduction of Discrete Time Systems using Stability Equation Method and weighted time Moments", IE Journal, vol 74 , Nov 1993, p 94-99.
- [9] S. Mukherjee, V. Kumar and R. Mitra' Ordered Reduction of Linear Discrete system using an Error Minimization Technique', Institute of Engineers, Vol 85, June 2004, p 68-76.
- [10] T. C. Chen, C .Y. Chang and K. W. Han, 'Reduction of Transfer functions by the Stability-Equation method', Journal of Franklin Inst, Vol 308, 1979, p 389.
- [11] Yoynseok Choo, 'Improved Bilinear Routh approximation for Discrete-Time Systems', ASME, J.Dyn.Syst, Meas, Control, 2001, Vol 112,p 125-127
- [12] Yoynseok Choo,'Sub-optimal Bilinear Routh approximation for Discrete-Time Systems', ASME, J.Dyn.Syst, Meas, Control, 2006, Vol 125,p 130-134
- [13] Y .P. Shih and W. T. Wu, 'Simplification of z-transfer functions by continued fractions', International Journal of Control systems, Vol 17, 1973, p 1089.